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CALCULATION OF HEAD LOSS (Pressure Drop)

The main purpose of this section is to evaluate the pressure changes that result from the steady flow of an incompressible and viscous fluid in a closed duct.

The changes in the pressure are due to the,

i) changes in the elevation

ii) Changes in the flow velocity due to the changes in the flow cross-sectional area

iii) viscous effects.

In an inviscid flow, the Bernoulli equation may be used in order to consider the effect of changes in the elevation and the velocity on pressure.

The prime concern in the analysis of real flows is to take the effect of viscosity into account. If a real fluid flows through a horizontal pipe of constant diameter, the pressure decreases in the direction of the fluid flow due to the friction exerted by the fluid against the pipe walls. Therefore the effect of friction is to decrease the pressure.

To simplify the analysis, the pressure or the head loss in closed conduits will be divided into two parts,

1. Major Losses : Head losses due to frictional effects in fully developed flow in constant area tubes.

2. Minor Losses (local losses) : Losses due to the flow through values, tees, elbows on other nonconstant cross-sectional area portions of the system.

Major Head Loss in the Laminar Flow

The pressure drop, ΔP, in steady, laminar and fully developed flow of an incompressible fluid through a horizontal pipe is computed by the solution of the Navier-Stokes equation an given by

$$\Delta P = f \frac{L}{d} \frac{\rho V^2}{2} \quad \text{or} \quad \Delta P = 4f \frac{L}{d} \frac{\rho V^2}{2}$$

where f is the friction factor, L is the length of the pipe d is the diameter of the pipe, ρ is the fluid density and V is the average velocity in the pipe.

For laminar flows, friction factor f is given by

$$f = \frac{64}{Re} \quad \text{or} \quad f = \frac{16}{Re}$$

where $Re = \frac{\rho V d}{\mu} = \frac{V d}{\nu}$ Reynolds number.

The pressure loss, ΔP, can also be expressed in terms of head loss as $h_f = \frac{\Delta P}{\rho g}$

$$h_f = f \frac{L}{d} \frac{V^2}{2g} \quad \text{or} \quad h_f = 4f \frac{L}{d} \frac{V^2}{2g}$$

This equation is often refered as the Darcy-Weisbach equation.

Major Head Loss in the Turbulent Flow

The pressure drop in the steady, turbulent and fully developed flow of an incompressible fluid through a horizontal pipe can not be evaluated analytically due to the complex nature of the governing equations. Therefore, it must be based on the experimental results and and the dimensional analysis should be used as an aid for correlating the experimental data.

Experiments show that the pressure loss ΔP is equal to

$$\Delta P = f\left(Re, \varepsilon/d\right) \frac{L}{d} \frac{\rho V^2}{2} \quad \text{or} \quad \Delta P = 4 f\left(Re, \varepsilon/d\right) \frac{L}{d} \frac{\rho V^2}{2}$$

Then the head loss

$$h_f = f\left(Re, \frac{\varepsilon}{d}\right) \frac{L}{d} \frac{V^2}{2g} \quad , \quad \frac{\varepsilon}{d} : \text{relative roughness}$$

Note: In laminar flow, friction factor f is function of only Reynolds number. However in turbulent flow the friction factor is affected by Reynolds number as well as the pipe roughness.

Friction factors f, as a function of Re and ε/d are given in charts. One of the most convenient charts for determining the friction factor in clean comercial pipes was constructed by L. F. Moody in 1944.

Experiments carried out by Nikarudse shawn that the friction factor becomes independent of the Reynolds number at sufficiently high Reynolds numbers.

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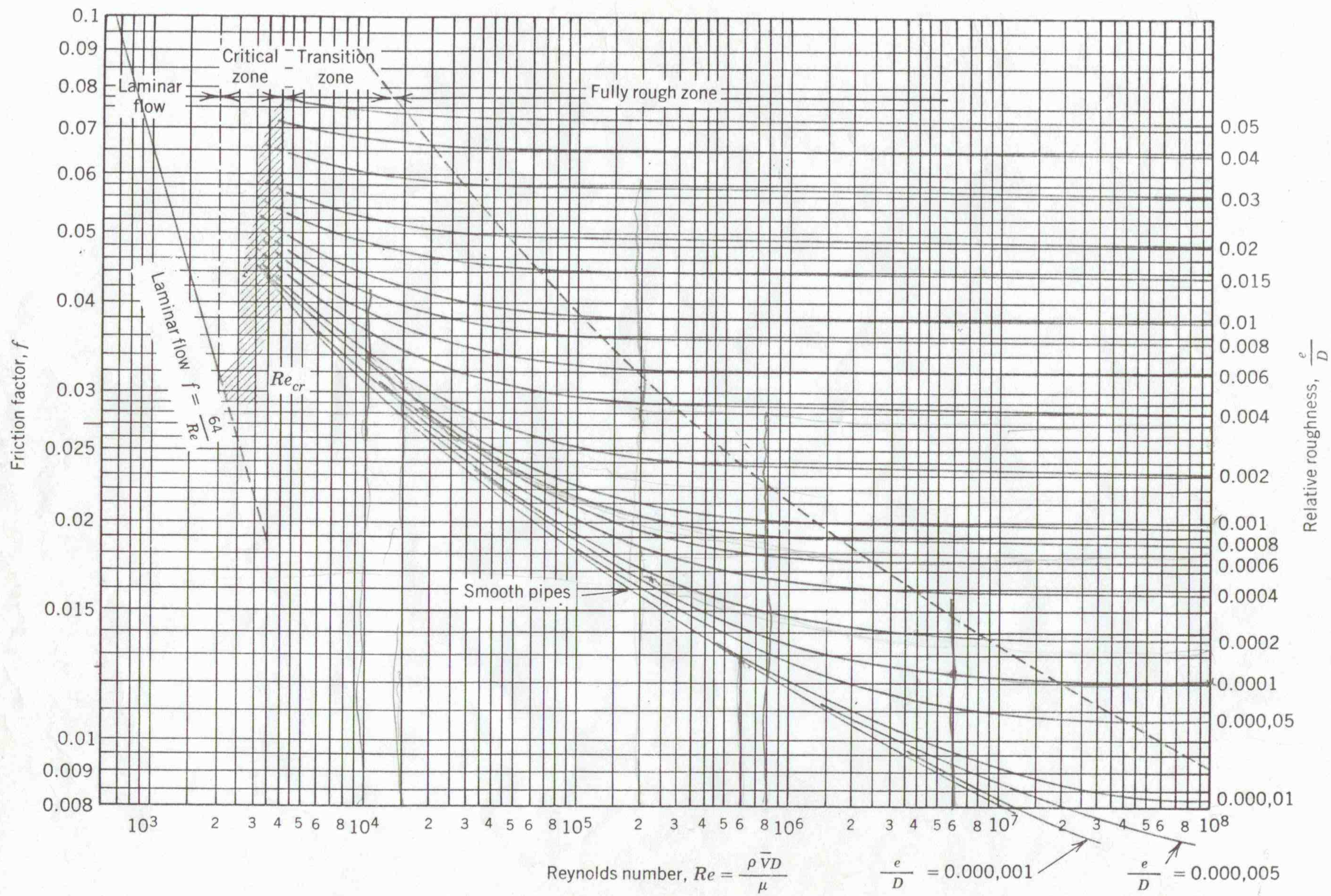


Fig. 8.14 Friction factor for fully developed flow in circular pipes. (Data from [6], used by permission.)

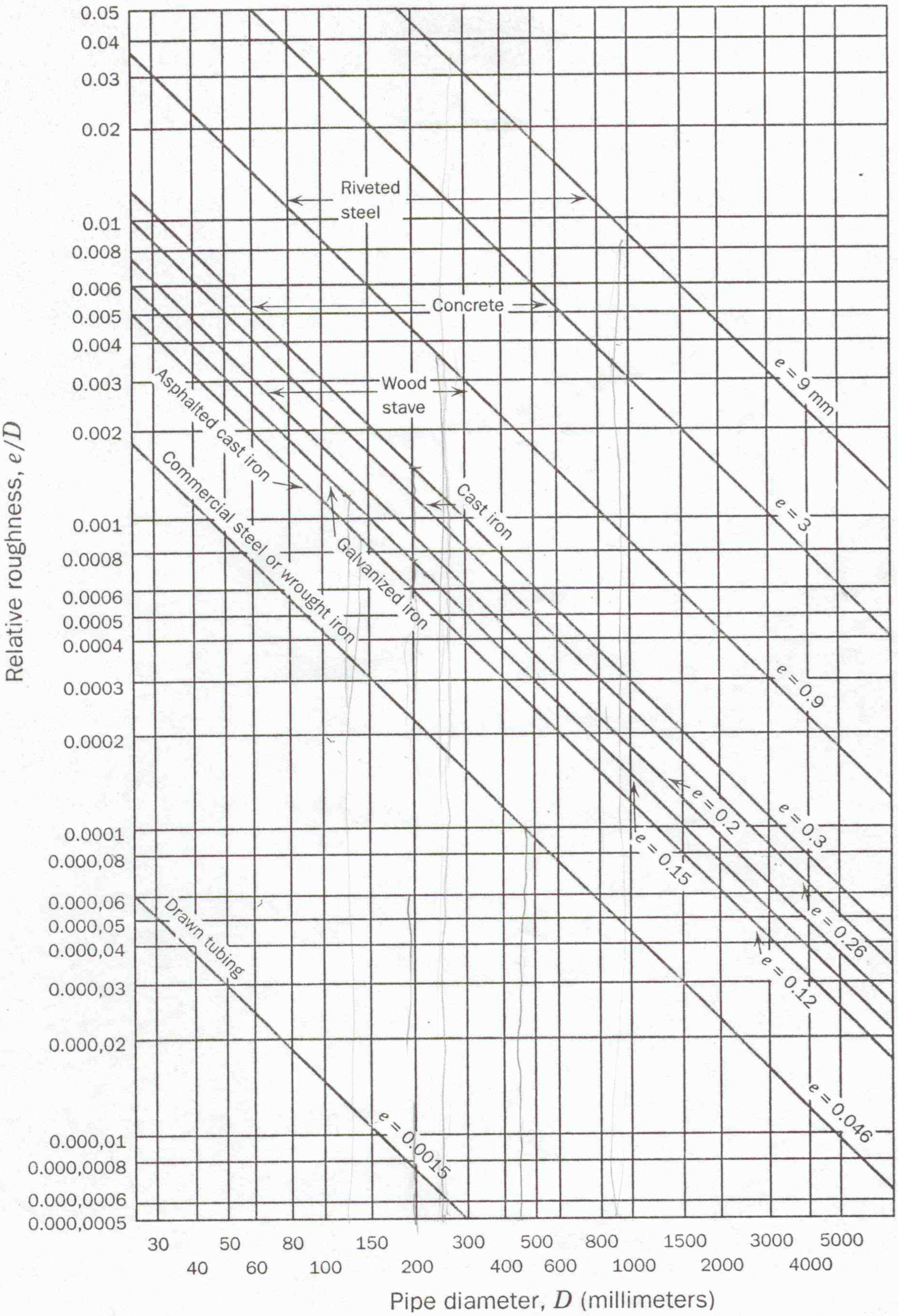


Fig. 8.15 Relative roughness for pipes of common engineering materials. (Data from [6], used by permission.)

MINOR (LOCAL) HEAD LOSSES

Minor Head Losses in Variable Area Parts

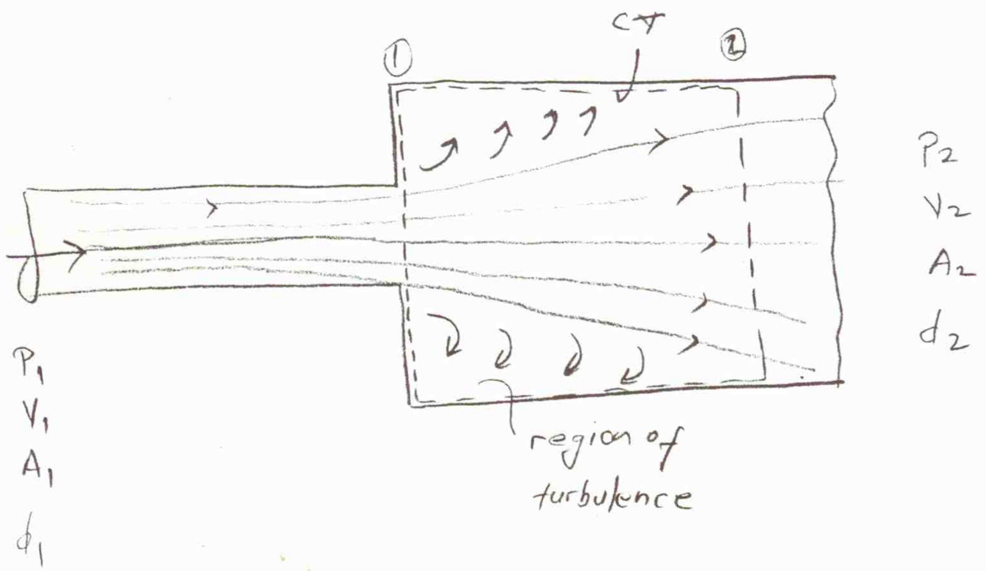
The minor head losses in variable area parts are proportional to the velocity head of the fluid, i.e.

$$h_f = k \frac{V^2}{2g}$$

where V is the average velocity, k is the constant of proportionality.

a) Sudden Enlargement Loss

As a fluid flows from a smaller pipe into a large pipe through a sudden enlargement, its velocity abruptly decreases causing turbulence which generates a head loss.



To determine the head loss, consider steady and turbulent flow of an incompressible fluid throug the control volume.

Applying the momentum equation

$$P_1 A_1 - P_2 A_2 = \rho V_2^2 A_2 - \rho V_1^2 A_1 \quad , \quad \underline{\text{Note:}} \text{ here } A_1 = A_2$$

Note: In the above equation, the friction effects are neglected since the pressure drop only due to sudden enlargement is required.

Applying extended Bernoulli equation along a stream line between sections 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

Continuity equation.

$$V_1 A_1 = V_2 A_2$$

Combining these equations and expressing V_2 interms of V_1 we obtain.

$$h_f = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2g} = \left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2 \frac{V_1^2}{2g} \quad //$$

Comparing the above equation with

$$h_f = k \frac{V^2}{2g}$$

we obtain head loss coefficient

$$k = \left(1 - \frac{A_1}{A_2}\right)^2 = \left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2$$

In table, theoretical head loss coefficients for the sudden expansion and also the experimental values of the head loss coefficients are given. One should note that the experimental and theoretical values of the head loss coefficient agree quite well at an upstream velocity of approximately 1.2 m/s. At higher upstream velocities, the actual values of the head loss coefficients are lower than theoretical ones. It is recommended that one should use the experimental values, if upstream velocity is known.

Table 10.1 Head loss coefficients for the sudden enlargement

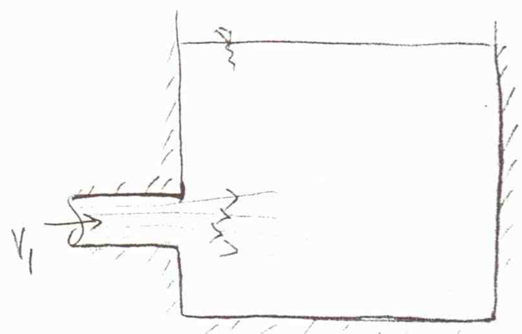
d_2/d_1	Theoretical k	Experimental k						
		Velocity, V_1 (m/s)						
		0.60	1.20	3.00	4.50	6.00	9.00	12.00
1.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.2	0.10	0.11	0.10	0.09	0.09	0.09	0.09	0.09
1.4	0.24	0.26	0.25	0.23	0.22	0.22	0.21	0.20
1.6	0.37	0.40	0.38	0.35	0.34	0.34	0.33	0.32
1.8	0.48	0.51	0.48	0.45	0.43	0.42	0.41	0.40
2.0	0.56	0.60	0.56	0.52	0.51	0.50	0.48	0.47
2.5	0.71	0.74	0.70	0.65	0.63	0.62	0.60	0.58
3.0	0.79	0.83	0.78	0.73	0.70	0.69	0.67	0.65
4.0	0.88	0.92	0.87	0.80	0.78	0.76	0.74	0.72
5.0	0.92	0.96	0.91	0.84	0.82	0.80	0.77	0.75
10.0	0.98	1.00	0.96	0.89	0.86	0.84	0.82	0.80
∞	1.00	1.00	0.98	0.91	0.88	0.86	0.83	0.81

b) Exit Loss

As a fluid flows from a pipe into a large reservoir or tank, its velocity is decreased to very nearly zero. During this process the kinetic energy or the velocity head, $V_1^2/2g$, which is possesed by the fluid, is dissipated. Then the head loss for an exit may be given as

$$h_f = k \frac{V_1^2}{2g} \quad \Bigg\} \Rightarrow h_f = \frac{V_1^2}{2g}.$$

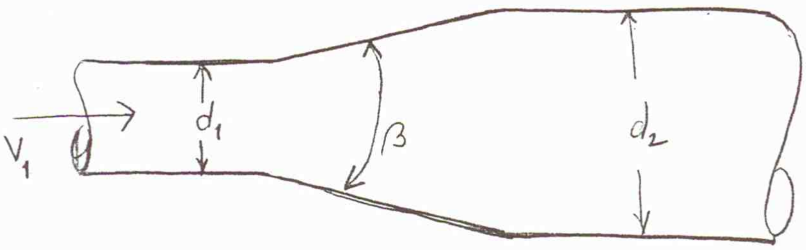
$$k \cong 1$$



c)

Gradual Enlargement Loss

If the transition from a smaller to a larger pipe is made less abrupt than the square-edged sudden enlargement, then the head loss will be reduced.



