ASENKRON MAKİNALAR

Doç. Dr. Orhan KAPLAN

AC Machines

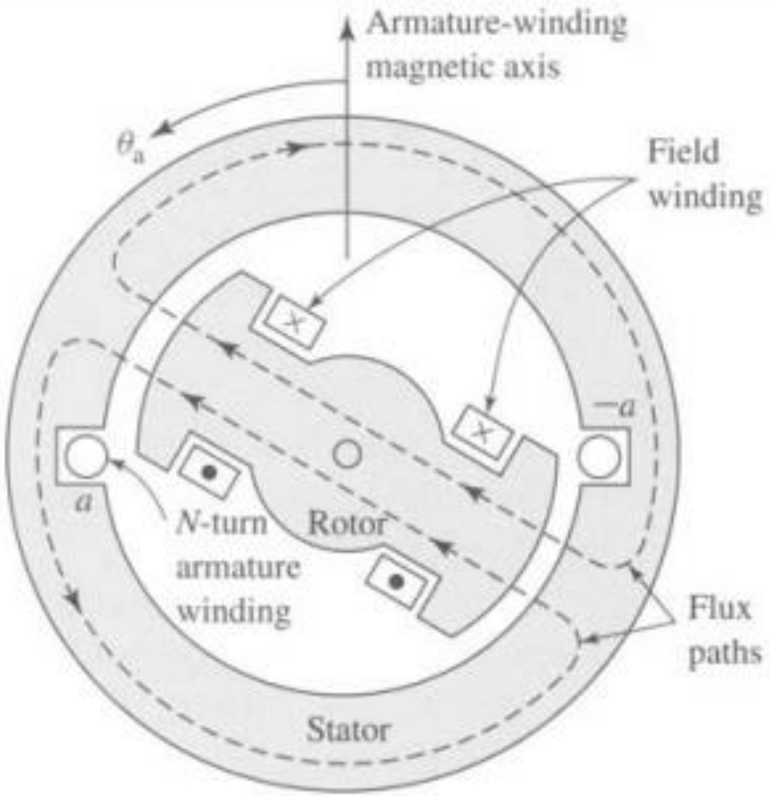


Figure 4.4 Schematic view of a simple, two-pole, single-phase synchronous generator.

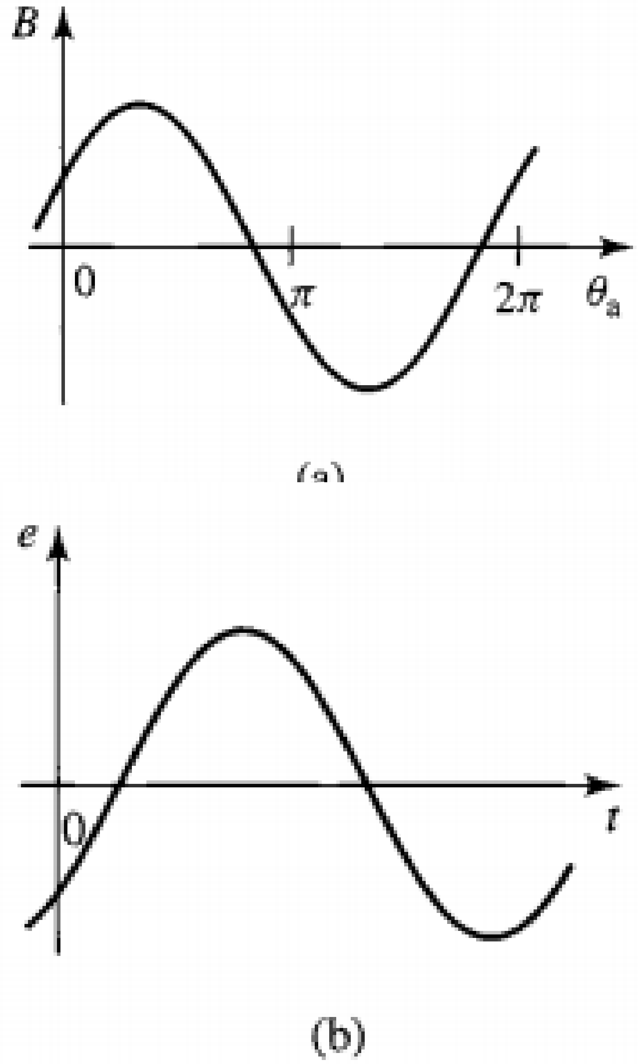


Figure 4.5 (a) Space distribution of flux density and (b) corresponding waveform of the generated voltage for the single-phase generator of Fig. 4.4.

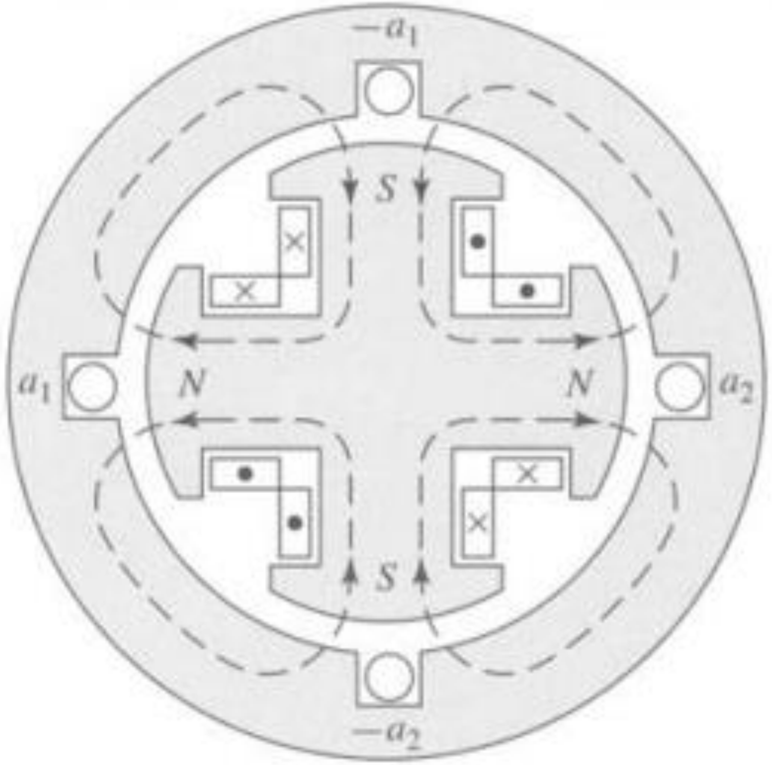


Figure 4.6 Schematic view of a simple, four-pole, single-phase synchronous generator.

