FLUID FLOW ABOUT IMMERSED BODIES

A body immersed in a moving stream (or, alternatively, a body moving through a still fluid) experiences a force. If the body is moving through in a viscous fluid, shear and pressure forces act on the body.

$$\vec{F} = \int_{\substack{\text{body}\\\text{surface}}} d\vec{F} = \int_{\substack{\text{body}\\\text{surface}}} d\vec{F}_{shear} + \int_{\substack{\text{body}\\\text{surface}}} d\vec{F}_{pressure}$$

The resultant force, $\vec{F}$, can be resolved into components parallel and perpendicular to the direction of motion. The component of force parallel to the direction of motion is the drag force, F_D, and the force component perpendicular to the direction of motion is the lift force, F_L.

Recognizing that

$$d\vec{F}_{shear} = \vec{\tau}_w \, dA$$

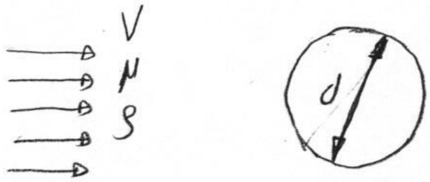
and

$$d\vec{F}_{pressure} = -p \, d\vec{A}$$

There are very few cases in which the lift and drag can be determined analytically. Flow seperation prohibits the analytical determination. Therefore, for most shapes of interest, the lift and drag are determined experimentally.

DRAG

Drag is the component of force on a body acting parallel to the direction of motion. We considered the problem of determining the drag force F_D, on a smooth sphere, moving through a viscous incompressible fluid.



The drag force, F_D was found in the functional form

$$F_D = f_1(d, V, \mu, \rho)$$

Application of the Buckingham Π theorem resulted

$$\frac{F_D}{\rho V^2 D^2} = f_2\left(\frac{\rho V d}{\mu}\right)$$

Note that d^2 is proportional to the cross-sectional area $A = \frac{\pi d^2}{4}$, and hence

$$\frac{F_D}{\rho V^2 A} = f_3\left(\frac{\rho V d}{\mu}\right) = f_3(Re)$$

Althoug this equation was obtained for a sphere, the form of the equation is valid for incompressible flow over any body. The characteristic length used in the Reynolds number depends on the body shape.

The drag coefficient, C_D, is defined as

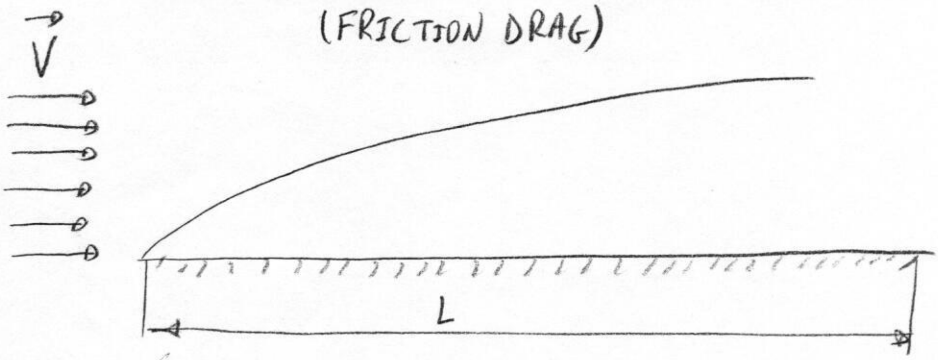
$$C_D \equiv \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

The number $\frac{1}{2}$ has been inserted to form the familiar dynamic pressure. Then

$$C_D = f(Re)$$

We have not considered compressibility or free surface effects in this discussion.

FLOW OVER A FLAT PLATE PARALLEL TO THE FLOW
(FRICTION DRAG)



Since the pressure gradient is zero, the total drag is equal to the friction drag. Thus

$$F_D = \int_{\substack{\text{plate}\\\text{surface}}} \tau_w \, dA$$

and

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{\int_{PS} \tau_w \, dA}{\frac{1}{2}\rho V^2 A}$$

For laminar flow over a flat plate, the shear stress coefficient was given by.