Head loss

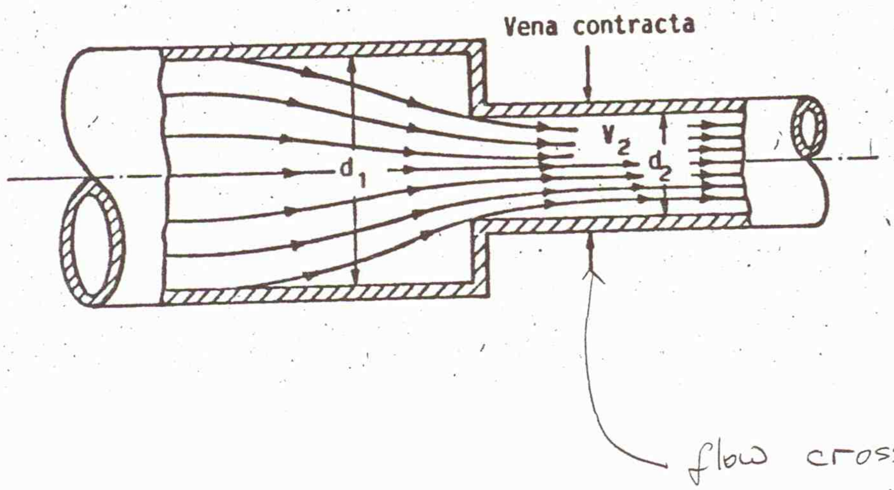
$$h_f = k\,\frac{V_1^2}{2g}$$

Head loss coefficients for gradual enlargement are given as a function cone angle β and the diameter ratio in the table below.

Table 10.2 Head loss coefficients for the gradual enlargement

d_2/d_1	Cone Angle, β										
	2°	6°	10°	15°	20°	25°	30°	35°	40°	45°	50°	60°
1.1	0.01	0.01	0.03	0.05	0.10	0.13	0.16	0.18	0.19	0.20	0.21	0.23
1.2	0.02	0.02	0.04	0.09	0.16	0.21	0.25	0.29	0.31	0.33	0.35	0.37
1.4	0.03	0.03	0.06	0.12	0.23	0.30	0.36	0.41	0.44	0.47	0.50	0.53
1.6	0.03	0.04	0.07	0.14	0.26	0.35	0.42	0.47	0.51	0.54	0.57	0.61
1.8	0.03	0.04	0.07	0.15	0.28	0.37	0.44	0.50	0.54	0.58	0.61	0.65
2.0	0.03	0.04	0.07	0.16	0.29	0.38	0.46	0.52	0.56	0.60	0.63	0.68
2.5	0.03	0.04	0.08	0.16	0.30	0.39	0.48	0.54	0.58	0.62	0.65	0.70
3.0	0.03	0.04	0.08	0.16	0.31	0.40	0.48	0.55	0.59	0.63	0.66	0.71
∞	0.03	0.05	0.08	0.16	0.31	0.40	0.49	0.56	0.60	0.64	0.67	0.72

d) Sudden Contraction Loss



flow cross-sectional area is smaller than the cross sectional area of the pipe.

Turbulence which is caused by the contraction and the subsequent expansion generates a head loss. This head loss is given by

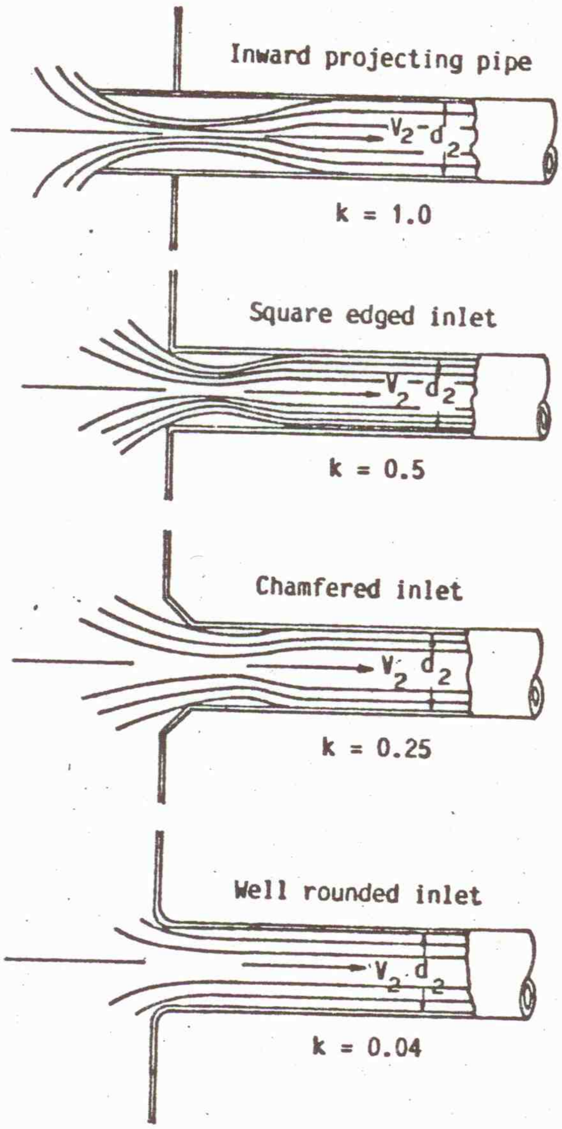
$$h_f = k \frac{V_2^2}{2g}$$

Table 10.3 Head loss coefficients for the sudden contraction

d_1/d_2	Velocity, V_2(m/s)									
	0.6	1.2	1.8	2.4	3.0	3.6	4.5	6.0	9.0	12.0
1.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.1	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.06
1.2	0.07	0.07	0.07	0.07	0.08	0.08	0.08	0.09	0.10	0.11
1.4	0.17	0.17	0.17	0.17	0.18	0.18	0.18	0.18	0.19	0.20
1.6	0.26	0.26	0.26	0.26	0.26	0.26	0.25	0.25	0.25	0.24
1.8	0.34	0.34	0.34	0.33	0.33	0.32	0.32	0.31	0.29	0.27
2.0	0.38	0.37	0.37	0.36	0.36	0.34	0.34	0.33	0.31	0.29
2.2	0.40	0.40	0.39	0.36	0.38	0.37	0.37	0.35	0.33	0.30
2.5	0.42	0.42	0.41	0.40	0.40	0.39	0.38	0.37	0.34	0.31
3.0	0.44	0.44	0.43	0.42	0.42	0.41	0.40	0.39	0.36	0.33
4.0	0.47	0.46	0.45	0.45	0.44	0.43	0.42	0.41	0.37	0.34
5.0	0.48	0.47	0.47	0.46	0.45	0.45	0.44	0.42	0.39	0.35
10.0	0.49	0.48	0.48	0.47	0.46	0.46	0.45	0.43	0.40	0.36
∞	0.49	0.48	0.48	0.47	0.47	0.46	0.45	0.44	0.41	0.38

e) Entrance Loss

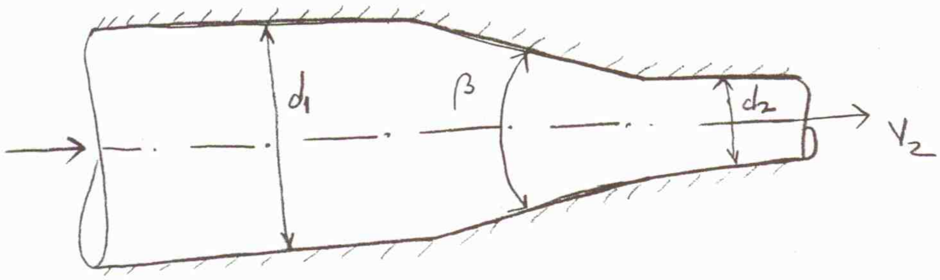
When a fluid flows from a relatively large reservoir into a pipe, a sepicial case of sudden contraction occurs.



$$h_f = k \frac{V_2^2}{2g}$$

Figure 10.10 Head loss coefficient for entrances

f) Gradual Contraction Loss



Head loss is given by $h_f = k \dfrac{V_2^2}{2g}$

The magnitute of the head loss coefficient, k, is dependent on the cone angle β, but it is not dependent on the diameter ratio d_2/d_1 for moderate ratios. The experimantal data for the head loss coefficient of a gradual contraction is quite sparse and a few representative values are presented in table below.

Table 10.4 Head loss coefficients for the gradual contraction

Cone Angle, β	Loss Coefficient, k
30	0.02
40	0.04
60	0.07

MINOR HEAD LOSSES IN VALVES, FITTINGS AND BENDS

For these type of devices, the minor head loss can be calculated by using the equivalent length technique, which may be given by the following equation

$$h_f = f \frac{Le}{d} \frac{V^2}{2g}$$

Le is known as the equivalent length and defined as the length of a straight pipe which would have the same total head loss as valves, fittings or bends. For many commercially available valves and fittings, the ratio of Le/d is approximately constant for a particular type of device regardless of its size. Therefore resistance to the flow varies with the friction factor, f.

Equivalent length can be converted to a head loss coefficient, i.e

$$k = f \frac{Le}{d}$$

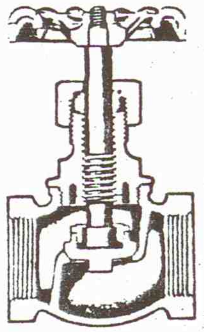
Note: V is the velocity in the pipe to which the valve, fitting or the bend is attached.

a) VALVES

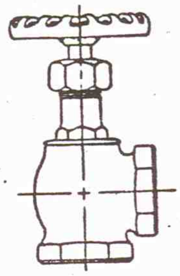
The resistance in some of values is given in terms of equivalent length in the table below.

Table 10.5 **Resistance in valves, which are expressed as the equivalent length in pipe diameters**

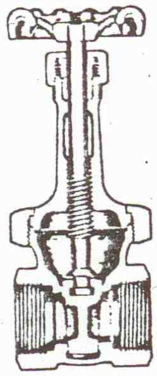
Type	Equivalent length in pipe diameters, L_e/d
Globe valve-fully open	340
Angle valve-fully open	145
Gate valve-fully open	13
-3/4 open	35
-1/2 open	160
-1/4 open	900
Butterfly valve-fully open	40
Check valve-ball type	150
Check valve-swing type	135



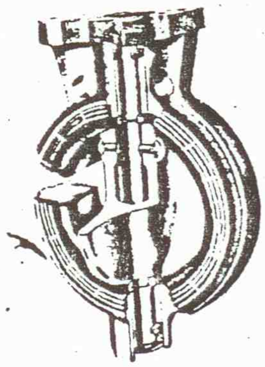
(a) Globe valve



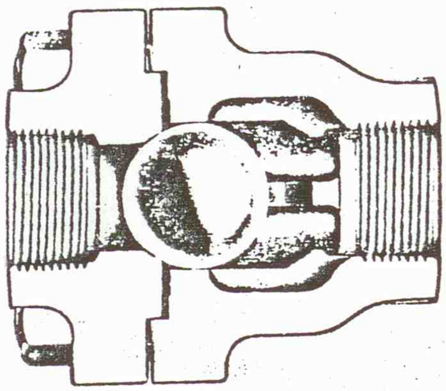
(b) Angle valve



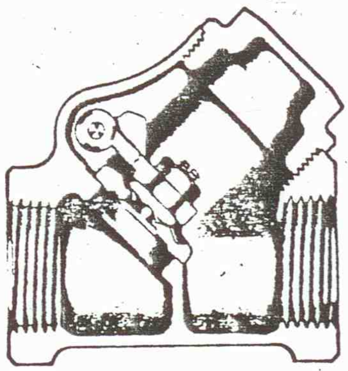
(b) Gate valve



(d) Butterfly valve



(e) Ball type check valve



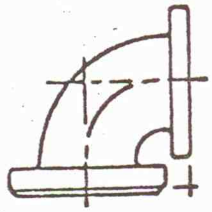
(f) Swing type check valve

Figure 10.12 Some typical valves.

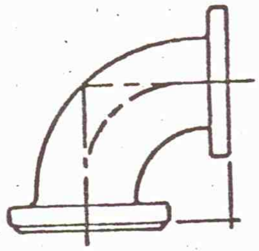
b) Fittings

Table 10.6 Resistance in fittings, which are expressed as the equivalent length in pipe diameters

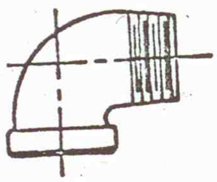
Type	Equivalent length in pipe diameters, L_e/d
90^0 standard elbow	30
90^0 long radius elbow	20
90^0 street elbow	50
45^0 standard elbow	16
45^0 street elbow	26
Close return bend	50
Standard tee-with flow through run	20
-with flow through branch	60



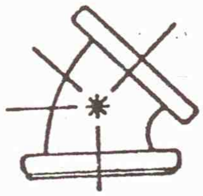
(a) 90^0 standard elbow



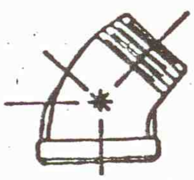
(b) 90^0 long radius elbow



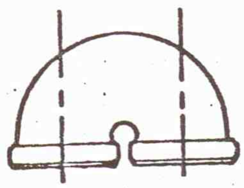
(c) 90^0 street elbow



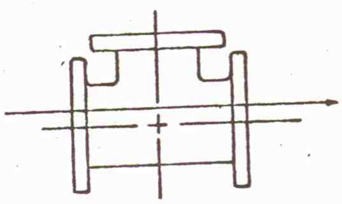
(d) 45^0 standard elbow



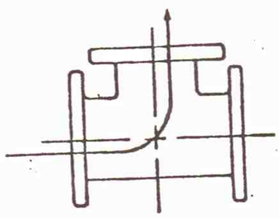
(e) 45^0 street elbow



(f) Return bend



(g) Standard tee
Flow through run



(h) Standard tee
Flow through branch

Figure 10.13 Some typical fittings

c) Bends

It is requently more convenient to bend a pipe or a tube rather than to install a commercially made elbow. The resistance to the fluid flow through a bend is dependent on the ratio of bend radius r to the pipe diameter d, which is known as the relative radius, r/d.