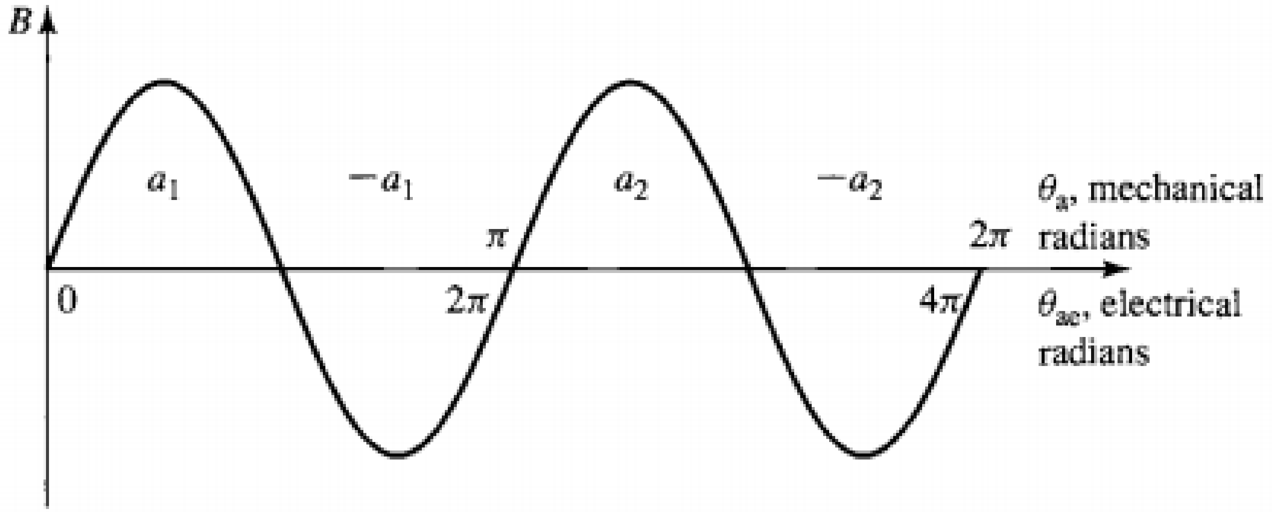


Figure 4.7 Space distribution of the air-gap flux density in a idealized, four-pole synchronous generator.

$$\theta_{ae} = \left(\frac{\text{poles}}{2}\right)\theta_a \qquad f_e = \left(\frac{\text{poles}}{2}\right)\frac{n}{60} \quad \text{Hz}$$

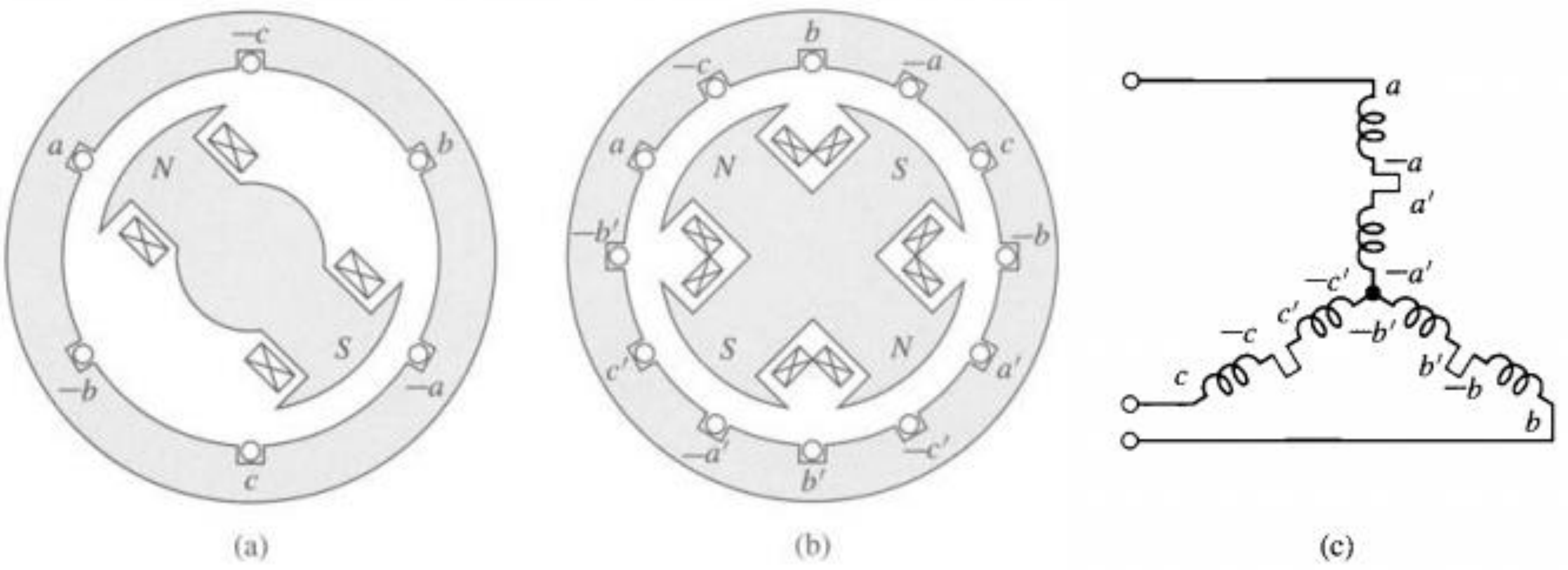


Figure 4.12 Schematic views of three-phase generators: (a) two-pole, (b) four-pole, and (c) Y connection of the windings.

INDUCTION MACHINES

Induction motors operate on the principle of interaction of the stator magnetic rotating field on currents induced in the rotor winding. It can be a cage rotor winding, wound-rotor winding with slip rings or winding in the shape of high-current conducting sleeve, i.e., solid rotor coated with copper layer.

