MM302 FORMULA SHEET

Vector Operators $\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$ $\nabla = \frac{\partial}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\vec{e}_\theta + \frac{\partial}{\partial z}\vec{e}_z$

Continuity equation $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$

$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$ $v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$ $v_\theta = -\frac{\partial \psi}{\partial r}$

Momentum Equation $\rho(\frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}) = -\nabla P + \mu(\frac{\partial^2 \vec{V}}{\partial x^2} + \frac{\partial^2 \vec{V}}{\partial y^2} + \frac{\partial^2 \vec{V}}{\partial z^2}) + \rho \vec{f}_B$

Euler's equation along a streamline $\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} = -\frac{1}{\rho}\frac{\partial P}{\partial s} - g\frac{\partial z}{\partial s}$ Euler's equation normal to the streamline

$\frac{1}{\rho}\frac{\partial P}{\partial n} + g\frac{\partial z}{\partial n} = \frac{V^2}{R}$

Velocity potential $\vec{V} = -\nabla \phi$ $v_r = -\frac{\partial \phi}{\partial r}$ $v_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}$

Uniform flow in x-direction $\psi = Uy$ $\phi = -Ux$ Source flow $\psi = \frac{q}{2\pi}\theta$ $\phi = -\frac{q}{2\pi}\ln r$

Sink flow $\psi = -\frac{q}{2\pi}\theta$ $\phi = \frac{q}{2\pi}\ln r$

Counter-clockwise vortex $\psi = -\frac{K}{2\pi}\ln r$ $\phi = -\frac{K}{2\pi}\theta$ Doublet flow $\psi = -\frac{A Sin\theta}{r}$ $\phi = -\frac{A Cos\theta}{r}$

Dimensionless numbers $\mathrm{Re} = \frac{\rho V D}{\mu}$ $Eu = \frac{\Delta P}{\frac{1}{2}\rho V^2}$ $Fr = \frac{V}{\sqrt{gL}}$ $We = \frac{\rho V^2 L}{\sigma}$ $M = \frac{V}{c}$

$c = \sqrt{kRT}$

Boundary layer displacement thickness $\delta^* = \int_0^\delta (1 - \frac{u}{U})dy$ Boundary layer momentum thickness $\theta = \int_0^\delta \frac{u}{U}(1 - \frac{u}{U})dy$

From Blasius solution: $\frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}}$ $\frac{\delta^*}{x} = \frac{1.72}{\sqrt{\mathrm{Re}_x}}$ $\frac{\theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$

$C_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$

Momentum Integral Equation $\tau_w = \frac{\partial}{\partial x}U^2\int_0^\delta \rho\frac{u}{U}(1 - \frac{u}{U})dy + U\frac{dU}{dx}\int_0^\delta \rho(1 - \frac{u}{U})dy$

In turbulent boundary layer: $\tau_w = 0.0233\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4}$

Drag Coefficient $C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$ For laminar flow over a flat palate $C_D = \frac{1.328}{\sqrt{\mathrm{Re}_L}}$

For Turbulent flow over a flat plate

$C_D = \frac{0.074}{(Re_L)^{1/5}} - \frac{1740}{Re_L}$ for $5x10^5 < Re_L < 10^7$, $C_D = \frac{0.455}{(\log_{10} Re_L)^{2.58}} - \frac{1610}{Re_L}$ for $5x10^5 < Re_L < 10^9$

Cosine Theorem: $a^2 = b^2 + c^2 - 2bcCos\theta$