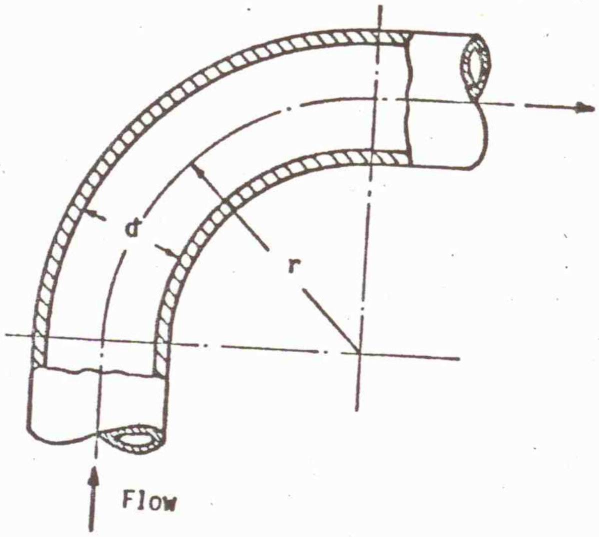


Figure 10.14 Flow through a bend

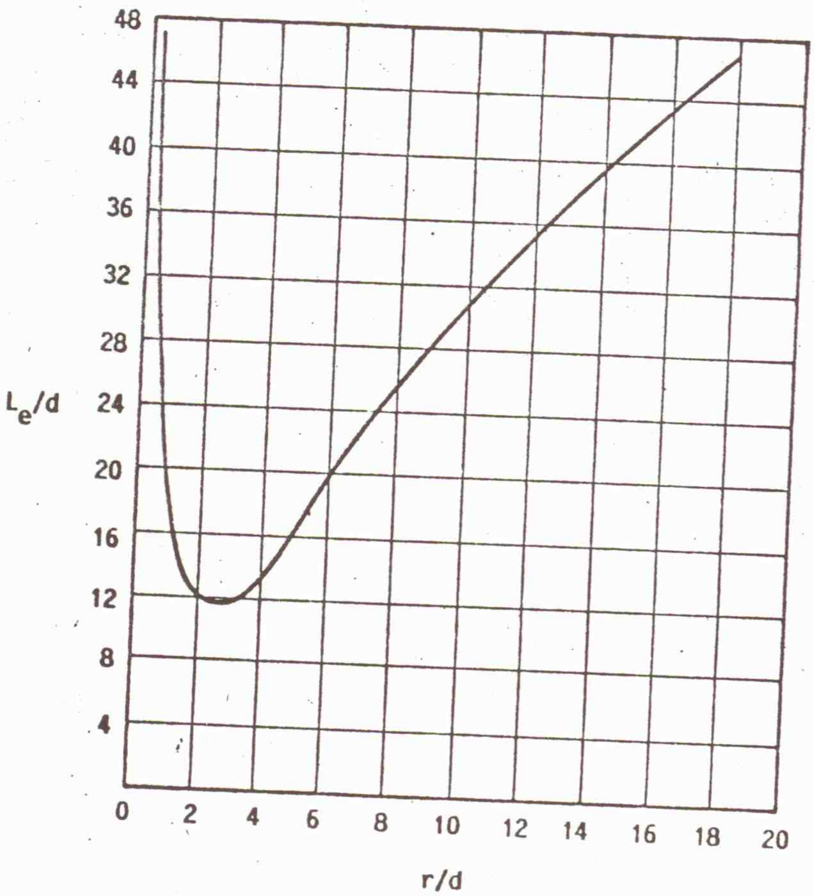


Figure 10.15 Resistance in 90° pipe bends

10.5 ANALYSIS OF VISCOUS FLOW IN PIPES

The viscous flow in pipe systems may be analysed by classifiying them in the following manner:

i) series system of pipes,
ii) parallel system of pipes,
iii) pipe networks, and
iv) interconnected reservoir systems.

In this section, the viscous flow in these pipe systems is discussed.

10.5.1 Series System of Pipes

If the pipe system is arranged such that the fluid flows through a continuous line without branching, it is referred as a series system of pipes. In a series system of pipes, which is shown in Figure 10.16, the

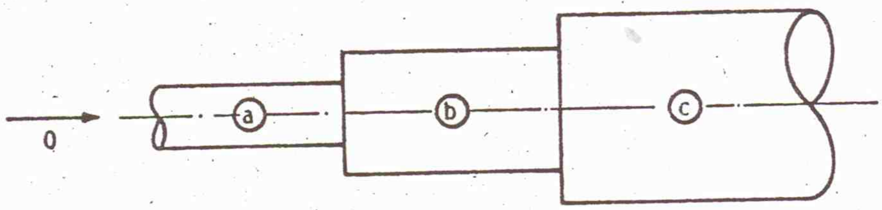


Figure 10.16 Series system of pipes

total head loss is the sum of the head losses in each serially connected pipes, that is

$$h_f = h_{fa} + h_{fb} + h_{fc} \qquad (10.18)$$

Also, one should observe that the volumetric flow rate in each of the serially connected pipes should be the same, so that

$$Q = Q_a = Q_b = Q_c \qquad (10.19)$$

In designing or analysing a pipe flow system, there are six primary parameters involved, which are

i) the total head loss of the system, h_f,
ii) the volumetric flow rate of the fluid, Q,
iii) the diameter of the pipe, d,
iv) the length of the pipe, L,
v) the roughness of the pipe, ϵ_s, and
vi) the absolute viscosity of the fluid, μ.

Usually, one of the first three parameters is to be determined, while the remaining ones are either known or can be specified by the designer. The method of performing the design or completing the analysis is different depending on what is unknown. These methods can be classified in the following manner:

i) Class I Systems for which the total head loss of the system is to be determined

ii) Class II Systems for which the volumetric flow rate of the fluid is to be determined

iii) Class III Systems for which the diameter of the pipe is to be determined.

10.5.1.1 Class I Systems

In Class I Systems, the volumetric flow rate of the fluid, Q, the diameter of the pipe, d, the length of the pipe, L, the roughness of the pipe, ε_s, and the absolute viscosity of the fluid, μ , are known, and the total head loss of the system, h_f, is to be determined.

The procedure for solving Class I problems may be given as follows:

i) Determine the relative roughness, ε_s/d, for each pipe from Figure 10.4 by using the pipe diameter, d, and the pipe material.

ii) Calculate the average velocity, $V = 4Q/(\pi d^2)$, in each pipe.

iii) Calculate the Reynolds number, $Re = \rho Vd/\mu$ for each pipe.

iv) Determine the friction factor, f, for each pipe from the Moody diagram in Figure 10.5 by using the Reynolds number, Re, and the relative roughness, ε_s/d.

v) Calculate the major head losses for each pipe by using the Darcy-Weisbach equation, $h_f = f(L/d)(V^2/2g)$.

vi) Calculate the minor head losses for the variable area portions in each pipe by using $h_f = kV^2/(2g)$. The head loss coefficient, k, can be determined by using Tables 10.1, 10.2, 10.3, and 10.4 and Figure 10.10.

vii) Calculate the minor head losses for valves, fittings and bends in each pipe by using $h_f = f(L_e/d)(V^2/2g)$. The equivalent length ratio, L_e/d, can be determined by using Tables 10.5 and 10.6 and Figure 10.15.

viii) Evaluate the total head loss, h_f, of the system by adding up the major and the minor head losses from steps (v), (vi) and (vii).

Example 10.1

Calculate the power supplied to the pump, which is shown in Figure 10.17, if its efficiency is 76 percent. Water at 20°C is flowing at the volumetric flow rate of 0.015 m^3/s. The length of the 4" commercial steel pipe in the suction line is 15 m, while the length of the 2" commercial steel pipe in the discharge line is 200 m. The entrance to the suction line from the reservoir is through a square edged inlet. 90° standard elbow is used and the globe valve is fully open.

Solution

The required pump head for transporting water between the two reservoirs may be determined by applying the extended Bernoulli equation

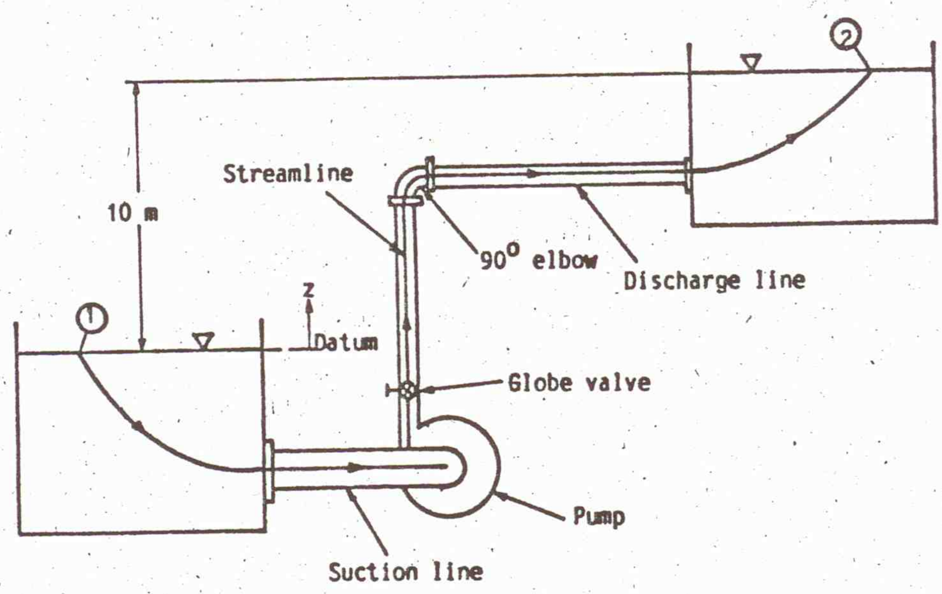


Figure 10.17 Sketch for Example 10.1

(7.28) between points 2 and 1 along the streamline, which is shown in Figure 10.17. Then

$$h_{t2} = h_{t1} + h_s - h_{f1-2}$$

or solving for the pump head, it is possible to obtain

$$h_s = h_{t2} - h_{t1} + h_{f1-2}$$

As long as the areas of the two reservoirs are very large, when compared to the cross-sectional areas of the suction and the discharge pipes, then the velocities at the free surfaces of the reservoirs are negligible, that is $V_1 \cong 0$, and $V_2 \cong 0$. Also, the free surfaces of the reservoirs are exposed to the atmosphere, so that $p_1 = p_2 = p_{atm}$. Therefore

$$h_{t1} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_{atm}}{\rho g}$$

$$h_{t2} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{P_{atm}}{\rho g} + z_2$$

according to the datum, which is shown in Figure 10.17. Therefore, the required pump head may be expressed as

$$h_s = h_{f1-2} + z_2.$$

The frictional head loss, h_{f1-2}, may now be evaluated by following the procedure which is presented in this section

i) The relative roughness for the 4" commercial steel suction pipe and the 2" commercial steel discharge pipe may be obtained from Figure 10.4 as follows:

$$(\varepsilon_s/d)_s = 0.00044$$

$$(\varepsilon_d/d)_d = 0.0009$$

ii) The average velocity in the suction and the discharge pipes may then be evaluated as follows:

$$V_s = \frac{4Q}{\pi d_s^2} = \frac{(4)(0.015\ m^3/s)}{(\pi)(0.1016\ m)^2} = 1.85\ m/s$$

$$V_d = \frac{4Q}{\pi d_d^2} = \frac{(4)(0.015\ m^3/s)}{(\pi)(0.0508\ m)^2} = 7.4\ m/s$$

iii) The kinematic viscosity of water at 20°C is presented in Table A.2 as $1.003\times10^{-6}\ m^2/s$. Therefore, the Reynolds number for the suction and the discharge pipes are as follows:

$$Re_s = \frac{V_s d_s}{\nu} = \frac{(1.85\ m/s)(0.1016\ m)}{(1.003\times10^{-6}\ m^2/s)} = 1.87\times10^5$$

$$Re_d = \frac{V_d d_d}{\nu} = \frac{(7.4\ m/s)(0.0508\ m)}{(1.003\times10^{-6}\ m^2/s)} = 3.75\times10^5$$

iv) Now, the friction factor for the suction pipe corresponding to a relative roughness of $(\varepsilon_s/d)_s = 0.00044$ and a Reynolds number of $Re_s = 1.87\times10^5$ from the Moody diagram in Figure 10.5 is

$$f_s = 0.019$$

Similarly, the friction factor for the discharge pipe corresponding to a relative roughness of $(\varepsilon_s/d)_d = 0.0009$ and a Reynolds number of $Re_d = 3.75\times10^5$ is

$$f_d = 0.020$$

v) The major head losses in the suction and the discharge pipes may be evaluated by using the Darcy-Weisbach equation as

$$h_{fs} = f_s \frac{L_s}{d_s} \frac{V_s^2}{2g} = (0.019) \frac{(15 \text{ m})}{(0.1016 \text{ m})} \frac{(1.85 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = 0.49 \text{ m}$$

$$h_{fd} = f_d \frac{L_d}{d_d} \frac{V_d^2}{2g} = (0.020) \frac{(200 \text{ m})}{(0.0508 \text{ m})} \frac{(7.4 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = 219.77 \text{ m}$$

vi) For the square edged inlet, the head loss coefficient, k_i, may be obtained from Figure 10.10 as 0.5. Then the minor head loss through the inlet is

$$h_{fi} = k_i \frac{V_s^2}{2g} = (0.5) \frac{(1.85 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = 0.09 \text{ m}$$

The head loss coefficient, k_o, for the outlet from the discharge pipe into the reservoir may be obtained from Equation (10.11) as 1.0, then the minor head loss is

$$h_{fo} = k_o \frac{V_d^2}{2g} = (1.0) \frac{(7.4 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = 2.79 \text{ m}$$

vii) The equivalent length ratio for the fully open globe valve is 340 from Table 10.5, so that the minor head loss through the globe valve is

$$h_{fv} = f_d \left(\frac{L_e}{d}\right)_v \frac{V_d^2}{2g} = (0.020)(340) \frac{(7.4 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = 18.98 \text{ m}$$

Similary, the equivalent length ratio for the 90^0 standard elbow is 30 from Table 10.6. Therefore the minor head loss through the elbow is

$$h_{fe} = f_d \left(\frac{L_e}{d}\right)_e \frac{V_d^2}{2g} = (0.020)(30) \frac{(7.4 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = 1.68 \text{ m}$$

viii) As long as the total head loss is the sum of the major and the minor head losses, then

$$h_{f1-2} = h_{fs} + h_{fd} + h_{fi} + h_{fo} + h_{fv} + h_{fe}$$

$$= 0.49 \text{ m} + 219.77 \text{ m} + 0.09 \text{ m} + 2.79 \text{ m} + 18.98 \text{ m} + 1.68 \text{ m}$$

$$= 243.8 \text{ m}$$

The required pump head may now be evaluated as

$$h_s = 243.8 \text{ m} + 10 \text{ m} = 253.8 \text{ m}$$

As long as the density of water at 20°C is 998.2 kg/m^3 from Table A.2, then the power required by the pump is

$$\mathbb{P}_p = \frac{\rho g Q h_s}{\eta_p} = \frac{(998.2\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.015\ \text{m}^3\text{/s})(253.8\ \text{m})}{(0.76)}$$

$$= \underline{49.05\ \text{kW}}$$

10.5.1.2 Class II Systems

In Class II Systems, the total head loss of the system, h_f, the diameter of the pipe, d, the length of the pipe, L, the roughness of the pipe, ε_s, and the absolute viscosity of the fluid, μ , are known, and the volumetric flow rate of the fluid, Q, is to be determined.

Whenever the volumetric flow rate in the system is unknown, then the system performance may be analysed by iteration. This is required because there are too many unknown quantities to use a direct solution procedure. Specifically, if the volumetric flow rate is unknown, then the velocity of flow is also unknown. It follows that the Reynolds number is unknown, since it depends on the velocity. If the Reynolds number cannot be evaluated, then the friction factor cannot be determined directly. Since the major and the minor head losses are dependent on both the velocity and the friction factor, then the value of these losses cannot be calculated directly. Iteration overcomes these difficulties. It is a type of trial-and-error solution method in which a trial value is assumed for the unknown friction factor, which allows the calculation of a corresponding velocity. The procedure provides a means of checking the accuracy of the trial value of the friction factor, and also indicates the new trial value to be used if an additional calculation is required. The procedure for solving Class II problems may be presented in step-by-step form as follows:

i) Determine the relative roughness, ε_s/d for each pipe from Figure 10.4 by using the pipe diameter, d, and the pipe material.

ii) Assume a friction factor, f, for each pipe from the Moody diagram in Figure 10.5 by using the relative roughness, ε_s/d, in the fully rough region. The reason for the assumption of the flow in the fully rough region is that no iteration is necessary if this assumption is true.

iii) Calculate the major head losses for each pipe by using the Darcy-Weisbach equation, $h_f = f(L/d)(V^2/2g)$ in terms of the unknown velocities.

iv) Calculate the minor head losses for the variable area portions in each pipe by using $h_f = kV^2/(2g)$ in terms of the unknown velocities. The head loss coefficient, k, can be determined by using Tables 10.1, 10.2, 10.3 and 10.4 and Figure 10.10.

v) Calculate the minor head losses for valves, fittings and bends in each pipe by using $h_f = f(L_e/d)(V^2/2g)$ in terms of the unknown velocities. The equivalent length ratio, L_e/d, can be determined by using Tables 10.5 and 10.6 and Figure 10.15.

vi) Evaluate the total head loss, h_f, of the system by adding up the major and the minor head losses from steps (iii), (iv) and (v)

vii) Relate the unknown velocities in each pipe by using the continuity equation.

viii) Express the total head loss, h_f, of the system in terms of the one of the unknown velocities, and then solve for the unknown velocities.

ix) Calculate the Reynolds number, $Re = \rho Vd/\mu$ for each pipe.