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho V^2} = \frac{0.664}{\sqrt{Re_x}} \Rightarrow \tau_w = \frac{0.664\left(\frac{1}{2}\rho V^2\right)}{\sqrt{Re_x}}$$

Thus, C_D can be determined for laminar flow, such that

$$C_D = \frac{\int_{PS} \frac{0.664\left(\frac{1}{2}\rho V^2\right)}{\sqrt{Re_x}}\, dA}{\frac{1}{2}\rho V^2 A} = \frac{1}{A}\int_A 0.664\, Re_x^{-0.5}\, dA$$

$$C_D = \frac{1}{bL}\int_0^L 0.664\left(\frac{V}{\nu}\right)^{-1/2} x^{-1/2}\, b\,dx = \frac{0.664}{L}\left(\frac{\nu}{V}\right)^{1/2}\left[\frac{x^{0.5}}{0.5}\right]_0^L$$

$$C_D = 1.328\left(\frac{\nu}{VL}\right)^{0.5} = \frac{1.328}{\sqrt{Re_L}}$$

Assuming the boundary layer is turbulent from the leading edge, the shear stress coefficient, based on the approximate analysis, is given by

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho V^2} = \frac{0.0594}{Re_x^{1/5}} \Rightarrow \tau_w = \frac{0.0594\left(\frac{1}{2}\rho V^2\right)}{Re_x^{1/5}}$$

Thus, C_D can be determined for turbulent flow, such that

$$C_D = \frac{\displaystyle\int_A \frac{0.0594\left(\frac{1}{2}\rho V^2\right)}{Re_x^{1/5}}\, dA}{\frac{1}{2}\rho V^2 A} = \frac{1}{A}\int_A 0.0594\, Re_x^{-0.2}\, dA$$

$$C_D = \frac{1}{bL}\int_0^L 0.0594\left(\frac{V}{\nu}\right)^{-0.2} x^{-0.2}\, b\, dx = \frac{0.0594}{L}\left(\frac{\nu}{V}\right)^{0.2}\left[\frac{x^{0.8}}{0.8}\right]_0^L$$

$$C_D = 0.074\left(\frac{\nu}{VL}\right)^{0.2} = \frac{0.074}{Re_L^{0.2}} \qquad 5\times10^5 < Re_L < 10^7$$

This is valid for $5\times10^5 < Re_L < 10^7$

$$C_D = \frac{0.455}{(\log_{10} Re_L)^{2.58}}$$ for $Re_L < 10^9$ (empirical equation)

For a boundary layer that is initially laminar and undergoes transition at some location on the plate, the turbulent drag coefficient must be adjusted to account for the laminar flow over the initial length. In this case

$$C_D = \frac{0.074}{Re_L^{1/5}} - \frac{1740}{Re_L} \qquad \text{for} \quad 5\times10^5 < Re_L < 10^7$$

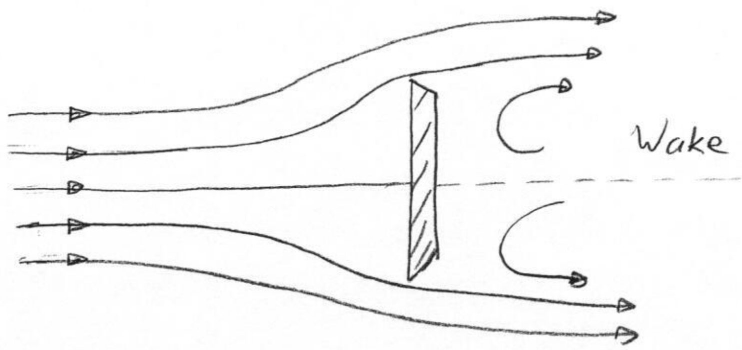
or

$$C_D = \frac{0.455}{(\log_{10} Re_L)^{2.58}} - \frac{1610}{Re_L} \qquad \text{for} \quad 5\times10^5 < Re_L < 10^9$$

The variation in drag coefficient for a flat plate parallel to the flow is shown in Fig. 9.8.

FLOW OVER A FLAT PLATE NORMAL TO THE FLOW: PRESSURE DRAG

In flow over a flate plate normal to the flow, the wall shear stress does not contribute to the drag force.