6.1 Construction

Generally, there are three-types of induction machines:

(a) Cage-rotor induction machines (Fig. 6.1);
(b) Wound-rotor (slip-ring) induction machines (Fig. 6.2);

The stator consists of a laminated core (Fig 6.4) and three-phase winding (Fig 6.5) embedded in slots. This winding, when energized by a three-phase source of power, provides a *rotating magnetic field* (Section 5.8).

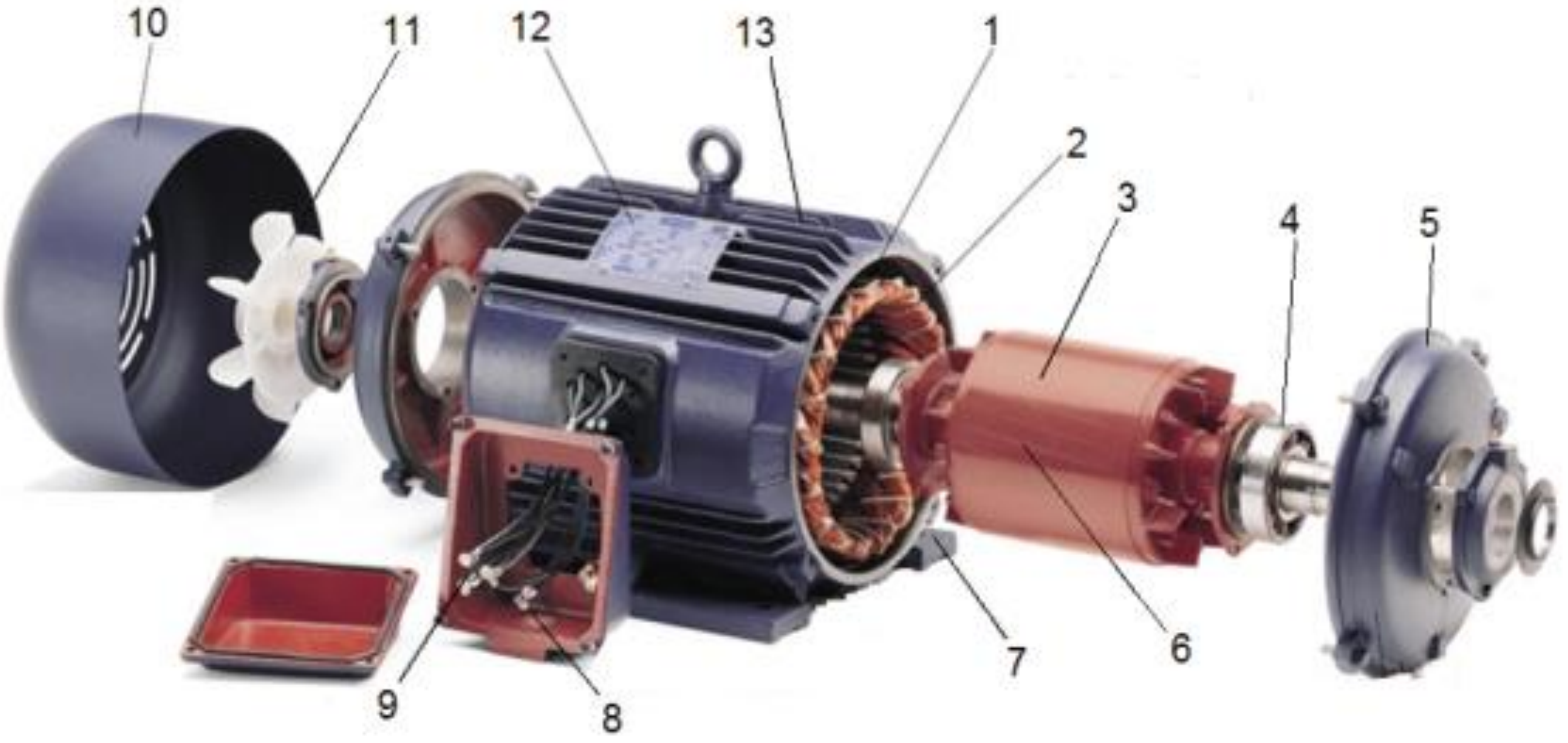


Fig. 6.1. Cage induction motor: 1 – stator core, 2 – stator winding, 3 – cage rotor, 4 – bearing, 5 – end plate (end bell), 6 – rotor bar, 7 – cast iron mounting feet, 8 – terminal box, 9 – terminal leads, 10 – fan cover, 11 – fan, 12 – nameplate, 13 – cast iron frame (housing).