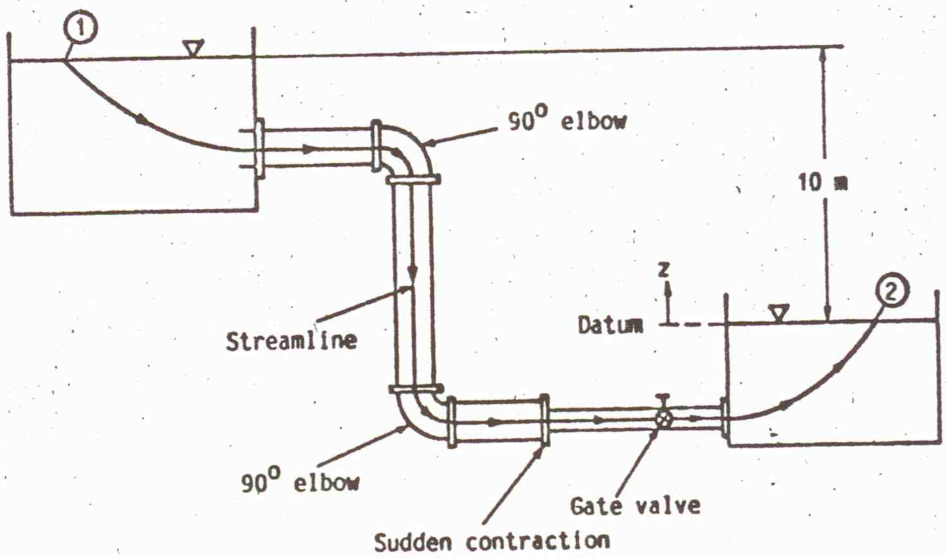
x) Determine the improved value of the friction factor, f, for each pipe from the Moody diagram in Figure 10.5 by using the Reynolds number, Re, and the relative roughness, ε_s/d.

xi) Compare the assumed and the improved values of the friction factor and repeat steps (iii) through (x) as many times as needed in order to obtain the desired accuracy in the friction factor.

xii) Evaluate the volumetric flow rate of the fluid, $Q = \pi d^2 V/4$.

Example 10.2

The piping system, which is shown in Figure 10.18, is used to transfer water at 25°C from one storage tank to the other. The larger pipe is a 6" commercial steel pipe having a total length of 30 m, while the smaller pipe is a 2" commercial pipe having a total length of 15 m. The entrance to the larger pipe from the storage tank is through an inward projecting pipe. 90° standard elbows are used, and the gate valve is half open. Determine the volumetric flow rate of water through the system.



Solution

The total head loss during the transfer of water from one storage tank to the other may be determined by applying the extended Bernoulli equation (7.28) between points 2 and 1 along the streamline which is shown in Figure 10.18. Then

$$h_{t2} = h_{t1} - h_{f1-2}$$

or solving for the total head loss

$$h_{f1-2} = h_{t1} - h_{t2}$$

As long as the areas of the two reservoirs are very large when compared to the cross-sectional areas of the larger and the smaller pipes, then the velocities at the free surfaces of the storage tanks are negligible that is $V_1 \approxeq 0$ and $V_2 \approxeq 0$. Also the free surfaces of both storage tanks are exposed to the atmosphere, so that $p_1 = p_2 = p_{atm}$, Therefore

$$h_{t1} = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_{atm}}{\rho g} + 10$$

$$h_{t2} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{p_{atm}}{\rho g}$$

according to the datum, which is shown in Figure 10.18. Therefore the total head loss of the system is

$$h_{f1-2} = 10 \text{ m}$$

The volumetric flow rate of water may now be evaluated by following the procedure which is presented in this section.

i) The relative roughness for the 6" commercial steel larger pipe and 2" commerçial steel smaller pipe may be obtained from Figure 10.4 as follows:

$$(\varepsilon_s/d)_l = 0.00028$$

$$(\varepsilon_s/d)_s = 0.0009$$

ii) The friction factor for the larger pipe corresponding to a relative roughness of $(\varepsilon_s/d)_l = 0.00028$ in the fully rough flow region may be obtained from the Moody diagram in Figure 10.5 as

$$f_l = 0.015$$

Similarly, the friction factor for the smaller pipe corresponding to a relative roughness of $(\varepsilon_s/d)_s = 0.0009$ in the fully rough flow region is

$$f_s = 0.019$$

iii) The major head losses in the larger and the smaller pipes may be evaluated by using the Darcy-Weisbach equation as

$$h_{fl} = f_l \frac{L_l V_l^2}{d_l 2g} = (0.015)\frac{(30\ \text{m})}{(0.1524\ \text{m})}\frac{V_l^2}{(2)(9.81\ \text{m/s}^2)} = 0.151\ V_l^2$$

$$h_{fs} = f_s \frac{L_s}{d_s}\frac{V_s^2}{2g} = (0.019)\frac{(15\ \text{m})}{(0.0508\ \text{m})}\frac{V_s^2}{(2)(9.81\ \text{m/s}^2)} = 0.286\ V_s^2$$

iv) For an inlet through an inward projecting pipe, the head loss coefficient, k_i, may be obtained from Figure 10.10 as 1.0. Then the minor head loss through the inlet is

$$h_{fi} = k_i \frac{V_l^2}{2g} = (1.0)\frac{V_l^2}{(2)(9.81\ \text{m/s}^2)} = 0.051\ V_l^2$$

Using $d_l/d_s = 0.1524\ \text{m}/0.0508\ \text{m} = 3$ and assuming that V_s is approximately 3 m/s, it is possible to obtain the head loss coefficient, k_r for the reducer as 0.42.

$$h_{fr} = k_r \frac{V_s^2}{2g} = (0.42)\frac{V_s^2}{(2)(9.81\ \text{m/s}^2)} = 0.021\ V_s^2$$

One should note that if V_s is much different from 3 m/s then k_r should be revaluated.

The head loss coefficient, k_o, for the outlet from the smaller pipe into the storage tank may be obtained from Equation (10.10) as 1.0, then the minor head loss is

$$h_{fo} = k_o \frac{V_s^2}{2g} = (1.0)\frac{V_s^2}{(2)(9.81\ \text{m/s}^2)} = 0.051\ V_s^2$$

v) The equivalent length ratio for the half open gate valve is 160 from Table 10.5, so that the minor head loss through the gate valve is

$$h_{fv} = f_s \left(\frac{L_e}{d}\right)_v \frac{V_s^2}{2g} = (0.019)(160)\frac{V_s^2}{(2)(9.81\ \text{m/s}^2)} = 0.155\ V_s^2$$

Similarly, the equivalent length ratio for the 90^{o} standard elbow is 30 from Table 10.6. Therefore, the minor head loss through two elbows is

$$h_{fe} = 2f_l \left(\frac{L_e}{d}\right)_e \frac{V_l^2}{2g} = (2)(0.015)(30)\frac{V_l^2}{(2)(9.81\ \text{m/s}^2)} = 0.046\ V_l^2$$

vi) As long as the total head loss is the sum of the major and the minor head losses, then

$$h_{f\,1-2} = h_{fl} + h_{fs} + h_{fi} + h_{fr} + h_{fo} + h_{fv} + h_{fe}$$

$$= 0.151\ V_1^2 + 0.286\ V_s^2 + 0.051\ V_1^2 + 0.021\ V_s^2 + 0.051\ V_s^2$$

$$+ 0.155\ V_s^2 + 0.046\ V_1^2 = 0.248\ V_1^2 + 0.513\ V_s^2$$

vii) The velocities in the larger and the smaller pipes may now be related by using the continuity equation as

$$\pi d_1^2 V_1/4 = \pi d_s^2 V_s/4$$

or

$$V_s = (d_1/d_s)^2 V_1 = (0.1524\ m/0.0508\ m)^2 V_1 = 9V_1$$

viii) Now, the equation for V_1 may be expressed as

$$10 = 0.248\ V_1^2 + 0.512(9\ V_1)^2$$

which may be solved for V_1 to yield

$$V_1 = 0.49\ m/s$$

$$V_s = 9\ V_1 = (9)(0.49\ m/s) = 4.41\ m/s$$

ix) The kinematic viscosity of water at 25°C is presented in Table A.2 as $0.893 \times 10^{-6}\ m^2/s$. Therefore, the Reynolds number for the larger and the smaller pipes are

$$Re_1 = \frac{V_1 d_1}{\nu} = \frac{(0.49\ m/s)(0.1524\ m)}{(0.893 \times 10^{-6}\ m^2/s)} = 8.36 \times 10^4$$

$$Re_s = \frac{V_s d_s}{\nu} = \frac{(4.47\ m/s)(0.0508\ m)}{(0.893 \times 10^{-6}\ m^2/s)} = 2.54 \times 10^5$$

x) Now, the improved value of the friction factor for the larger pipe corresponding to a relative roughness of $(\varepsilon_s/d)_1 = 0.00028$ and a Reynolds number of $Re_1 = 8.36 \times 10^4$ from the Moody diagram in Figure 10.5 is

$$f_1 = 0.02$$

Similarly, the improved value of the friction factor for the smaller pipe corresponding to a relative roughness of $(\varepsilon_s/d)_s = 0.0009$ and a Reynolds number of $Re_s = 2.54 \times 10^5$ is

$$f_s = 0.0205$$

xi) As long as the assumed and the improved values of the friction factors are not the same, then the calculations in steps (iii) through (x) must be repeated. The summary of these iterations is presented in Table 10.6.

xii) Now, the volumetric flow rate of water may be evaluated as

$$Q = V_1 \pi d_1^2/4 = (0.473\ m/s)(\pi)(0.1524\ m)^2/4 = \underline{8.63\times10^{-3}\ m^3/s}$$

Table 10.6 Iterations for Example 10.2

Iteration	1	2
f_1(assumed)	0.015	0.02
f_s(assumed)	0.019	0.0205
h_{f1}(m)	0.151 V_1^2	0.201 V_1^2
h_{fs}(m)	0.286 V_s^2	0.309 V_s^2
h_{fi}(m)	0.051 V_1^2	0.051 V_1^2
h_{fr}(m)	0.021 V_s^2	0.020 V_s^2
h_{fo}(m)	0.051 V_s^2	0.051 V_s^2
h_{fv}(m)	0.155 V_s^2	0.167 V_s^2
h_{fe}(m)	0.046 V_1^2	0.061 V_1^2
h_{f1-2}(m)	0.248 V_1^2 + 0.513 V_s^2	0.313 V_1^2 + 0.547 V_s^2
V_1(m/s)	0.49	0.473
V_s(m/s	4.41	4.26
Re_1	8.36×10^4	8.07×10^4
Re_s	2.51×10^5	2.42×10^5
f_1(improved)	0.02	0.02
f_s(improved)	0.0205	0.0205

10.5.1.3 Class III Systems

Systems that fall into Class III are true design problems. The requirements placed on the system are specified in terms of an allowable total head loss, h_f, a desired volumetric flow rate, Q, the length of the pipe, L, the absolute viscosity of the fluid, μ, and the roughness of the pipe, ϵ_s. Then the proper diameter of the pipe, d, which will meet these requirements is to be determined.

Iteration is required to solve Class III design problems because there are too many unknowns to allow a direct solution. The average velocity, the Reynolds number and the relative roughness are all dependent on the pipe diameter. Therefore, the friction factor cannot be determined directly. The procedure for solving Class III problems may be given as follows:

i) Assume the diameter, d of each pipe

ii) Determine the relative rouhness, ϵ_s/d for each pipe from Figure 10.4 by using the pipe diameter, d, and the pipe material.

iii) Calculate the average velocity, $V = 4Q/(\pi d^2)$, in each pipe

iv) Calculate the Reynolds number, $Re = \rho Vd/\mu$ for each pipe.

v) Determine the friction factor, f, for each pipe from the Moody diagram in Figure 10.5 by using the Reynolds number, Re, and the relative roughness, ϵ_s/d.

vi) Calculate the major head losses for each piped by using the Darcy-Weisbach equation, $h_f = f(L/d)(V^2/2g)$.

vii) Calculate the minor head losses for the variable area portions in each pipe by using $h_f = kV^2/(2g)$. The head loss coefficient, k, can be determined by using Tables 10.1, 10.2, 10.3 and 10.4 and Figure 10.10.

viii) Calculate the minor head losses for valves, fittings and bends in each pipe by using $h_f = f(L_e/d)(V^2/2g)$. The equivalent length ratio, L_e/d, can be determined by using Tables 10.5 and 10.6 and Figure 10.15.

ix) Evaluate the total head loss, h_f, of the system by adding up the major and the minor head losses from steps (vi), (vii) and (viii).

x) Compare the calculated head loss with the given one. If the calculated head loss is less than the given one, then decrease the pipe diameters. If the calculated head loss is more than the given one, then increase the pipe diameters. Repeat steps (ii) through (ix) as many times as needed.

Example 10.3

In a chemical processing system, benzene at 20°C is taken from the bottom of a large tank and transferred by gravity to another part of the system, as shown in Figure 10.19. The length of the line between two tanks is 7 m. A filter is installed in the line, and is known to have a loss coefficient of 8.5. The gate valve is fully open. Commercial steel piping is to be used for the transfer line. Determine the standard size of piping which would allow a volumetric flow rate of 0.0025 m^3/s through this system.

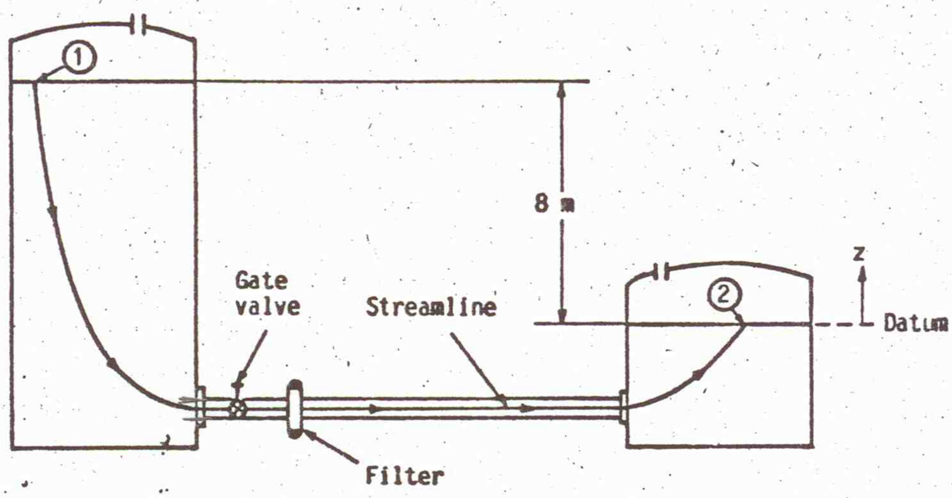


Figure 10.19 Sketch for Example 10.3

Solution

The total head loss during the transportation of benzene from one tank to the other may be determined by applying the extended Bernoulli equation (7.28) between points 2 and 1 along the streamline which is shown in Figure 10.19. Then

$$h_{t2} = h_{t1} - h_{f1-2}$$

or solving for the total head loss

$$h_{f1-2} = h_{t1} - h_{t2}$$

As long as the areas of the two reservoirs are very large when compared to the cross-sectional area of the transfer pipe, then the velocities at the free surfaces of the tanks are negligible, that is $V_1 \doteq 0$ and $V_2 \doteq 0$. Also the free surfaces of both tank are exposed to the atmosphere, so that $p_1 = p_2 = p_{atm}$. Therefore

$$h_{t1} = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_{atm}}{\rho g} + 8$$

$$h_{t2} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{p_{atm}}{\rho g}$$

according to the datum, which is shown in Figure 10.19. Therefore the total head loss of the system is

$$h_{f1-2} = 8 \text{ m}$$

The diameter of the transfer line may now be determined by following the procedure which is presented in this section.

i) In order to start the iteration procedure, a pipe diameter of 2" may be assumed for the transfer line.

ii) The relative roughness for the 2" commercial steel pipe may be obtained from Figure 10.4 as

$$\varepsilon_s/d = 0.0009$$

iii) The average velocity in the transfer may then be evaluated as

$$V = \frac{4Q}{\pi d^2} = \frac{(4)(0.0025 \text{ m}^3/\text{s})}{(\pi)(0.0508 \text{ m})} = 1.23 \text{ m/s}$$

iv) The density and the absolute viscosity of benzene at 20°C are given as 895 kg/m³ and 6.5x10⁻⁴ Pa.s in Table A.1. Therefore, the Reynolds number is

$$Re = \frac{\rho V d}{\mu} = \frac{(895 \text{ kg/m}^3)(1.23 \text{ m})(0.0508 \text{ m})}{(6.5\text{x}10^{-4} \text{ Ns/m}^2)} = 8.6\text{x}10^4$$

v) Now, the friction factor for the transfer pipe corresponding to a relative roughness of $\varepsilon_s/d = 0.0009$ and a Reynolds number of $Re = 8.6\text{x}10^4$ from the Moody diagram in Figure 10.5 is

$$f = 0.022$$

vi) The major head loss in the transfer pipe may now be evaluated by using the Darcy-Weisbach equation as

$$h_{fp} = f \frac{L}{d} \frac{V^2}{2g} = (0.022) \frac{(7 \text{ m})}{(0.0508 \text{ m})} \frac{(1.23 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = 0.234 \text{ m}$$

vii) For an inlet through an inward projecting pipe, the head loss coefficient, k_i, may be obtained from Figure 10.10 as 1.0. Then the minor head loss through the inlet is

$$h_{fi} = k_i \frac{V^2}{2g} = (1.0) \frac{(1.23 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = 0.077 \text{ m}$$

The minor head loss through the filter is

$$h_{ff} = k_f \frac{V^2}{2g} = (8.5)\,\frac{(1.23\ m/s)^2}{(2)(9.81\ m/s^2)} = 0.655\ m$$

The head loss coefficient, k_o for the outlet from the discharge pipe into the reservoir may be obtained from Equation (10.10) as 1.0, then the minor head loss is

$$h_{fo} = k_o \frac{V^2}{2g} = (1.0)\,\frac{(1.23\ m/s)^2}{(2)(9.81\ m/s^2)} = 0.077\ m$$

viii) The equivalent length ratio for the fully open gate valve is 13 from Table 10.5, so that the minor head loss through the gate valve is

$$h_{fv} = f\left(\frac{L_e}{d}\right)_v \frac{V^2}{2g} = (0.022)(13)\,\frac{(1.23\ m/s)^2}{(2)(9.81\ m/s^2)} = 0.022\ m$$

ix) As long as the total head loss is the sum of the major and the minor head losses, then

$$h_{f\,1-2} = h_{fp} + h_{fi} + h_{ff} + h_{fo} + h_{fv}$$

$$= 0.234\ m + 0.077\ m + 0.655\ m + 0.077\ m + 0.022\ m$$

$$= 1.065\ m$$

x) As long as the given and the calculated values of the total head loss are not the same, then the calculations in step (ii) through (ix) must be repeated. The summary of these iterations are presented in Table 10.7.