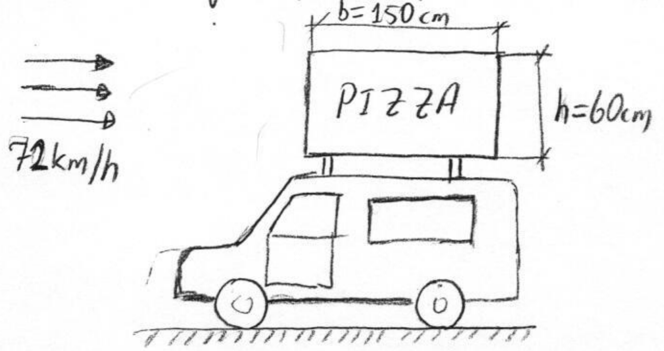
The drag is given by

$$F_D = \int_{surface} p \, dA$$

For this geometry the flow separetes from the edges of the plate, therefore drag coefficient cannot be determined analytically. The variation of C_D with the ratio of plate width to height (b/h) is shown in Fig. 9. 10.

The drag coefficient for all objects with sharp edges is essentially independent of Reynolds number (for $Re \geqslant 1000$) because the separation points are fixed by the geometry of the object. Drag coefficients for a few selected objects are given in Table 9.3. Note that, the drag coefficient for flow over an immersed object usually is based on the frontal area (or projected area) of the object.

<u>Example:</u> A thin, rectangular plastic sign on top of a delivery van is placed as shown in the figure. The sign measures 60 cm by 150 cm. Estimate the extra power required to drive the van in still air at 72 km/h if the sign faces forward rather than sideways. (The density of air 1.2 kg/m³ and kinematic viscosity of air $\nu = 1.5 \times 10^{-5}\, m^2/sn$)



We first consider the sign placed sideways. The Reynolds number

$$Re_L = \frac{VL}{\nu} = \frac{\frac{72 \times 10^3}{3600} \times 1.5}{1.5 \times 10^{-5}} = 2 \times 10^6 \qquad V = \frac{72 \times 10^3}{3600} = 20\, m/s$$

From Fig. 9.8, the drag coefficient $C_D = 0.0031$, or from the equation

$$C_D = \frac{0.455}{(\log_{10} Re_L)^{2.58}} - \frac{1610}{Re_L} \qquad 5 \times 10^5 < Re_L < 10^9$$

$$C_D = \frac{0.455}{(\log_{10} 2 \times 10^6)^{2.58}} - \frac{1610}{2 \times 10^6} = 0.0031$$

Drag coefficient is given by

$$C_D = \frac{F_D}{\frac{1}{2} \rho V^2 A} \quad \text{and thus,} \quad F_D = C_D A \frac{1}{2} \rho V^2$$

$$F_D = 0.0031 * 2*(0.6 * 1.5) * \frac{1}{2} * 1.2 * 20^2 = 1.3392\ N$$

The power required to overcome this air drag is the product of the drag force and the van speed.

$$\dot{W}_1 = F_D \cdot V = 1.3392 * 20 = 26.784\ W$$

$$\dot{W}_1 = \frac{26.784}{746} \cong 0.036\ hp$$

We now consider forward-facing sign

$$\text{Aspect ratio} = \frac{b}{h} = \frac{1.5}{0.6} = 2.5$$

From Fig 9.10., the drag coefficient $C_D \cong 1.2$

Thus $$F_D = 1.2 * (0.6 * 1.5) * \frac{1}{2} * 1.2 * 20^2 = 259.2\ N$$

The power required to overcome this air drag is

$$\dot{W}_2 = F_D \cdot V = 259.2 * 20 = 5184\ W$$

$$\dot{W}_2 = \frac{5184}{746} \cong 6.95\ hp$$

The extra power required to drive the van if the sign faces forward rather than sideways is