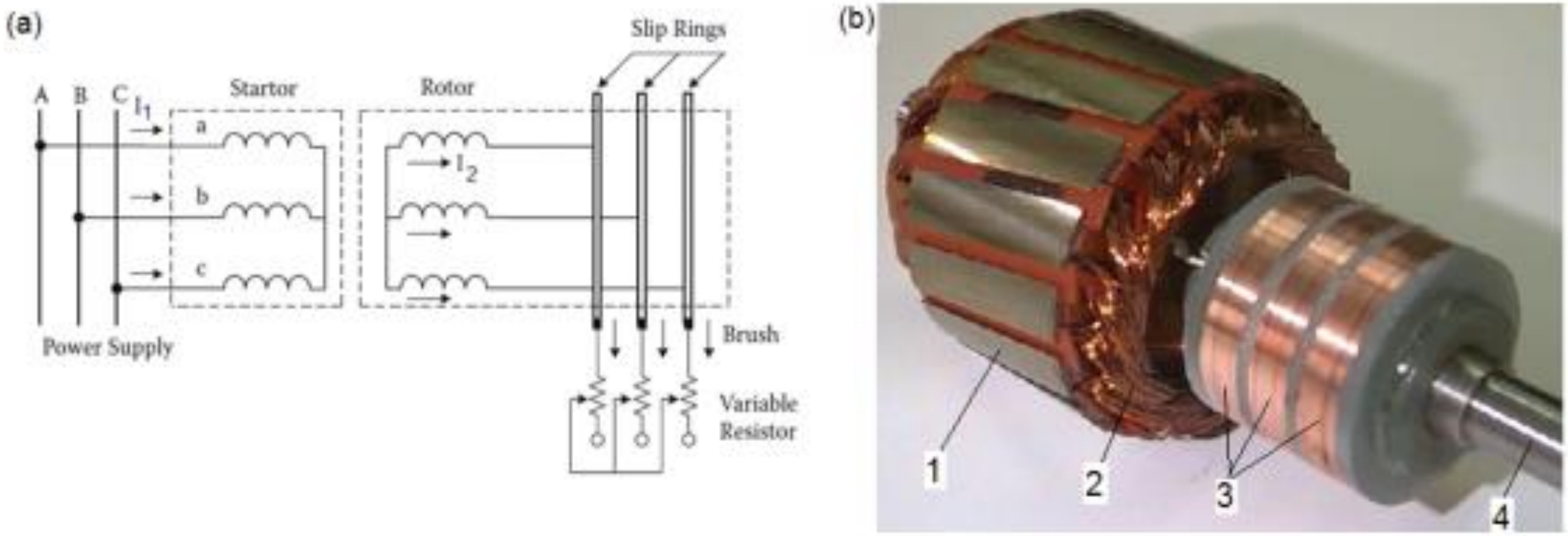


Fig. 6.2. Wound-rotor (slip-ring rotor): (a) stator and rotor connection diagram; (b) construction of rotor. 1 – rotor core, 2 – rotor winding, 3 – slip rings, 4 – shaft.

The stator consists of a laminated core (Fig 6.4) and three-phase winding (Fig 6.5) embedded in slots. This winding, when energized by a three-phase source of power, provides a *rotating magnetic field* (Section 5.8).

The rotor windings are also contained in slots in a laminated core which is mounted on the shaft. In small motors, the rotor-lamination stack is pressed directly on the shaft. In larger machines, the core is mechanically connected to the shaft through a set of spokes called a "spider."

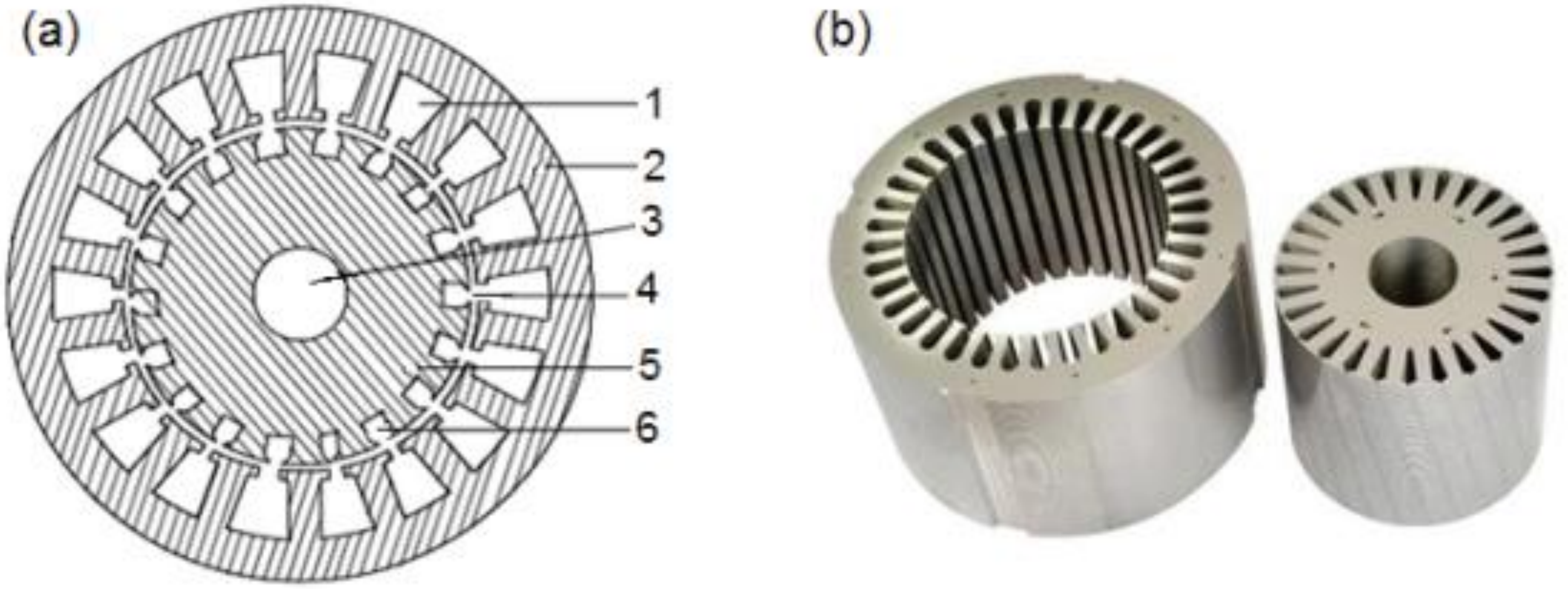


Fig. 6.4. Laminated cores of induction motors: (a) stator and rotor single laminations; (b) stator and rotor laminated stacks. 1 – stator slot, 2 – stator lamination, 3 – shaft, 4 – air gap, 5 – rotor lamination, 6 – shaft.

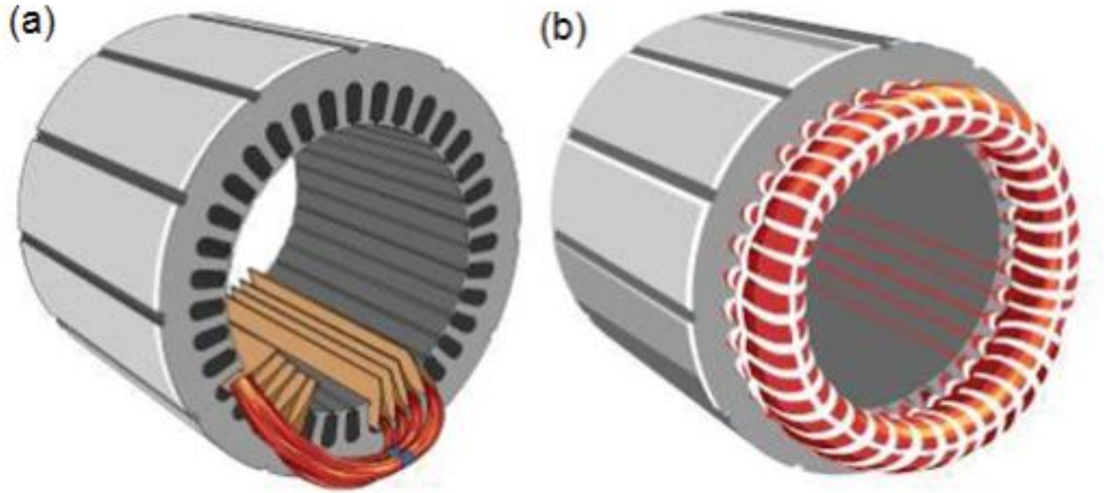


Fig. 6.5. Stator core with winding: (a) partially wound stator core; (b) stator winding completed.