Table 10.7 Iterations for Example 10.3

Iteration	1	2	3	4
d (assumed)	2"	1 1/2"	1"	1 1/4"
ϵ_s/d	0.0009	0.0012	0.0018	0.0015
V(m/s)	1.23	2.19	4.93	3.16
Re	8.6×10^4	11.49×10^4	17.24×10^4	13.82×10^4
f	0.022	0.0225	0.024	0.023
h_{fp}(m)	0.234	1.011	8.194	2.581
h_{fi}(m)	0.077	0.245	1.239	0.509
h_{ff}(m)	0.655	2.078	10.530	4.326
h_{fo}(m)	0.077	0.245	1.239	0.509
h_{fv}(m)	0.022	0.072	0.387	0.152
$h_{f\,1-2}$(m)	1.065	3.651	21.589	8.077
Change in d	decrease	decrease	increase	O.K.

10.5.2 Parallel System of Pipes

If the pipe system causes the flow to branch into two or more lines, then it is referred as a **parallel system of pipes**. A typical parallel system of pipes is shown in Figure 10.20. The flow in the main line splits into three branches at section 1, and then rejoins at section 2. The head loss in each branch between sections 1 and 2 must be equal, that is

$$h_{f\,1-2} = h_{fa} = h_{fb} = h_{fc} \tag{10.20}$$

Also, one should observe that the volumetric flow rate in the main line is equal to the sum of the volumetric flow rates through each branch. Hence

$$Q = Q_a + Q_b + Q_c \tag{10.21}$$

Usually, two types of problems occur in parallel system of pipes. These are:

i) Class I Systems for which the head loss in each branch is known, and the volumetric flow rate in each branch and in the main line are to be determined.

ii) Class II Systems for which the total volumetric flow rate is known, and the volumetric flow rate and the head loss in each branch are to be determined.

10.5.2.1 Class I Systems

In Class I problems, the pressure drop or the head loss across the parallel branches is known, and it is desirable to determine the volumetric flow rates in each branch and in the main line.

The procedure for solving Class I problems may be given as follows:

i) Determine the relative roughness, ε_s/d, for each branch from Figure 10.4 by using the pipe diameter, d, and the pipe material.

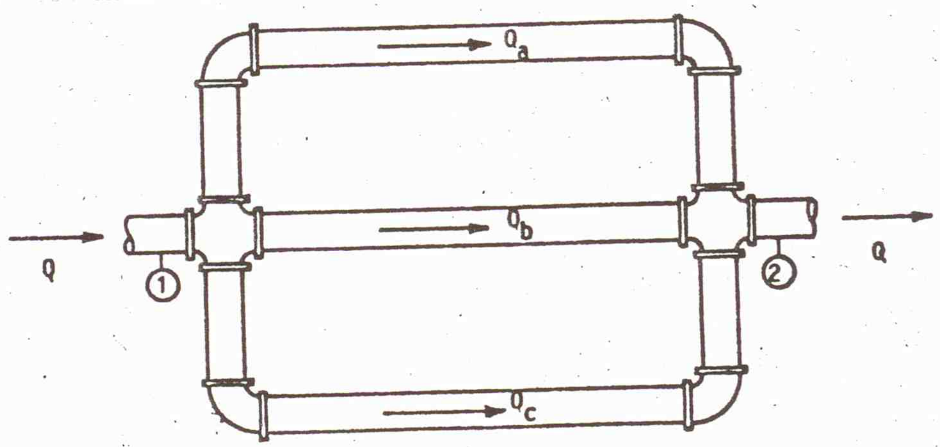


Figure 10.20 Parallel system of pipes

ii) Assume a friction factor, f, for each branch from the Moody diagram in Figure 10.5 by using the relative roughness, ε_s/d, in the fully rough region. The reason for the assumption of the flow in the fully rough region is that no iteration is necessary, if this assumption is true.

iii) Calculate the major head losses for each branch by using the Darcy-Weisbach equation, $h_f = f(L/d)(V^2/2g)$ in terms of the unknown branch velocities.

iv) Calculate the minor head losses for the variable area portions in each branch by using $h_f = kV^2/(2g)$ in terms of the unknown branch velocities. The head loss coefficient, k, can be determined by using Tables 10.1, 10.2, 10.3 and 10.4 and Figure 10.10.

v) Calculate the minor head losses for valves, fittings and bends in each pipe by using $h_f = f(L_e/d)(V^2/2g)$ in terms of the unknown branch velocities. The equivalent length ratio, L_e/d, can be determined by using Tables 10.5 and 10.6 and Figure 10.15.

vi) Evaluate the total head loss, h_f, of each branch by adding up the major and the minor head losses from steps (iii), (iv) and (v).

vii) Equate this head loss to the given head loss across the parallel branches and solve for the unknown branch velocities.

viii) Calculate the Reynolds number, $Re = \rho Vd/\mu$ for each branch.

ix) Determine the improved value of the friction factor, f, for each branch from the Moody diagram in Figure 10.5 by using the Reynolds number, Re, and the relative roughness, ε_s/d.

x) Compare the assumed and the improved values of the friction factor for each branch and repeat steps (iii) through (ix) as many times as needed in order to obtain the desired accuracy in the friction factor.

xi) Evaluate the volumetric flow rate of the fluid, $Q = \pi d^2 V/4$, in each branch and in the main line.

Example 10.4

The arrangement, which is shown in Figure 10.21, is used to supply lubricating oil to the bearing of a large machine. The bearings act as restrictions to the flow. The head loss coefficients for the journal bearings in branches x and y are 11 and 4 respectively. The length of 1" commercial steel branch is 10 m, while the length of 1 1/2" commercial steel branch y is 5 m. Each of the four bends in the tubing has a radius of 100 mm. The pressures before and after the branching are 275 kPa and 195 kPa respectively. The density and the absolute viscosity of the lubricating oil are 881 kg/m^3 and 2.2×10^{-3} Pa.s. Determine

a) the volumetric flow rate through each bearing, and
b) the total volumetric flow rate.

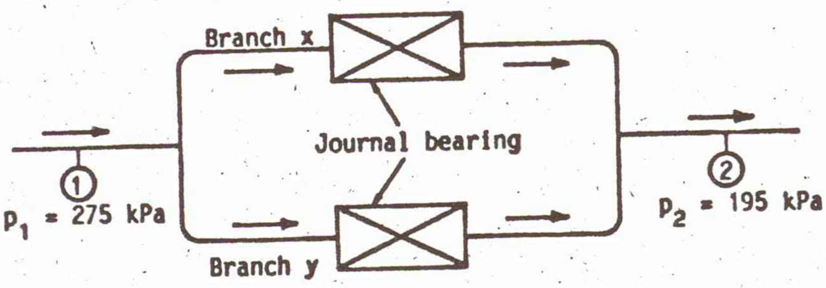


Figure 10.21 Sketch for Example 10.4

Solution

The head loss across the parallel branches may be determined by applying the extended Bernoulli equation between sections 2 and 1 in Figure 10.21 as

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 - h_{f1-2}$$

However, one should note that $V_1 = V_2$ and $z_1 = z_2$, so that the head loss between sections 1 and 2 may be evaluated as

$$h_{f1-2} = \frac{p_1 - p_2}{\rho g} = \frac{275000\ \text{N/m}^2 - 195000\ \text{N/m}^2}{(881\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = 9.26\ \text{m}$$

a) The volumetric flow rate through each branch may now be determined by applying the procedure which is presented in this section.

i) The relative roughness for the 1" and 1 1/2" commercial steel pipes may be obtained from Figure 10.4 as follows:

$$(\epsilon_s/d)_x = 0.0018$$

$$(\epsilon_s/d)_y = 0.0012$$

ii) The friction factors in branches x and y corresponding to relative roughness values of $(\epsilon_s/d)_x = 0.0018$ and $(\epsilon_s/d)_y = 0.0012$ in the fully rough flow region may be obtained from the Moody diagram in Figure 10.6 as

$$f_x = 0.0225$$

$$f_y = 0.02$$

iii) The major head loss in branches x and y may be evaluated by using the Darcy-Weisbach equation as

$$h_{fpx} = f_x \frac{L_x}{d_x} \frac{V_x^2}{2g} = (0.0225) \frac{(10\ m)}{(0.0254\ m)} \frac{V_x^2}{(2)(9.81\ m/s^2)}$$

$$= 0.452\ V_x^2$$

$$h_{fpy} = f_y \frac{L_y}{d_y} \frac{V_y^2}{2g} = (0.02) \frac{(5\ m)}{(0.0381\ m)} \frac{V_y^2}{(2)(9.81\ m/s^2)}$$

$$= 0.134\ V_y^2$$

iv) The minor head losses in the journal bearings are

$$h_{fjx} = k_{jx} \frac{V_x^2}{2g} = (11) \frac{V_x^2}{(2)(9.81\ m/s^2)} = 0.561\ V_x^2$$

$$h_{fjy} = k_{jy} \frac{V_y^2}{2g} = (4) \frac{V_y^2}{(2)(9.81\ m/s^2)} = 0.204\ V_y^2$$

v) The equivalent length ratio $(L_e/d)_t$ for standard tees with flow through the branch is 60 from Table 10.6. Then the minor head losses in branches x and y are

$$h_{ftx} = 2f_x (\frac{L_e}{d})_t \frac{V_x^2}{2g} = (2)(0.0225)(60) \frac{V_x^2}{(2)(9.81\ m/s^2)}$$

$$= 0.138\ V_x^2$$

$$h_{fty} = 2f_y (\frac{L_e}{d})_t \frac{V_y^2}{2g} = (2)(0.02)(60) \frac{V_y^2}{(2)(9.81\ m/s^2)}$$

$$= 0.122\ V_y^2$$

The relative radius of bends in branch x is $r/d_x = 0.10\ m/0.0254\ m = 3.94$, so that the equivalent length ratio is 14 from Figure 10.5. Then the minor head loss is

$$h_{fbx} = 2f_x (\frac{L_e}{d})_b \frac{V_x^2}{2g} = (2)(0.225)(14) \frac{V_x^2}{(2)(9.81\ m/s^2)}$$

$$= 0.032\ V_x^2$$

Similarly, the relative radius of bends in branch y is $r/d_y = 0.10\ m/0.0381\ m = 2.63$, so that the equivalent length ratio is 11.5 from Figure 10.5. Therefore, the minor head loss is

$$h_{fby} = 2f_y (\frac{L_e}{d})_d \frac{V_y^2}{2g} = (2)(0.02)(11.5) \frac{V_y^2}{(2)(9.81\ m/s^2)}$$

$$= 0.024\ V_y^2$$

vi) Then the head losses in branches x any y may be evaluated as

$$h_{fx} = h_{fpx} + h_{fjx} + h_{ftx} + h_{fbx}$$

$$= 0.452\ V_x^2 + 0.561\ V_x^2 + 0.138\ V_x^2 + 0.032\ V_x^2 = 1.183\ V_x^2$$

$$h_{fy} = h_{fpy} + h_{fjy} + h_{fty} + h_{fby}$$

$$= 0.134\ V_y^2 + 0.204\ V_y^2 + 0.122\ V_y^2 + 0.024\ V_y^2 = 0.484\ V_y^2$$

vii) The velocity in branch x is

$$V_x = \left(\frac{h_{fx}}{1.183}\right)^{1/2} = \left(\frac{9.26\ \text{m}}{1.183}\right)^{1/2} = 2.80\ \text{m/s}$$

and the velocity in branch y is

$$V_y = \left(\frac{h_{fy}}{0.484}\right)^{1/2} = \left(\frac{9.26\ \text{m}}{0.484}\right)^{1/2} = 4.37\ \text{m/s}$$

viii) Now, the Reynolds number in branches x and y may be calculated as

$$Re_x = \frac{\rho V_x d_x}{\mu} = \frac{(881\ \text{kg/m}^3)(2.80\ \text{m/s})(0.0254\ \text{m})}{(2.2\text{x}10^{-3}\ \text{N.s/m}^2)} = 2.85\text{x}10^4$$

$$Re_y = \frac{\rho V_y d_y}{\mu} = \frac{(881\ \text{kg/m}^3)(4.37\ \text{m/s})(0.0381\ \text{m})}{(2.2\text{x}10^{-3}\ \text{N.s/m}^2)} = 6.67\text{x}10^4$$

ix) Now, the improved value of the friction factor for branch x corresponding to a relative roughness of $(\epsilon_s/d)_x = 0.0018$ and a Reynolds number of $Re_x = 2.81\text{x}10^4$ from the Moody diagram in Figure 10.5 is

$$f_x = 0.028$$

Similarly, the improved value of the friction factor for branch y corresponding to a relative roughness of $(\epsilon_s/d)_y = 0.0012$ and a Reynolds of $Re_y = 6.67\text{x}10^4$ is

$$f_y = 0.024$$

x) As long as the assumed and the improved values of the friction factors are not the same, then the calculations in steps (iii) through (vii) must be repeated. The summary of these iterations is presented in Table 10.8.

xi) The volumetric flow rate through branches x and y may now be evaluated as

$$Q_x = \pi d_x^2 V_x/4 = (\pi)(0.0254\ \text{m})^2(2.64\ \text{m/s})/4 = \underline{1.34\text{x}10^{-3}\ \text{m}^3/\text{s}}$$

$$Q_y = \pi d_y^2 V_y/4 = (\pi)(0.0381\ \text{m})^2(4.14\ \text{m/s})/4 = \underline{4.72\text{x}10^{-3}\ \text{m}^3/\text{s}}$$

b) The volumetric flow rate through the main line is the sum of the volumetric flow rates in branches x and y, that is

$$Q = Q_x + Q_y = 1.34\times10^{-3}\ m^3/s + 4.72\times10^{-3}\ m^3/s = \underline{6.06\times10^{-3}\ m^3/s}$$

Table 10.8 Iterations for Example 10.4

Iteration	1	2
f_x(assumed)	0.0225	0.028
f_y(assumed)	0.02	0.024
h_{fpx}(m)	$0.452\ V_x^2$	$0.562\ V_x^2$
h_{fpy}(m)	$0.134\ V_y^2$	$0.161\ V_y^2$
h_{fjx}(m)	$0.561\ V_x^2$	$0.561\ V_x^2$
h_{fjy}(m)	$0.204\ V_y^2$	$0.204\ V_y^2$
h_{ftx}(m)	$0.138\ V_x^2$	$0.171\ V_x^2$
h_{fty}(m)	$0.122\ V_y^2$	$0.147\ V_y^2$
h_{fbx}(m)	$0.032\ V_x^2$	$0.040\ V_x^2$
h_{fby}(m)	$0.024\ V_y^2$	$0.028\ V_y^2$
h_{fx}(m)	$1.183\ V_x^2$	$1.334\ V_x^2$
h_{fy}(m)	$0.484\ V_y^2$	$0.540\ V_y^2$
V_x(m/s)	2.80	2.64
V_y(m/s)	4.37	4.14
Re_x	2.85×10^4	2.69×10^4
Re_y	6.67×10^4	6.32×10^4
f_x(improved)	0.028	0.028
f_y(improved)	0.024	0.024

10.5.2.2 Class II Systems

In Class II problems, the volumetric flow rate in the main line is known, and it is desirable to determine the volumetric flow rates in each branch and the head loss across the parallel branches.