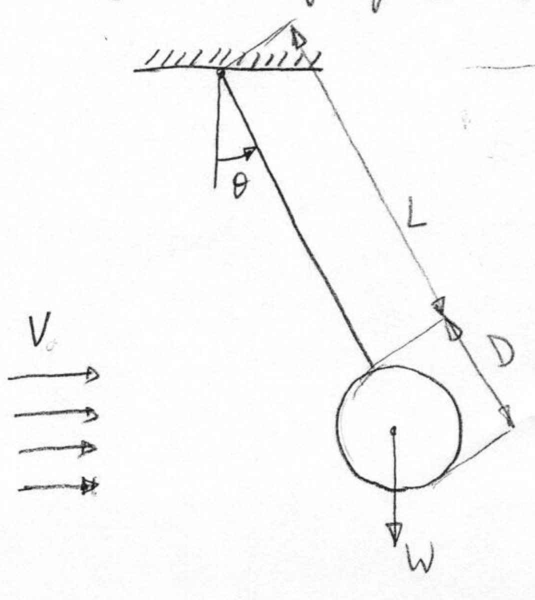
$$\dot{W}_2 - \dot{W}_1 = 6.95 - 0.036 = 6.914\ hp.$$

FLOW OVER A SPHERE AND CYLINDER: FRICTION AND PRESSURE DRAG

In the case of flow over a sphere and cylinder, both friction and pressure drag contribute to total drag. Drag coefficient of a smooth sphere as a function of Reynolds number is shown in Fig. 9.11. Also in Fig 9.12, pressure distribution around a smooth sphere is shown for laminar, turbulent and ideal flow.

The drag coefficients for a circular cylinder is plotted in Fig. 9.13.

Example: The smooth sphere shown in the figure has weight W and hangs by a weightless wire of length L. A wind of velocity V_∞ blows over the sphere. Develop an expression relating the drag coefficient C_D of the sphere, the velocity V_∞, and the angle θ. The air density is ρ and the sphere diameter is D. Use the resulting expression to calculate the angle θ, for a 15 cm diameter sphere, weighing 3.5 N, a wind velocity of 17 m/sn, and a 60 cm wire. (The density of air $\rho = 1.2\ kg/m^3$ and kinematic viscosity of air, $\nu = 1.51 \times 10^{-5}\ m^2/s$)



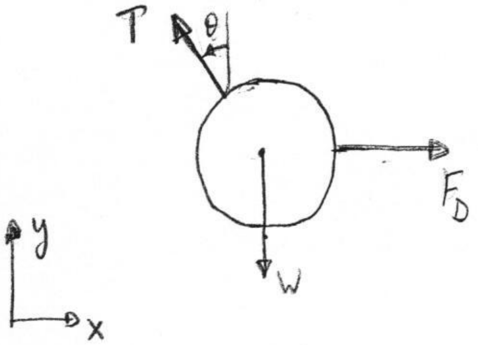
Given:

$W = 3.5\,N$

$D = 15\,cm$

$V_\infty = 17\ m/sn$

$L = 60\ cm$



Using the free-body diagram, we get

$$F_D = T \sin\theta$$

where T is unknown, but can be found by applying Newton's first law in vertical (y) direction

$$W = T\cos\theta, \quad \text{so} \quad T = \frac{W}{\cos\theta}$$

$$\therefore \quad F_D = T\sin\theta = \frac{W}{\cos\theta}\sin\theta = W\tan\theta.$$

The drag coefficient is given by $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$

and thus $\quad F_D = C_D A \frac{1}{2}\rho V^2$

$$\therefore \quad W\tan\theta = C_D A \frac{1}{2}\rho V^2$$

where A is the frontal area of sphere $A = \dfrac{\pi D^2}{4}$

Solving for θ gives

$$\theta = \arctan\left(\frac{C_D A \rho V^2}{2W}\right) = \arctan\left(\frac{C_D \pi D^2 \rho V^2}{8W}\right)$$

$$Re = \frac{VD}{\nu} = \frac{17 * 0.15}{1.51 * 10^{-5}} = 1.69 * 10^5$$

From Fig 9.11, $C_D = 0.42$. The numerical values give

$$\theta = \arctan\left[\frac{0.42 * \pi * 0.15^2 * 1.2 * 17^2}{8 * 3.5}\right] = 20.2^\circ$$