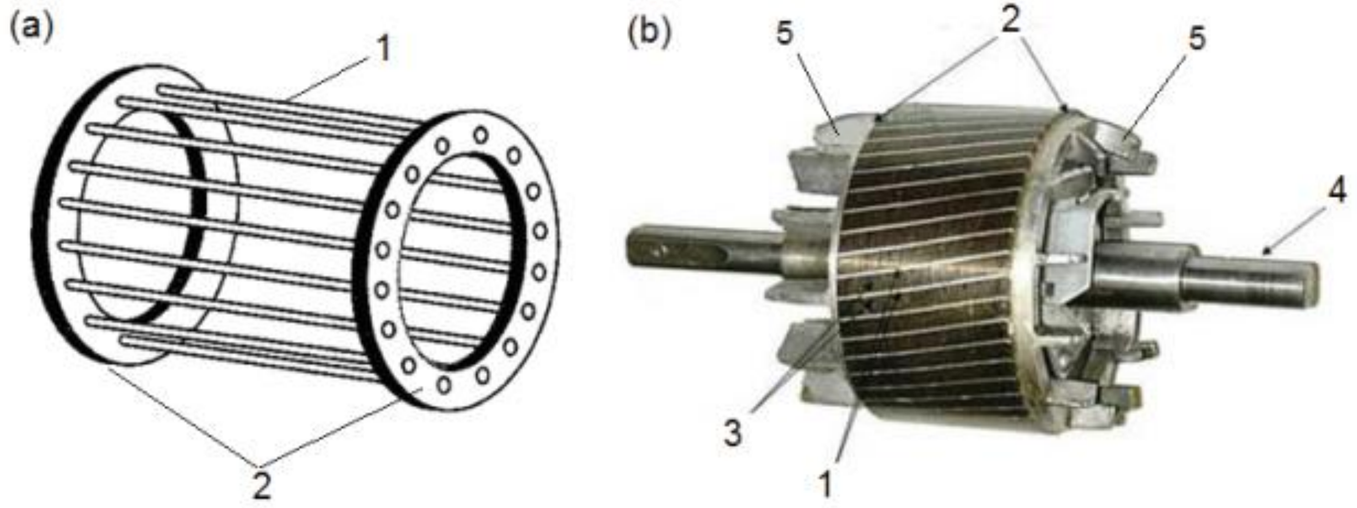


Fig. 6.6. Cage winding: (a) cage; (b) complete cage rotor. 1 – rotor bar, 2 – rotor end ring, 3 – rotor core, 4 – shaft, 5 – cooling fan blades.

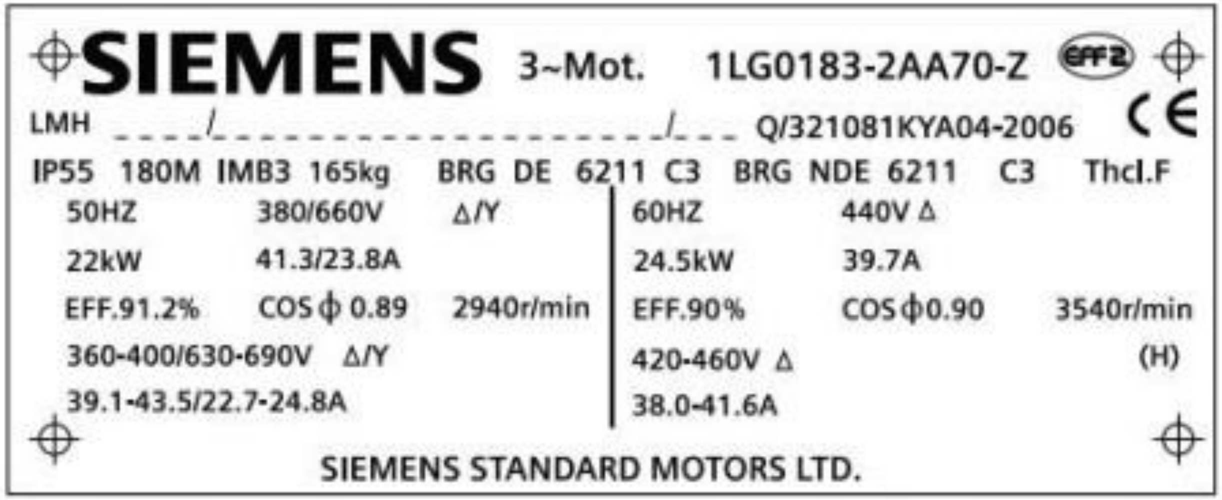
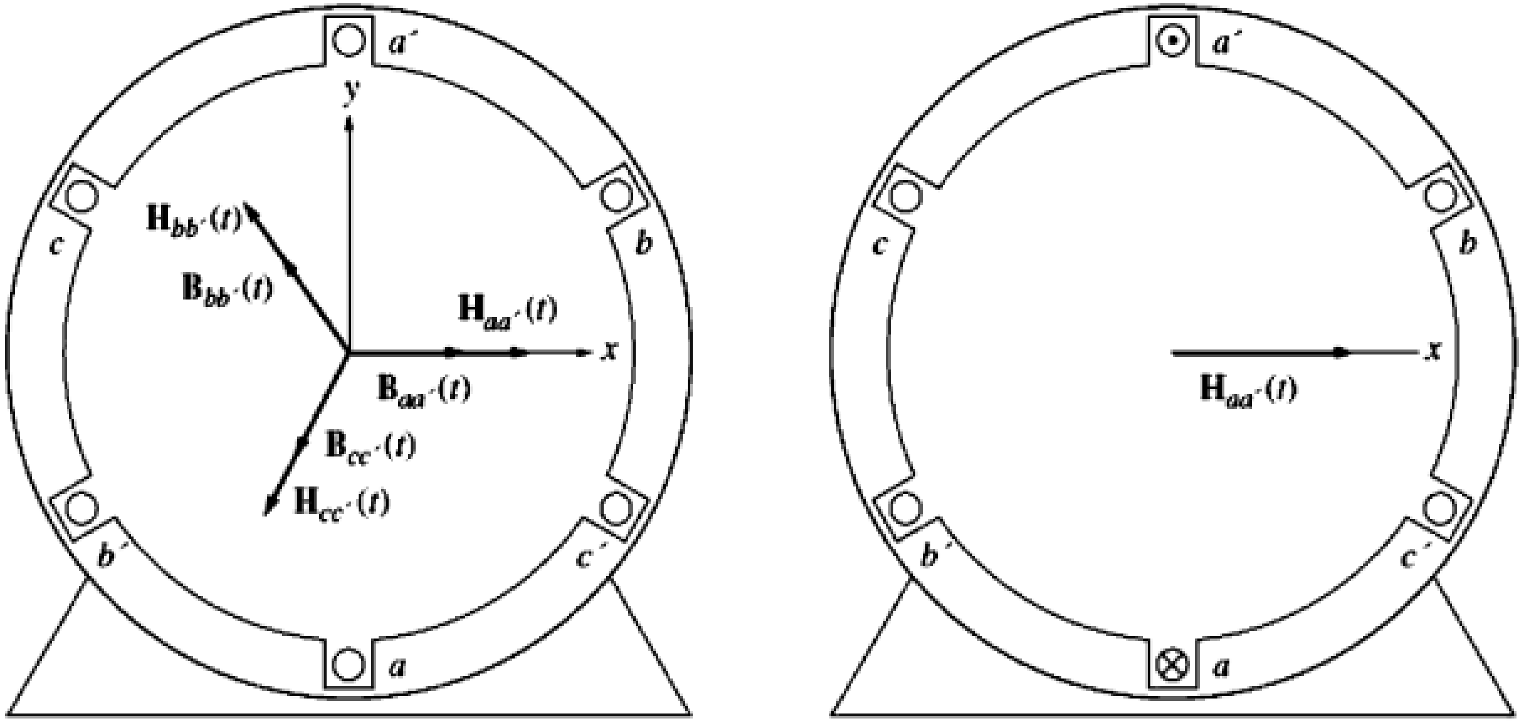