The procedure for solving Class II problems may be given as follows

i) Determine the relative roughness, ε_s/d, for each branch from Figure 10.4 by using the pipe diameter, d, and the pipe material

ii) Assume a friction factor, f, for each branch from the Moody diagram in Figure 10.5 by using the relative roughness, ε_s/d in the fully rough region. The reason for the assumption of the flow in the fully rough region is that no iteration is necessary, if this assumption is true.

iii) Calculate the major head losses for each branch by using the Darcy-Weisbach equation, $h_f = f(L/d)(V^2/2g)$ in terms of the unknown branch velocities.

iv) Calculate the minor head losses for the variable area portions in each branch by using $h_f = kV^2/2g$ in terms of the unknown branch velocities. The head loss coefficient, k, can be determined from Tables 10.1, 10.2, 10.3, and 10.4 and Figure 10.10.

v) Calculate the minor head losses for valves, fittings and bends in each pipe by using $h_f = f(L_e/d)(V^2/2g)$ in terms of the unknown branch velocities. The equivalent length ratio, L_e/d, can be determined by using Tables 10.5 and 10.6, and Figure 10.15.

vi) Evaluate the total head loss, h_f, of each branch by adding up the major and the minor head losses from steps (iii), (iv) and (v).

vii) As long as the total head loss of each branch must be the same, then express all unknown branch velocities in terms of one unknown branch velocity.

viii) As long as the volumetric flow rate in the main line is the sum of the volumetric flow rates of each branch, then express it in terms of the unknown branch velocities.

ix) Solve for the unknown branch velocities.

x) Calculate the Reynolds number, $Re = \rho Vd/\mu$, for each branch.

xi) Determine the improved value of the friction factor, f, for each branch from the Moody diagram in Figure 10.5 by using the Reynolds number, Re, and the relative roughness, ε_s/d.

xii) Compare the assumed and the improved values of the friction factor for each branch and repeat steps (iii) through (xi) as many times as needed in order to obtain the desired accuracy in the friction factor.

xiii) Evaluate the volumetric flow rate of the fluid, $Q = \pi d^2 V/4$, in each branch.

Example 10.5

0.0063 m^3/s of water at 15°C is flowing in a 2" commercial steel pipe at section 1, as shown in Figure 10.22. The heat exchanger in branch x has a head loss coefficient of 12.5. Branch y is a bypass line, which is composed of 1 1/4" commercial steel pipe with a length of 6 m. All three valves are wide open, and the elbows are standard. Determine

a) the volumetric flow rate of water in each branch and

b) the pressure drop between points 1 and 2.

Solution

a) To determine the volumetric flow rate of water in each branch one may apply the procedure which is presented in this section.

i) The relative roughness for the 2" and 1 1/4" commercial steel pipes may be obtained from Figure 10.4 as follows:

$$(\varepsilon_s/d)_x = 0.0009$$

$$(\varepsilon_s/d)_y = 0.0014$$

ii) The friction factor in branches x and y corresponding to relative roughness values of $(\varepsilon_s/d)_x = 0.0009$ and $(\varepsilon_s/d)_y = 0.0014$ in the fully rough flow region may be obtained from the Moody diagram in Figure 10.5 as

$$f_x = 0.019$$

$$f_y = 0.021$$

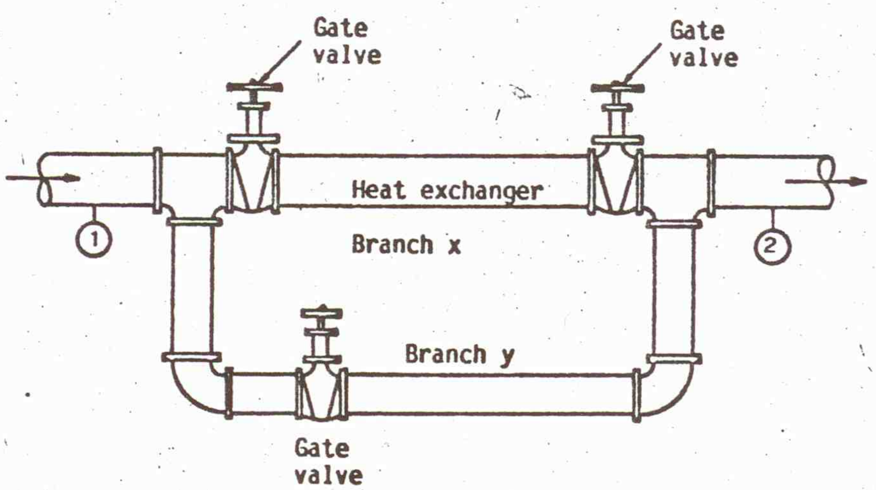


Figure 10.22 Sketch for Example 10.5

iii) The major head loss in branch y may be evaluated by using the Darcy-Weisbach equation as

$$h_{fpy} = f_y \frac{L_y}{d_y} \frac{V_y^2}{2g} = (0.021) \frac{(6\ m)}{(0.03175\ m)} \frac{V_y^2}{(2)(9.81\ m/s^2)}$$

$$= 0.202\ V_y^2$$

iv) The minor head loss for the heat exchanger is

$$h_{fhx} = k_h \frac{V_x^2}{2g} = (12.5) \frac{V_x^2}{(2)(9.81\ m/s^2)} = 0.637\ V_x^2$$

v) The equivalent length ratio $(L_e/d)_v$ for the fully open gate valve is 13 from Table 10.6. Then the minor head losses in branches x and y are

$$h_{fvx} = 2f_x(\frac{L_e}{d})_v \frac{V_x^2}{2g} = (2)(0.019)(13) \frac{V_y^2}{(2)(9.81\ m/s^2)}$$

$$= 0.025\ V_x^2$$

$$h_{fvy} = f_y(\frac{L_e}{d})_v \frac{V_y^2}{2g} = (0.021)(13) \frac{V_y^2}{(2)(9.81\ m/s^2)}$$

$$= 0.014\ V_y^2$$

The equivalent length ratios $(L_e/d)_{t1}$ and $(L_e/d)_{t2}$ for standard tees with flow through run and with flow through branch are 20 and 60 respectively, so that the minor head losses in branches x and y are

$$h_{ftx} = 2f_x(\frac{L_e}{d})_{t1} \frac{V_x^2}{2g} = (2)(0.019)(20) \frac{V_x^2}{(2)(9.81\ m/s^2)}$$

$$= 0.039\ V_x^2$$

$$h_{fty} = 2f_y(\frac{L_e}{d})_{t2} \frac{V_y^2}{2g} = (2)(0.021)(60) \frac{V_y^2}{(2)(9.81\ m/s^2)}$$

$$= 0.128\ V_y^2$$

Finally, the equivalent length ratio $(L_e/d)_e$ for the standard elbow is 30, so that the minor head loss in branch y is

$$h_{fey} = 2f_y(\frac{L_e}{d})_e \frac{V_y^2}{2g} = (2)(0.021)(30) \frac{V_y^2}{(2)(9.81\ m/s^2)}$$

$$= 0.064\ V_y^2$$

vi) Now, the total head losses in branches x and y are

$$h_{fx} = h_{fhx} + h_{fvx} + h_{ftx}$$

$$= 0.637\ V_x^2 + 0.025\ V_x^2 + 0.039\ V_x^2 = 0.701\ V_x^2.$$

$$h_{fy} = h_{fpy} + h_{fvy} + h_{fty} + h_{fey}$$

$$= 0.202\ V_y^2 + 0.014\ V_y^2 + 0.128\ V_y^2 + 0.064\ V_y^2 = 0.408\ V_y^2$$

vii) As long as the total head loss in each branch must be equal then

$$h_{fx} = h_{fy} = 0.701\ V_x^2 = 0.408\ V_y^2$$

or

$$V_x = 0.763\ V_y$$

viii) The volumetric flow rate in the main line may now be expressed as

$$Q = \pi\, d_x^2 V_x/4 + \pi\, d_y^2 V_y/4$$

Now, the numerical valves may now be substituted to obtain

$$(\pi)(0.0508\ \text{m})^2 V_x/4 + (\pi)(0.03175\ \text{m})^2 V_y/4 = 0.0063\ \text{m}^3/\text{s}$$

or

$$2.027\ V_x + 0.792\ V_y = 6.3$$

ix) The velocities in branches x and y may now be evaluated as

$$V_x = 2.05\ \text{m/s}$$

$$V_y = 2.69\ \text{m/s}$$

x) The kinematic viscosity of water at 15°C is $1.139\text{x}10^{-6}\ \text{m}^3/\text{s}$ from Table A.2. Then the Reynolds number in branches x and y may now be evaluated as

$$Re_x = \frac{V_x d_x}{\nu} = \frac{(2.05\ \text{m/s})(0.0508\ \text{m})}{(1.139\text{x}10^{-6}\ \text{m}^2/\text{s})} = 9.14\text{x}10^4$$

$$Re_y = \frac{V_y d_y}{\nu} = \frac{(2.69\ \text{m/s})(0.03175\ \text{m})}{(1.139\text{x}10^{-6}\ \text{m}^2/\text{s}} = 7.50\text{x}10^4$$

xi) Now, the improved value of the friction factor for branch x corresponding to a relative roughness of $(\varepsilon_s/d)_x = 0.0009$ and a Reynolds number of $Re_x = 9.19\times10^4$ from the Moody diagram in Figure 10.5 is

$$f_x = 0.022$$

Similarly, the improved value of the friction factor for branch y corresponding to a relative roughness of $(\varepsilon_s/d)_y = 0.0014$ and a Reynolds number of $Re_y = 7.50\times10^4$ is

$$f_y = 0.024$$

xii) As long as the assumed and the improved values of the friction factors are not the same, then the calculations in steps (iii) through (xii) must be repeated. The summary of these iterations is presented in Table 10.9.

xiii) The volumetric flow rate through branches x and y may now be evaluated as

$$Q_x = \pi d_x^2 V_x/4 = (\pi)(0.0508\ \text{m})^2(2.10\ \text{m/s})/4 = \underline{4.26\times10^{-3}\ \text{m}^3/\text{s}}$$

$$Q_y = \pi d_y^2 V_y/4 = (\pi)(0.03175\ \text{m})^2(2.59\ \text{m/s})/4 = \underline{2.05\times10^{-3}\ \text{m}^3/\text{s}}$$

b) The pressure drop between sections 1 and 2 may be evaluated by using the extended Bernoulli equation as

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2} + z_2 = \frac{p_1}{\rho g} + \frac{V_1^2}{2} + z_1 - h_{f1-2}$$

At this point, one should note that $V_1 = V_2$ and $z_1 = z_2$. Also the frictional head loss between sections 1 and 2 is

$$h_{f1-2} = h_{fx} = h_{fy} = 0.711\ V_x^2 = (0.711)(2.10\ \text{m/s})^2 = 3.14\ \text{m}$$

The density of water is 999.1 kg/m^3 from Table A.2, therefore

$$p_1 - p_2 = \rho g h_{f1-2} = (999.1\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3.14\ \text{m})$$

$$= \underline{30.78\ \text{kPa}}$$

Table 10.9 Iterations for Example 10.5

Iteration	1	2
f_x(assumed)	0.019	0.022
f_y(assumed)	0.021	0.024
h_{fpy}(m)	0.202 V_y^2	0.231 V_y^2
h_{fhx}(m)	0.637 V_x^2	0.637 V_x^2
h_{fvx}(m)	0.025 V_x^2	0.029 V_x^2
h_{fvy}(m)	0.014 V_y^2	0.016 V_y^2
h_{ftx}(m)	0.039 V_x^2	0.045 V_x^2
h_{fty}(m)	0.128 V_y^2	0.147 V_y^2
h_{fey}(m)	0.064 V_y^2	0.073 V_y^2
h_{fx}(m)	0.701 V_x^2	0.711 V_x^2
h_{fy}(m)	0.408 V_y^2	0.467 V_y^2
V_x(m/s)	2.05	2.10
V_y(m/s)	2.69	2.59
Re_x	9.14×10^4	9.37×10^4
Re_y	7.50×10^4	7.22×10^4
f_x(improved)	0.022	0.022
f_y(improved)	0.024	0.024

10.5.3 Pipe Networks

In municipal water distribution systems, the pipelines are frequently branching and looping in a complex manner which is often referred as a pipe network. A simple pipe network is shown in Figure 10.23. The most important factor which makes the analysis of pipe networks difficult, is the uncertainity about the direction of the flow in different pipes one should note that, it is almost impossible to tell the direction of the flow, such as in pipe c of Figure 10.23.

The fluid flow in a pipe network must satisfy the basic principles of the conservation of mass and the conservation of energy. These principles can be summarized as follows

i) At each junction of the pipe network, the sum of the volumetric flow rates into the junction must be equal to the sum of the volumetric flow rates out of the junction.

ii) The algebraic sum of the head losses around any closed loop must be zero.

The pipe network problems are generally solved by a trial-and-error procedure. The most practical and widely used method of flow analysis in pipe networks is that of successive approximations, which is developed by Hardy Cross. In the solution procedure, which is outlined below, the pipe network should be divided into several closed loops

i) Determine the relative roughness, ϵ_s/d, for each pipe from Figure 10.4 by using the pipe diameter, d, and the pipe material

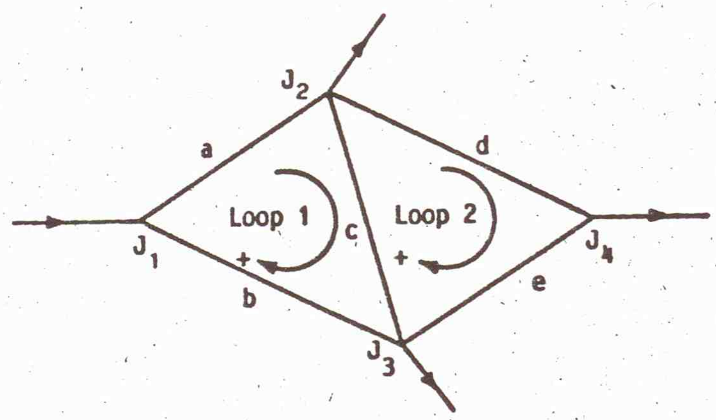


Figure 10.23 A simple pipe network

ii) Assume a friction factor, f, for each pipe from the Moody diagram in Figure 10.5 by using the relative roughness, ϵ_s/d in the fully rough flow region. The reason for the assumption of the flow in the fully roughn region is that no iteration is necessary. if this assumption is true

iii) Express the total head loss, h_f, for each pipe in the form of

$$h_f = KQ^2 \tag{10.22}$$

where K is the equivalent resistance to the flow in the pipe, and Q is the unknown volumetric flow rate through the pipe.

iv) Assume a value for the volumetric flow rate in each pipe such that the sum of the volumetric flow rates into each junction is equal to the sum of the volumetric flow rates out of the junction. Note that the fluid tends to follow the path of least resistance through the pipe network, so that the pipes having a high values of K have relatively low volumetric flow rates.

v) Determine the algebraic sum of total head losses, Σh_f, for each closed loop by using the following sign convention. If the flow is clockwise, then the total head loss and the volumetric flow rate are positive. However, if the flow is counterclockwise, then the total head loss and the volumetric flow rate are negative.

vi) Determine the arithmetic sum of 2KQ, that is $\Sigma(2KQ)$ for each closed loop. During this summation, consider all 2KQ values as positive.

vii) Calculate the correction for the volumetric flow rate, ΔQ, for each loop from

$$\Delta Q = \frac{h_f}{\Sigma(2KQ)} \tag{10.23}$$

viii) Calculate the improved values of the volumetric flow rate, Q', by using

$$Q' = Q - \Delta Q \tag{10.24}$$

ix) Repeat calculations in steps (v) through (viii) until ΔQ from step (vii) becomes negligibly small. The Q' value is used for the next cycle of iteration.

x) Calculate the average velocity, $V = 4Q/(\pi d^2)$ for each pipe.

xi) Calculate the Reynolds number, $Re = \rho Vd/\mu$ for each pipe.

xii) Determine the improved value of the friction factor, f, for each pipe from the Moody diagram in Figure 10.5 by using the Reynolds number, Re, and the relative roughness, ε_s/d.

xiii) Compare the assumed and the improved values of the friction factor for each pipe and repeat steps (iii) through (xii) as many times as needed in order to obtain the desired accuracy in the friction factor.