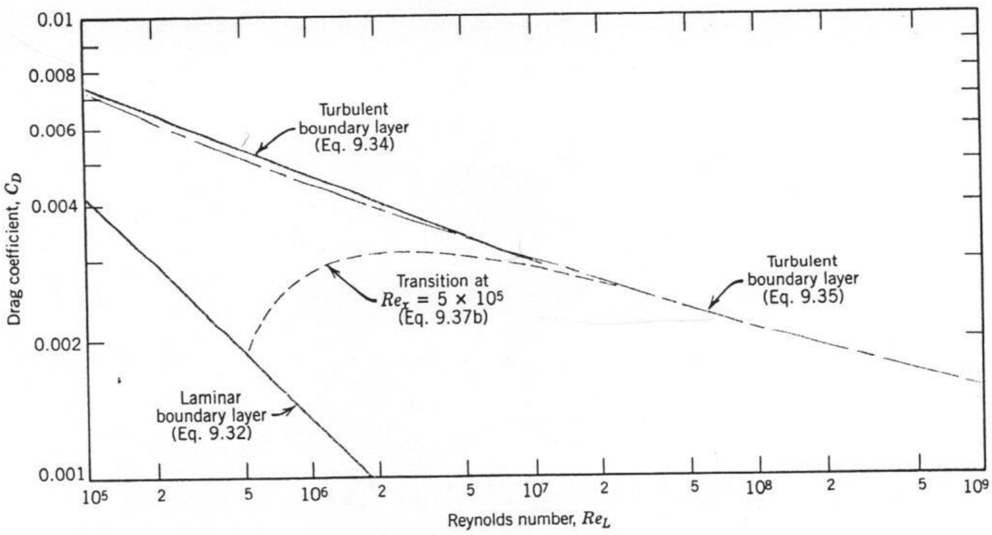


Fig. 9.8 Variation of drag coefficient with Reynolds number for a smooth flat plate parallel to the flow.

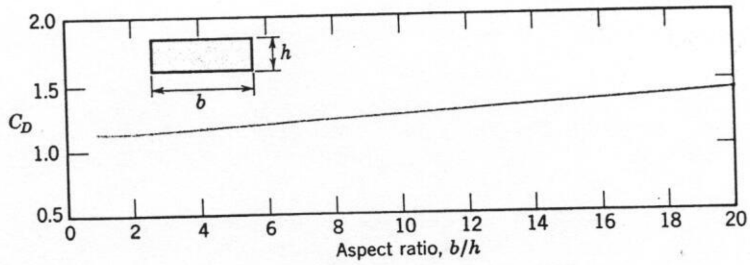
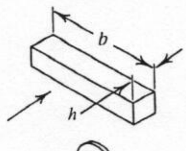
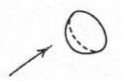
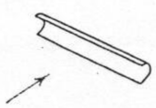
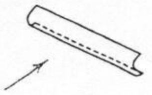


Fig. 9.10 Variation of drag coefficient with aspect ratio for a flat plate of finite width normal to the flow with $Re_h > 1000$ [14].

Table 9.3 Drag Coefficient Data for Selected Objects ($Re \gtrsim 10^3$)[a]

Object	Diagram		$C_D(Re \gtrsim 10^3)$
Square prism		$b/h = \infty$ $b/h = 1$	2.05 1.05
Disk			1.17
Ring			1.20[b]
Hemisphere (open end facing flow)			1.42
Hemisphere (open end facing downstream)			0.38
C-section (open side facing flow)			2.30
C-section (open side facing downstream)			1.20

[a] Data from [14].
[b] Based on ring area.