Fig. 6.8. Name plate of induction motor.

- Serial number;
- Rated (nominal power);
- Class of insulation;
- Year of manufacturing;
- Number of phases;
- Frequency;
- Stator line-to-line voltage;
- Stator current;
- Stator winding conenction (Y or Δ);
- Power factor $\cos\varphi$;
- Internal protection IP;
- Duty cycle;
- Nominal (rated) speed;
- Ambient temperature of operation;
- Mass;
- Number of standard the machine meets.

THE ROTATING MAGNETIC FIELD



(a)

(b)

$$i_{aa'}(t) = I_M \sin \omega t \qquad \text{A}$$

$$i_{bb'}(t) = I_M \sin(\omega t - 120°) \qquad \text{A}$$

$$i_{cc'}(t) = I_M \sin(\omega t - 240°) \qquad \text{A}$$

$$\mathbf{H}_{aa'}(t) = H_M \sin \omega t \angle 0° \qquad \text{A} \cdot \text{turns / m}$$

$$\mathbf{H}_{bb'}(t) = H_M \sin(\omega t - 120°) \angle 120° \qquad \text{A} \cdot \text{turns / m}$$

$$\mathbf{H}_{cc'}(t) = H_M \sin(\omega t - 240°) \angle 240° \qquad \text{A} \cdot \text{turns / m}$$

The flux densities resulting from these magnetic field intensities are given by Equation (1–21):

$$\mathbf{B} = \mu\mathbf{H} \tag{1–21}$$

They are

$$\mathbf{B}_{aa'}(t) = B_M \sin \omega t \angle 0^\circ \quad \text{T} \tag{4–23a}$$

$$\mathbf{B}_{bb'}(t) = B_M \sin (\omega t - 120^\circ) \angle 120^\circ \quad \text{T} \tag{4–23b}$$

$$\mathbf{B}_{cc'}(t) = B_M \sin (\omega t - 240^\circ) \angle 240^\circ \quad \text{T} \tag{4–23c}$$

For example, at time $\boxed{\omega t = 0^\circ}$, the magnetic field from coil aa' will be

$$\mathbf{B}_{aa'} = 0 \tag{4–24a}$$

The magnetic field from coil bb' will be

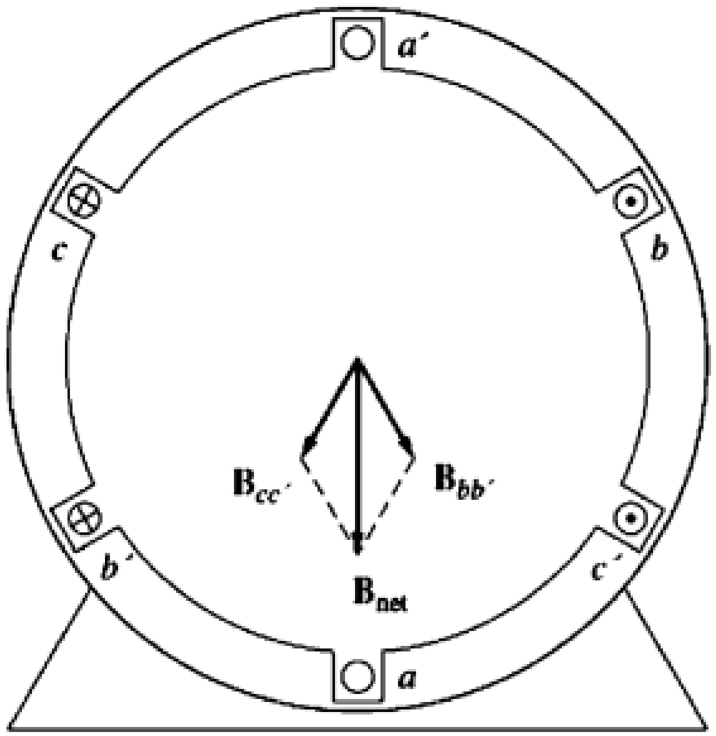
$$\mathbf{B}_{bb'} = B_M \sin (-120^\circ) \angle 120^\circ \tag{4–24b}$$

and the magnetic field from coil cc' will be

$$\mathbf{B}_{cc'} = B_M \sin (-240^\circ) \angle 240^\circ \tag{4–24c}$$

The total magnetic field from all three coils added together will be

$$\begin{aligned}\mathbf{B}_{\text{net}} &= \mathbf{B}_{aa'} + \mathbf{B}_{bb'} + \mathbf{B}_{cc'} \\ &= 0 + \left(-\frac{\sqrt{3}}{2}B_M\right)\angle 120^\circ + \left(\frac{\sqrt{3}}{2}B_M\right)\angle 240^\circ \\ &= 1.5B_M \angle -90^\circ\end{aligned}$$