Example 10.6

Water at 20°C is flowing through the pipe network, as shown in Figure 10.24. All pipes are 1" cast iron, and the elbows are standard. Determine the volumetric flow rate of water in pipes a,b,c,d and e.

Solution

The volumetric flow rate in each pipe of the network may be determined by applying the procedure which is presented in this section as follows:

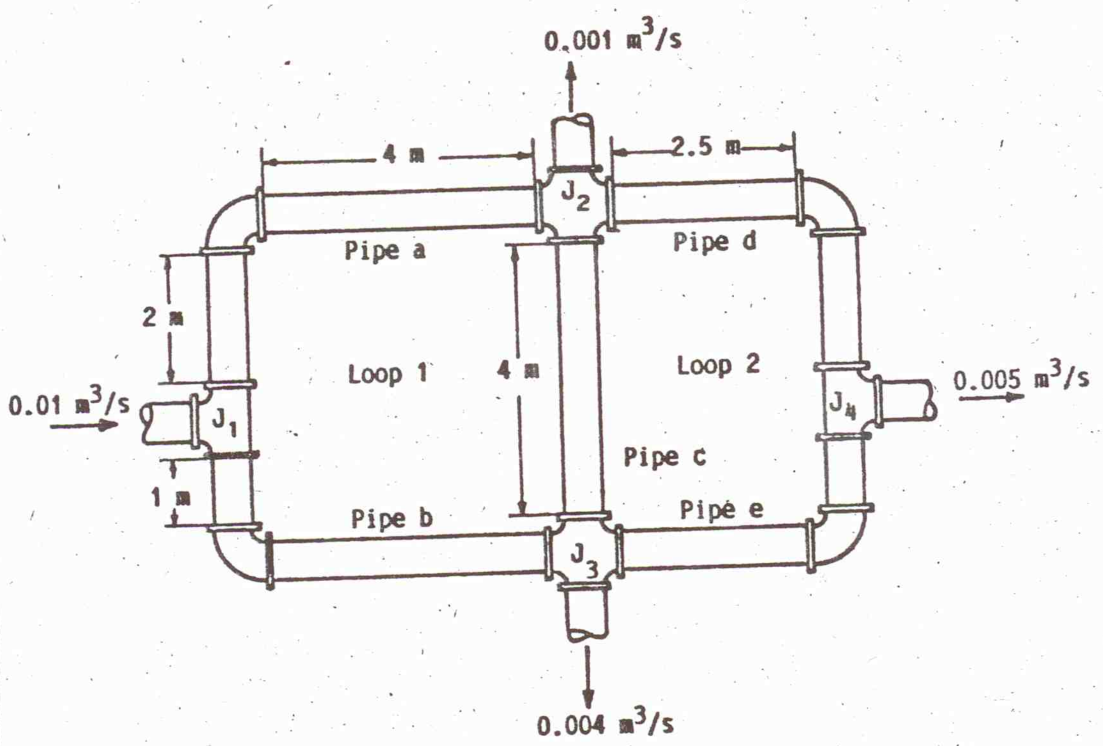


Figure 10.24 Sketch for Example 10.6

i) The relative roughness for the 1" cast iron pipes may be obtained from Figure 10.4 as

$$\varepsilon_s/d = 0.01$$

ii) The friction factor corresponding to a relative roughness of $\varepsilon_s/d = 0.01$ in the fully rough flow region may be obtained from the Moody diagram in Figure 10.5 as

$$f = 0.038$$

iii) As long as the equivalent length ratios for the standard elbow, the standard tee with flow through run and the standard tee with flow through branch are $(L_e/d)_e = 30$, $(L_e/d)_{t1} = 20$ and (L_e/d) : 60 respectively, the total head loss in pipes a,b,c,d and [illegible] be expressed as

$$h_{fa} = f[\frac{L_a}{d} + (\frac{L_e}{d})_{t2} + (\frac{L_e}{d})_e + (\frac{L_e}{d})_{t1}]\frac{1}{2g}[\frac{4Q_a}{\pi d^2}]^2$$

$$= (0.038)[\frac{6\ m}{0.0254\ m} + 60 + 30 + 20]\frac{1}{(2)(9.81\ m/s^2)}$$

$$x[\frac{4Q_a}{(\pi)(0.0254\ m)^2}]^2 = 2.61x10^6\ Q_a^2$$

$$h_{fb} = f[\frac{L_b}{d} + (\frac{L_e}{d})_{t2} + (\frac{L_e}{d})_e + (\frac{L_e}{d})_{t1}]\frac{1}{2g}[\frac{4Q_b}{\pi d^2}]^2$$

$$= (0.038)[\frac{5\ m}{0.0254\ m} + 60 + 30 + 20]\frac{1}{(2)(9.81\ m/s^2)}$$

$$x[\frac{4Q_b}{(\pi)(0.0254\ m)^2}] = 2.32x10^6\ Q_b^2$$

$$h_{fc} = f[\frac{L_c}{d} + 2(\frac{L_e}{d})_{t2}\frac{1}{2g}[\frac{4Q_c}{\pi d^2}]^2$$

$$= (0.038)[\frac{4\ m}{0.0254\ m} + 2(60)]\frac{1}{(2)(9.81\ m/s^2)}[\frac{4Q_c}{(\pi)(0.0254\ m)^2}]^2$$

$$= 2.09x10^6\ Q_c^2$$

$$h_{fd} = f[\frac{L_d}{d} + (\frac{L_e}{d})_{t1} + (\frac{L_e}{d})_e + (\frac{L_e}{d})_{t2}]\frac{1}{2g}[\frac{4Q_b}{\pi d^2}]^2$$

$$= (0.038)[\frac{4.5\ m}{0.0254\ m} + 20 + 30 + 60]\frac{1}{(2)(9.81\ m/s^2)}$$

$$x[\frac{4Q_d}{(\pi)(0.0254\ m)^2}]^2 = 2.17x10^6\ Q_d^2$$

$$h_{fe} = f[\frac{L_e}{d} + (\frac{L_e}{d})_{t1} + (\frac{L_e}{d})_e + (\frac{L_e}{d})_{t2}]\frac{1}{2g}[\frac{4Q_e}{\pi d^2}]^2$$

$$= (0.038)[\frac{3.5\ m}{0.0254\ m} + 20 + 30 + 60]\frac{1}{(2)(9.81\ m/s^2)}$$

$$x[\frac{4Q_e}{(\pi)(0.0254\ m)^2}]^2 = 1.87x10^6\ Q_e^2$$

iv) As long as the K value for pipe a is larger than the K value for pipe b, then the volumetric flow rate in pipe b will be larger. The continuity equation at junction J yields

$$Q_a + Q_b = 0.01\ m^3/s$$

therefore it is possible to assume Q_a and Q_b as

$$Q_a = 0.004\ m^3/s$$

$$Q_b = 0.006\ m^3/s$$

Now, the continuity equation may be applied to junction J2 as

$$Q_c + Q_d = Q_a - 0.001\ m^3/s = 0.003\ m^3/s$$

when the flow in pipe C is assumed to proceed from junction J2 to junction J3. Since the K value for pipe C is smaller than the K value in pipe d, then it is probable that the volumetric flow rate in pipe c will be larger. Hence, Q_c and Q_d may be assumed as

$$Q_c = 0.002\ m^3/s$$

$$Q_d = 0.001\ m^3/s$$

Finally, the continuity equation for junction J_3 is

$$Q_e = Q_b + Q_c - 0.004\ m^3/s = 0.004\ m^3/s$$

v) The sum of the total head losses in loop 1 may now be calculated as

$$\Sigma h_{f1} = h_a + h_c - h_b = K_aQ_a^2 + K_cQ_c^2 - K_bQ_b^2 = (2.61\times10^6)(0.004)^2$$
$$+ (2.09\times10^6)(0.002)^2 - (2.32\times10^6)(0.006)^2 = -33.4\ m$$

Similarly, the sum of the head losses in loop 2 is

$$\Sigma h_{f2} = h_d - h_e - h_c = K_dQ_d^2 - K_eQ_e^2 - K_cQ_c^2 = (2.17\times10^6)(0.001)^2$$
$$- (1.87\times10^6)(0.004)^2 - (2.09\times10^6)(0.002)^2 = -36.11\ m$$

vi) The sum of the values of 2KQ for loop 1 is

$$\Sigma(2KQ)_1 = 2[K_aQ_a + K_cQ_c + K_bQ_b] = 2[(2.61\times10^6)(0.004)$$
$$+ (2.09\times10^6)(0.002) + (2.32\times10^6)(0.006)] = 57080\ s/m^2$$

Similarly, the sum of the values of 2KQ for loop 2 is

$$\Sigma(2KQ)_2 = 2[K_dQ_d + K_eQ_e + K_cQ_c] = 2[(2.17\times10^6)(0.001)$$
$$+ (1.87\times10^6)(0.004) + (2.09\times10^6)(0.002)] = 27640\ s/m2$$

vii) The estimates of error in the assumed values of the volumetric flow rates for loops 1 and 2 are

$$\Delta Q_1 = \frac{\Sigma h_1}{\Sigma(2KQ)_1} = \frac{-33.4\ m}{57080\ s/m^2} = -0.59\times10^{-3}\ m^3/s$$

$$\Delta Q_2 = \frac{\Sigma h_2}{\Sigma(2KQ)_2} = \frac{-36.11\ m}{27640\ s/m^2} = -1.31\times10^{-3}\ m^3/s$$

viii) The volumetric flow rates in pipes a, b and c of loop 1 may now be corrected as

$$Q'_a = Q_a - \Delta Q_1 = 4\times10^{-3}\ m^3/s - (-0.59\times10^{-3}\ m^3/s)$$

$$= 4.59\times10^{-3}\ m^3/s$$

$$Q'_b = Q_b - \Delta Q_1 = -6\times10^{-3}\ m^3/s - (-0.59\times10^{-3}\ m^3/s)$$

$$= -5.41\times10^{-3}\ m^3/s$$

$$Q'_c = Q_c - \Delta Q_1 = 2\times10^{-3}\ m^3/s - (-0.59\times10^{-3}\ m^3/s)$$

$$= 2.59\times10^{-3}\ m^3/s$$

Similarly, the correction may be applied to the volumetric flow rates of pipes c, d and e as

$$Q''_c = Q'_c - \Delta Q_2 = -2.59\times10^{-3}\ m^3/s - (-1.31\times10^{-3}\ m^3/s)$$

$$= -1.28\times10^{-3}\ m^3/s$$

$$Q'_d = Q_d - \Delta Q_2 = 1\times10^{-3}\ m^3/s - (-1.31\times10^{-3}\ m^3/s)$$

$$= 2.31\times10^{-3}\ m^3/s$$

$$Q'_e = Q_e - \Delta Q_2 = -4.10\times10^{-3}\ m^3/s - (-1.31\times10^{-3}\ m^3/s)$$

$$= -2.69\times10^{-3}\ m^3/s$$

ix) In order to make ΔQ_1 and ΔQ_2 negligibly small, it is necessary to repeat the calculations. These iterations are presented in Table 10.10.

x) The velocity in each pipe may now be evaluated as

$$V_a = \frac{4Q_a}{\pi d^2} = \frac{(4)(4.79\times10^{-3}\ m^3/s)}{(\pi)(0.0254\ m)^2} = 9.45\ m/s$$

$$V_b = \frac{4Q_b}{\pi d^2} = \frac{(4)(5.21\times10^{-3}\ m^3/s)}{(\pi)(0.0254\ m)^2} = 10.28\ m/s$$

$$V_c = \frac{4Q_c}{\pi d^2} = \frac{(4)(1.23\text{x}10^{-3}\ m^3/s)}{(\pi)(0.0254\ m)^2} = 2.43\ m/s$$

$$V_d = \frac{4Q_d}{\pi d^2} = \frac{(4)(2.56\text{x}10^{-3}\ m^3/s)}{(\pi)(0.0254\ m)^2} = 5.05\ m/s$$

$$V_e = \frac{4Q_e}{\pi d^2} = \frac{(4)(2.44\text{x}10^{-3}\ m^3/s)}{(\pi)(0.0254\ m)^2} = 4.82\ m/s$$

xi) The kinematic viscosity of water at 20^oC is $1.003\text{x}10^{-6}$ m^2/s from Table A.2, so that the Reynolds number for each pipe may be calculated as

$$Re_a = \frac{V_a d}{\nu} = \frac{(9.45\ m/s)(0.0254\ m)}{(1.003\text{x}10^{-6}\ m^2/s)} = 2.39\text{x}10^5$$

$$Re_b = \frac{V_b d}{\nu} = \frac{(10.28\ m/s)(0.0254\ m)}{(1.003\text{x}10^{-6}\ m^2/s)} = 2.60\text{x}10^5$$

$$Re_c = \frac{V_c d}{\nu} = \frac{(2.43\ m/s)(0.0254\ m)}{(1.003\text{x}10^{-6}\ m^2/s)} = 6.15\text{x}10^4$$

$$Re_d = \frac{V_d d}{\nu} = \frac{(5.05\ m/s)(0.0254\ m)}{(1.003\text{x}10^{-6}\ m^2/s)} = 1.28\text{x}10^5$$

$$Re_e = \frac{V_e d}{\nu} = \frac{(4.82\ m/s)(0.0254\ m)}{(1.003\text{x}10^{-6}\ m^2/s)} = 1.22\text{x}10^5$$

xii) The friction factor corresponding to a relative roughness value of ε_s/d = 0.01 and Reynolds numbers in the range $6.15\text{x}10^4$ to $2.60\text{x}10^5$ may be determined from the Moody diagram in Figure 10.5 as

$$f = 0.038$$

xiii) As long as the assumed and the improved values of the friction factor are the same, then there is no need to iterate.