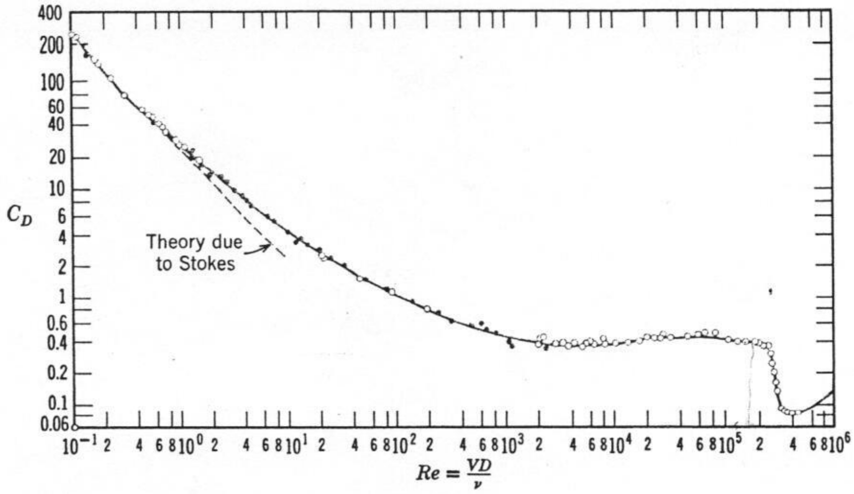


Fig. 9.11 Drag coefficient of a smooth sphere as a function of Reynolds number [3].

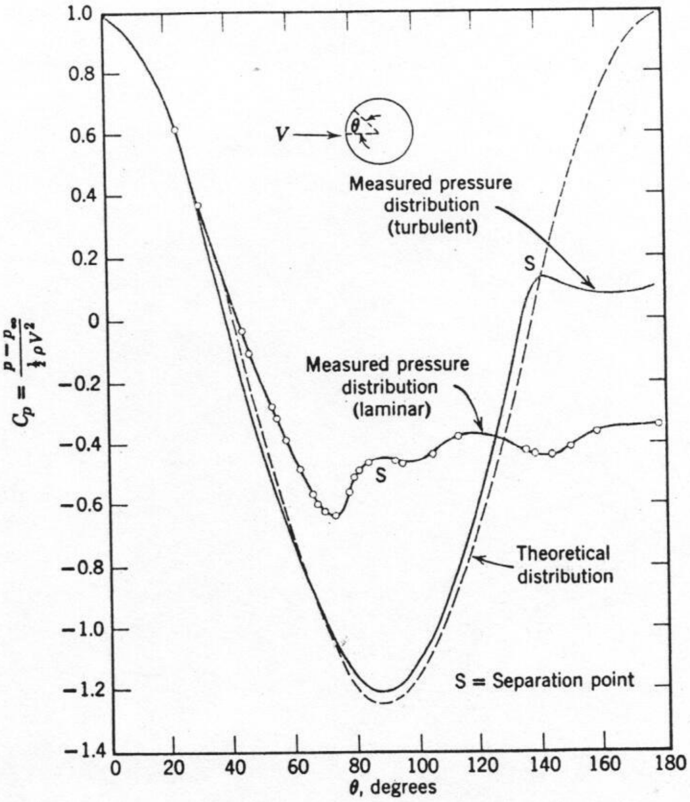


Fig. 9.12 Pressure distribution around a smooth sphere for laminar and turbulent boundary-layer flow, compared with inviscid flow [16].

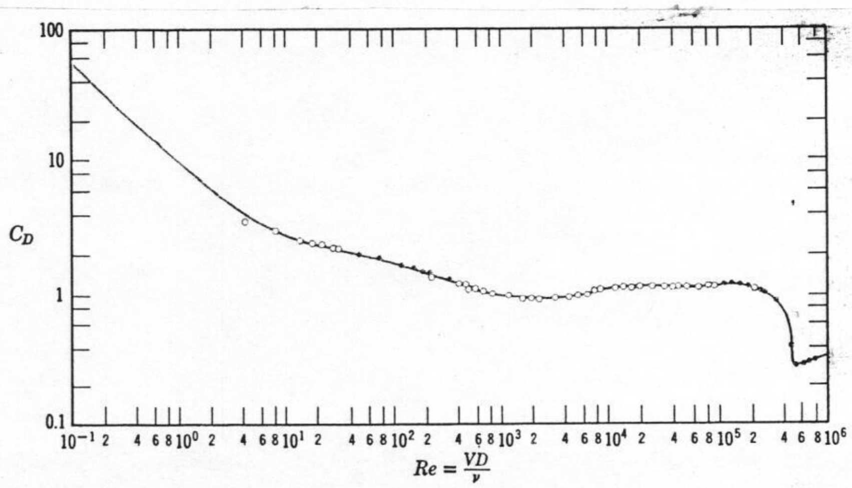


Fig. 9.13 Drag coefficient for a smooth circular cylinder as a function of Reynolds number [3].