$\omega t = 0^\circ$

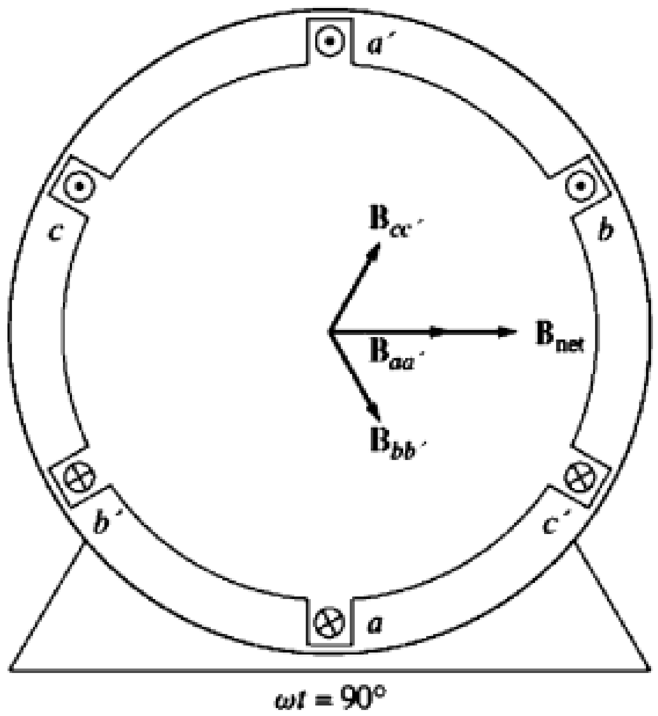
As another example, look at the magnetic field at time $\omega t = 90°$. At that time, the currents are

$$i_{aa'} = I_M \sin 90° \qquad \text{A}$$
$$i_{bb'} = I_M \sin(-30°) \qquad \text{A}$$
$$i_{cc'} = I_M \sin(-150°) \qquad \text{A}$$

$$\mathbf{B}_{aa'} = B_M \angle 0°$$
$$\mathbf{B}_{bb'} = -0.5\, B_M \angle 120°$$
$$\mathbf{B}_{cc'} = -0.5\, B_M \angle 240°$$

The resulting net magnetic field is

$$\begin{aligned}\mathbf{B}_{\text{net}} &= \mathbf{B}_{aa'} + \mathbf{B}_{bb'} + \mathbf{B}_{cc'} \\ &= B_M \angle 0° + (-0.5B_M) \angle 120° + (-0.5B_M) \angle 240° \\ &= 1.5\, B_M \angle 0°\end{aligned}$$



(b)

Proof of the Rotating Magnetic Field Concept

The net magnetic flux density in the stator is given by

$$\mathbf{B}_{net}(t) = \mathbf{B}_{aa'}(t) + \mathbf{B}_{bb'}(t) + \mathbf{B}_{cc'}(t)$$
$$= B_M \sin \omega t \angle 0° + B_M \sin(\omega t - 120°) \angle 120° + B_M \sin(\omega t - 240°) \angle 240° \text{ T}$$

Each of the three component magnetic fields can now be broken down into its x and y components.

$$\mathbf{B}_{net}(t) = B_M \sin \omega t\, \hat{\mathbf{x}}$$
$$- [0.5B_M \sin(\omega t - 120°)]\hat{\mathbf{x}} + \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 120°)\right]\hat{\mathbf{y}}$$
$$- [0.5B_M \sin(\omega t - 240°)]\hat{\mathbf{x}} - \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 240°)\right]\hat{\mathbf{y}}$$

Combining x and y components yields

$$\mathbf{B}_{net}(t) = [B_M \sin \omega t - 0.5B_M \sin(\omega t - 120°) - 0.5B_M \sin(\omega t - 240°)]\,\hat{\mathbf{x}}$$
$$+ \left[\frac{\sqrt{3}}{2} B_M \sin(\omega t - 120°) - \frac{\sqrt{3}}{2} B_M \sin(\omega t - 240°)\right]\hat{\mathbf{y}}$$

By the angle-addition trigonometric identities,

$$\mathbf{B}_{net}(t) = \left[B_M \sin \omega t + \frac{1}{4}B_M \sin \omega t + \frac{\sqrt{3}}{4}B_M \cos \omega t + \frac{1}{4}B_M \sin \omega t - \frac{\sqrt{3}}{4}B_M \cos \omega t\right]\hat{\mathbf{x}}$$
$$+ \left[-\frac{\sqrt{3}}{4}B_M \sin \omega t - \frac{3}{4}B_M \cos \omega t + \frac{\sqrt{3}}{4}B_M \sin \omega t - \frac{3}{4}B_M \cos \omega t\right]\hat{\mathbf{y}}$$

$$\boxed{\mathbf{B}_{net}(t) = (1.5B_M \sin \omega t)\hat{\mathbf{x}} - (1.5B_M \cos \omega t)\hat{\mathbf{y}}} \qquad (4\text{–}25)$$

Equation (4–25) is the final expression for the net magnetic flux density. Notice that the magnitude of the field is a constant $1.5B_M$ and that the angle changes continually in a counterclockwise direction at angular velocity ω. Notice also that at