Consequently, the volumetric flow rate in each pipe may be given as follows:

$$Q_a = \underline{4.79\text{x}10^{-3}\ m^3/s}$$

$$Q_b = \underline{5.21\text{x}10^{-3}\ m^3/s}$$

$$Q_c = \underline{1.23\text{x}10^{-3}\ m^3/s}$$

$$Q_d = \underline{2.56\text{x}10^{-3}\ m^3/s}$$

$$Q_e = \underline{2.44\text{x}10^{-3}\ m^3/s}$$

Table 10.10 Iterations for Example 10.6

Iteration	1	2	3	4
$Q_a (m^3/s)$	4.00×10^{-3}	4.59×10^{-3}	4.76×10^{-3}	4.79×10^{-3}
$Q_b (m^3/s)$	-6.00×10^{-3}	-5.41×10^{-3}	-5.24×10^{-3}	-5.21×10^{-3}
$Q_c (m^3/s)$	2.00×10^{-3}	1.28×10^{-3}	1.24×10^{-3}	1.23×10^{-3}
$Q_c (m^3/s)$	-2.00×10^{-3}	-1.28×10^{-3}	-1.24×10^{-3}	-1.23×10^{-3}
$Q_d (m^3/s)$	1.00×10^{-3}	2.31×10^{-3}	2.52×10^{-3}	2.56×10^{-3}
$Q_e (m^3/s)$	-4.00×10^{-3}	-2.69×10^{-3}	-2.48×10^{-3}	-2.44×10^{-3}
$\Sigma h_1 (m)$	-33.40	-9.49	-1.35	-0.07
$\Sigma h_2 (m)$	-36.11	-5.38	-0.93	-0.07
$\Sigma(2KQ)_1 (s/m^2)$	57080	54413	54344	54320
$\Sigma(2KQ)_2 (s/m^2)$	27640	25436	25395	25377
$\Delta Q_1 (m^3/s)$	-0.59×10^{-3}	-0.17×10^{-3}	-0.03×10^{-3}	-0.001×10^{-3}
$\Delta Q_2 (m^3/s)$	-1.31×10^{-3}	-0.21×10^{-3}	-0.04×10^{-3}	-0.003×10^{-3}
$Q'_a (m^3/s)$	4.59×10^{-3}	4.76×10^{-3}	4.79×10^{-3}	4.79×10^{-3}
$Q'_b (m^3/s)$	-5.41×10^{-3}	-5.24×10^{-3}	-5.21×10^{-3}	-5.21×10^{-3}
$Q'_c (m^3/s)$	2.59×10^{-3}	1.45×10^{-3}	1.27×10^{-3}	1.23×10^{-3}
$Q''_c (m^3/s)$	-1.28×10^{-3}	-1.24×10^{-3}	-1.23×10^{-3}	-1.23×10^{-3}
$Q'_d (m^3/s)$	2.31×10^{-3}	2.52×10^{-3}	2.56×10^{-3}	2.56×10^{-3}
$Q'_e (m^3/s)$	-2.69×10^{-3}	-2.48×10^{-3}	-2.44×10^{-3}	-2.44×10^{-3}

10.5.4 Interconnected Reservoir Systems

Another example of pipe systems, which has practical importance in water supply systems is when three or more reservoirs at various elevations are interconnected at a joint. Figure 10.24 shows three interconnected reservoirs. Reservoir A is at the highest elevation, reservoir B is at the intermediate elevation and reservoir C is at the lowest elevation. Three pipes 1, 2 and 3 meet at a common junction J, and Q_1, Q_2 and Q_3 are the volumetric flow rates in the respective pipes. It is obvious that the flow takes place from the highest

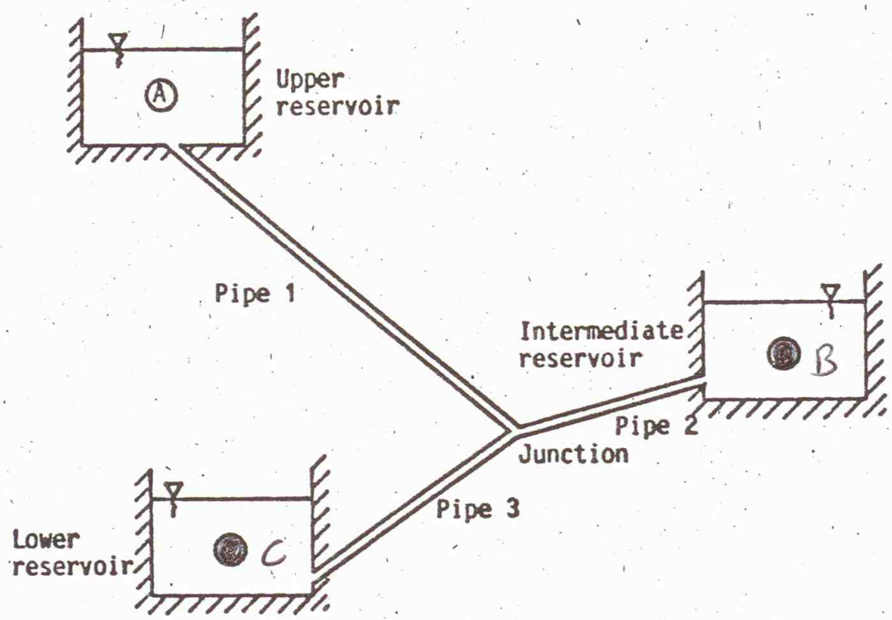


Figure 10.25 Three interconnected reservoirs

reservoir, that is reservoir A, towards the junction, and then from the junction to the lowermost reservoir, that is reservoir C. However, the direction of the flow in the second pipe is still uncertain. If the total head at the free surface of reservoir B is greater than the total head at junction J, then the flow takes place from reservoir B towards junction J. In this case, the continuity equation at junction J becomes

$$Q_1 + Q_2 = Q_3 \qquad \text{when} \qquad h_{tB} > h_{tJ} \tag{10.24}$$

However, when the total head at the free surface of the reservoir is smaller than the total head at junction J, then the flow takes place from junction J towards reservoir B. Hence, the continuity equation at junction J is

$$Q_1 = Q_2 + Q_3 \qquad \text{when} \qquad h_{tB} < h_{tJ} \tag{10.25}$$

The procedure for determining the volumetric flow rate for each pipe in the interconnected reservoir system may be given as follows:

i) Calculate the total head, h_t, at the free surface of each reservoir.

ii) Determine the relative roughness, ε_s/d, for each pipe from Figure 10.4 by using the pipe diameter, d, and the pipe material.

iii) Assume a friction factor, f, for each pipe from the Moody diagram in Figure 10.5 by using the relative roughness, ε_s/d, in the fully rough flow region. The reason for the assumption of the flow in the fully rough region is that no iteration is necessary if this assumption is true.

iv) Express the total head loss, h_f, for each pipe in the form of $h_f = KQ^2$, where K is the equivalent resistance to the flow and Q is the unknown volumetric flow rate through the pipe.

v) Assume the total head for the junction, and determine the direction of flow in pipes which are connected to intermediate reservoirs. Write the continuity equation for the junction.

vi) Write the extended Bernoulli equation along the streamlines between the free surface of each reservoir and the junction.

vii) Determine the volumetric flow rate, Q, for each pipe.

viii) Determine whether the continuity equation is satisfied at the junction or not. If the continuity equation is not satisfied, then repeat the calculations in steps (v) through (vii) as many times as necessary.

ix) Calculate the average velocity, $V = 4Q/(\pi d^2)$ for each pipe.

x) Calculate the Reynolds number, $Re = \rho Vd/\mu$ for each pipe.

xi) Determine the improved value of the friction factor, f, for each pipe from the Moody diagram in Figure 10.5 by using the Reynolds number, Re, and the relative roughness, ε_s/d.

xii) Compare the assumed and the improved values of the friction factor for each pipe and rereat steps (iv) through (xi) as many times as needed in order to obtain the desired accuracy in the friction factor.

Example 10.7

Water at 20^{o}C is flowing through the three reservoir system, which is shown in Figure 10.26. All pipes are cast iron. Neglecting all minor head losses, determine the volumetric flow rate for each pipe.

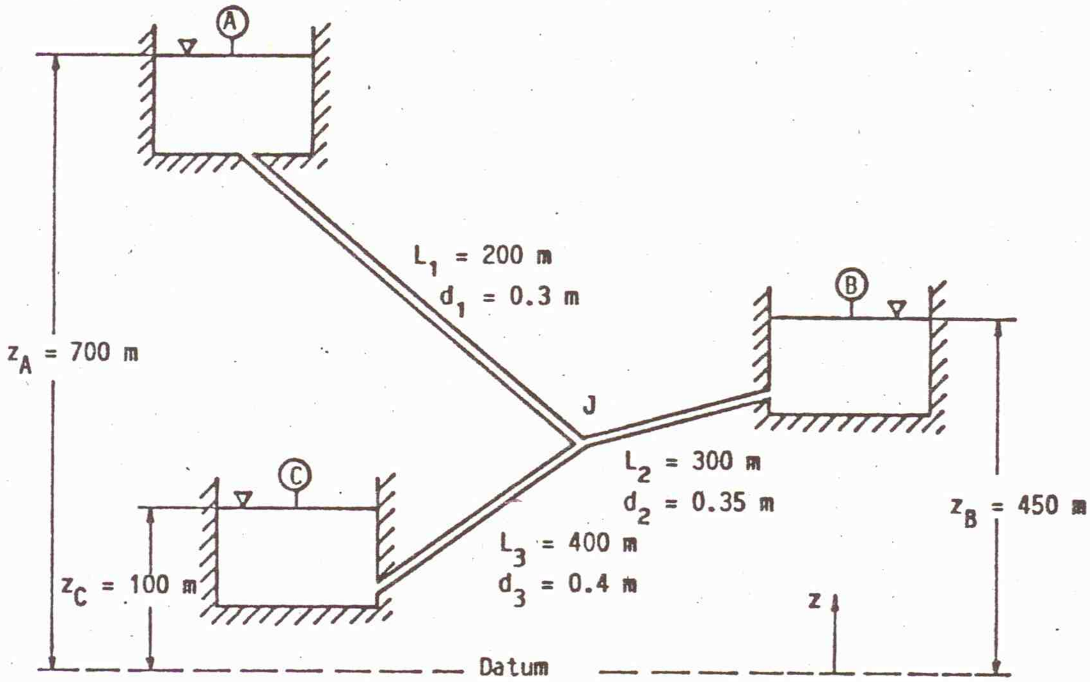


Figure 10.26 Sketch for Example 10.7

Soultion

i) As long as the cross-sectional area of the reservoirs are very large when compared to the cross-sectional area of pipes, then the velocities at the free surface of three reservoirs are negligible, that is V_A = 0, V_B = 0 and V_C = 0. Therefore, the total head at the free surface of reservoirs are

$$h_{tA} = \frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{101325\ \text{N/m}^2}{(998.2\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + 700\ \text{m}$$

$$= 710.35\ \text{m}$$

$$h_{tB} = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B = \frac{101325\ \text{N/m}^2}{(998.2\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + 450\ \text{m}$$

$$= 460.35\ \text{m}$$

$$h_{tC} = \frac{p_C}{\rho g} + \frac{V_C^2}{2g} + z_C = \frac{101325\ \text{N/m}^2}{(998.2\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} + 100\ \text{m}$$

$$= 110.35\ \text{m}$$

ii) The relative roughness for cast iron pipes with diameters of $d_1 = 0.3$ m, $d_2 = 0.35$ m and $d_3 = 0.4$ m may be obtained from Figure 10.4 as follows:

$$(\varepsilon_s/d)_1 = 0.00084$$

$$(\varepsilon_s/d)_2 = 0.00072$$

$$(\varepsilon_s/d)_3 = 0.00062$$

iii) The friction factors corresponding to relative roughness values of $(\varepsilon_s/d)_1 = 0.00084$, $(\varepsilon_s/d)_2 = 0.00072$ and $(\varepsilon_s/d)_3 = 0.00062$ in the fully rough region are

$$f_1 = 0.0185$$

$$f_2 = 0.018$$

$$f_3 = 0.0175$$

from the Moody diagram in Figure 10.5.

iv) As long as the minor head losses are neglected, then the total head loss for pipes 1, 2 and 3 may be evaluated from the Darcy-Weisbach equation as follows:

$$h_{f1} = f_1 \frac{L_1}{d_1} \frac{1}{2g}\left[\frac{4Q_1}{\pi d_1^2}\right]^2$$

$$= (0.0185)\left(\frac{200 \text{ m}}{0.3 \text{ m}}\right)\frac{1}{(2)(9.81 \text{ m/s}^2)}\left[\frac{4Q_1}{(\pi)(0.3 \text{ m})^2}\right]^2 = 125.81\ Q_1^2$$

$$h_{f2} = f_2 \frac{L_2}{d_2} \frac{1}{2g}\left[\frac{4Q_2}{\pi d_2^2}\right]^2$$

$$= (0.018)\left(\frac{300 \text{ m}}{0.35 \text{ m}}\right)\frac{1}{(2)(9.81 \text{ m/s}^2)}\left[\frac{4Q_2}{(\pi)(0.35 \text{ m})^2}\right]^2 = 84.95\ Q_2^2$$

$$h_{f3} = f_3 \frac{L_3}{d_3} \frac{1}{2g}\left[\frac{4Q_3}{\pi d_3^2}\right]^2$$

$$= (0.0175)\left(\frac{400 \text{ m}}{0.4 \text{ m}}\right)\frac{1}{(2)(9.81 \text{ m/s}^2)}\left[\frac{4Q_3}{(\pi)(0.4 \text{ m})^2}\right]^2 = 56.48\ Q_3^2$$

v) The total head at junction J may now be assumed as 400 m, that is $h_{tJ} = 400$ m. In this case,

$$h_{tA} > h_{tB} > h_{tJ} > h_{tC}$$

and water is flowing from reservoirs A and B into reservoir C. As a result, the continuity equation at junction J becomes

$$Q_1 + Q_2 = Q_3$$

vi) The extended Bernoulli equation may now be applied along the streamlines between the free surfaces of three reservoirs and junction J as

$$h_{tA} - h_{tJ} = h_{f1}$$

$$h_{tB} - h_{tJ} = h_{f2}$$

$$h_{tJ} - h_{tC} = h_{f3}$$

vii) The volumetric flow rate in each pipe may now be determined as

$$Q_1 = \left(\frac{h_{tA} - h_{tJ}}{125.81}\right)^{1/2} = \left(\frac{710.35\ \text{m} - 400\ \text{m}}{125.81}\right)^{1/2} = 1.57\ \text{m}^3/\text{s}$$

$$Q_2 = \left(\frac{h_{tB} - h_{tJ}}{84.95}\right)^{1/2} = \left(\frac{460.35\ \text{m} - 400\ \text{m}}{84.95}\right)^{1/2} = 0.84\ \text{m}^3/\text{s}$$

$$Q_3 = \left(\frac{h_{tJ} - h_{tC}}{56.48}\right)^{1/2} = \left(\frac{400\ \text{m} - 110.35\ \text{m}}{56.48}\right)^{1/2} = 2.27\ \text{m}^3/\text{s}$$

viii) Therefore, at junction J

$$Q_1 + Q_2 = 1.57\ \text{m}^3/\text{s} + 0.84\ \text{m}^3/\text{s} = 2.41\ \text{m}^3/\text{s} > Q_3 = 2.27\ \text{m}^3/\text{s}$$

Now, in order to decrease the volumetric flow rates from reservoirs A and B to junction J, and to increase the volumetric flow rate from junction J to reservoir C, one must increase the assumed value of the total head at junction J, and must repeat the calculations in step (vii). The summary of iterations is presented in Table 10.11. As a result, the volumetric flow rate in pipe 1, 2 and 3 are

$$Q_1 = 1.54\ \text{m}^3/\text{s}$$

$$Q_2 = 0.76\ \text{m}^3/\text{s}$$

$$Q_3 = 2.30\ \text{m}^3/\text{s}$$

ix) Now, the velocity in each pipe may be evaluated as follows:

$$V_1 = \frac{4Q_1}{\pi d_1^2} = \frac{(4)(1.54\ \text{m}^3/\text{s})}{(\pi)(0.30\ \text{m})^2} = 21.79\ \text{m/s}$$

$$V_2 = \frac{4Q_2}{\pi d_2^2} = \frac{(4)(0.76\ \text{m}^3/\text{s})}{(\pi)(0.35\ \text{m})^2} = 7.90\ \text{m/s}$$

$$V_3 = \frac{4Q_3}{\pi d_3^2} = \frac{(4)(2.30\ \text{m}^3/\text{s})}{(\pi)(0.40\ \text{m})^2} = 18.30\ \text{m/s}$$

x) As long as the kinematic viscosity of water at 20^oC is $1.003x10^{-6}\ m^2/s$, then the Reynolds number for each pipe may be evaluated as

$$Re_1 = \frac{V_1 d_1}{\nu} = \frac{(21.79\ m/s)(0.30\ m)}{(1.003x10^{-6}\ m^2/s)} = 6.52x10^6$$

$$Re_2 = \frac{V_2 d_2}{\nu} = \frac{(7.90\ m/s)(0.35\ m)}{(1.003x10^{-6}\ m^2/s)} = 2.76x10^6$$

$$Re_3 = \frac{V_3 d_3}{\nu} = \frac{(18.30\ m/s)(0.40\ m)}{(1.003x10^{-6}\ m^2/s)} = 7.30x10^6$$

xi) The friction factors corresponding to the relative roughness and the Reynolds number for pipes 1, 2 and 3 may be determined from the Moody diagram as follows:

$$f_1 = 0.0185$$

$$f_2 = 0.018$$

$$f_3 = 0.0175$$

xii) As long as the assumed and the improved values of friction factors are the same for each pipe, then there is no need to iterate.

Hence, the volumetric flow rates for pipes 1, 2 and 3 may be given as

$$Q_1 = \underline{1.54\ m^3/s}$$

$$Q_2 = \underline{0.76\ m^3/s}$$

$$Q_3 = \underline{2.30\ m^3/s}$$

Iteration	1	2	3	
$h_J(m)$	400	420	410	411
$Q_1(m^3/s)$	1.57	1.52	1.55	1.54
$Q_2(m^3/s)$	0.84	0.69	0.77	0.76
$Q_3(m^3/s)$	2.27	2.34	2.30	2.30
$Q_1 + Q_2 - Q_3(m^3/s)$	0.14	-0.13	0.02	0.00
Change in $h_J(m)$	increase	decrease	increase	O.K

Table 10.11 Iterations for Example 10.7