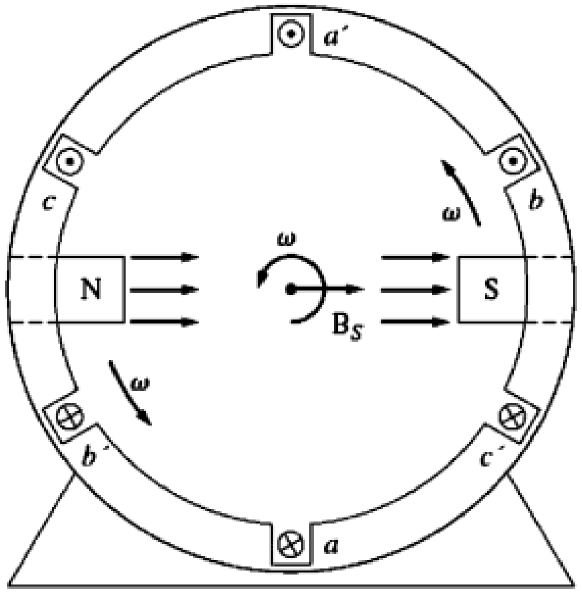


FIGURE 4–10
The rotating magnetic field in a stator represented as moving north and south stator poles.

$\omega t = 0°$, $\mathbf{B}_{net} = 1.5B_M \angle -90°$ and that at $\omega t = 90°$, $\mathbf{B}_{net} = 1.5B_M \angle 0°$. These results agree with the specific examples examined previously.

INDUCED VOLTAGES

Assume that the rotor winding is wound-type, wye-connected, and open-circuited. Since the rotor winding is open-circuited, no torque can develop. This represents the *standstill operation* of a three-phase induction motor. The application of a three-phase voltage to the three-phase stator winding results in a rotating magnetic field that "cuts" both the stator and rotor windings at the supply frequency f_1. Hence, the rms value of the induced voltage per phase of the rotor winding can be expressed as

$$E_2 = \frac{2\pi}{\sqrt{2}} f_1 N_2 \phi k_{\omega 2} \tag{6.28}$$

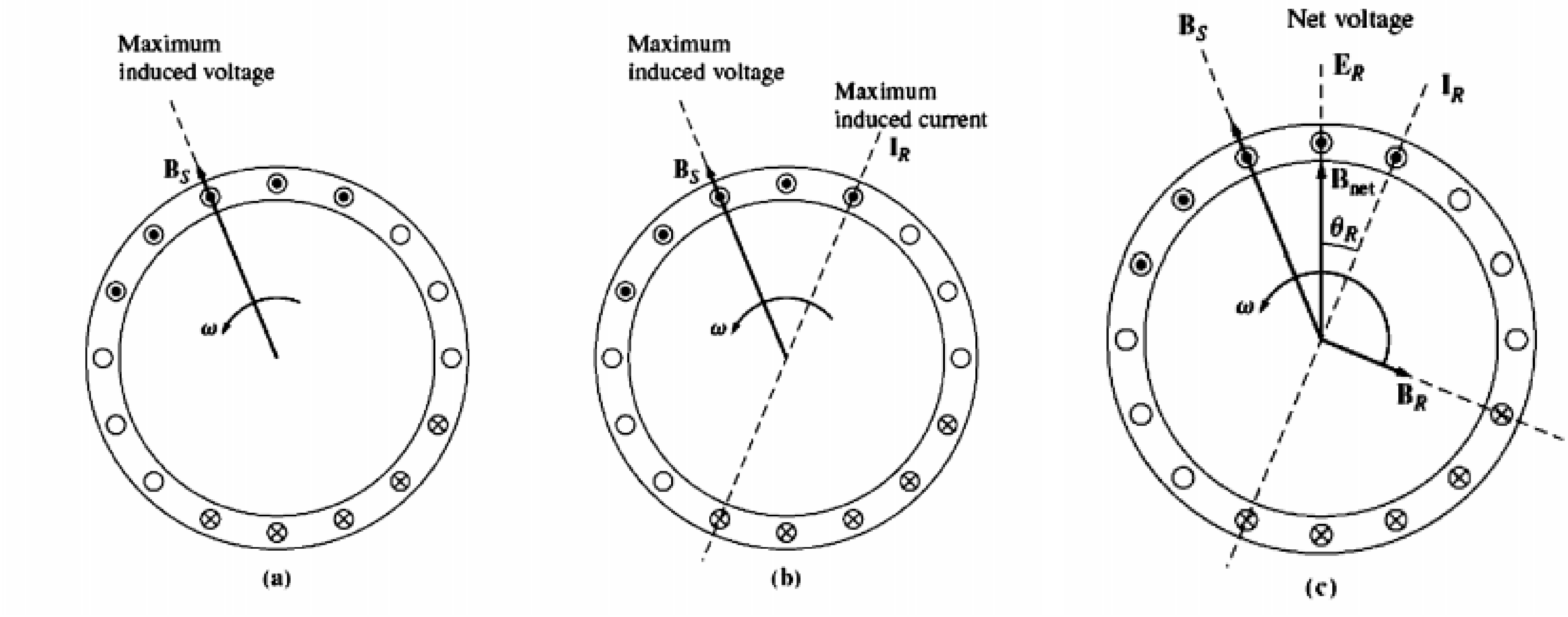
$$E_2 = 4.44 f_1 N_2 \phi k_{\omega 2} \tag{6.29}$$

where the subscripts 1 and 2 are used to denote stator- and rotor-winding quantities, respectively. Since the rotor is at standstill, the stator frequency f_1 is used in Equations 6.28 and 6.29. Here, the flux ϕ is the mutual flux per pole involving both the stator and rotor windings. Similarly, the rms value of the induced voltage per phase of the stator winding can be expressed as

$$E_1 = 4.44 f_1 N_1 \phi k_{\omega 1} \tag{6.30}$$

Thus, it can be shown that

$$\frac{E_1}{E_2} = \frac{N_1 k_{\omega 1}}{N_2 k_{\omega 2}} \longrightarrow \frac{E_1}{E_2} = \frac{N_1}{N_2} = a$$



Finally, since the induced torque in the machine is given by

$$\tau_{ind} = k\mathbf{B}_R \times \mathbf{B}_S$$

6.5 CONCEPT OF ROTOR SLIP

In the event that the stator windings are connected to a three-phase supply and the rotor circuit is closed, the induced voltages in the rotor windings produce three-phase rotor currents. These currents in turn cause another rotating magnetic field to develop in the air gap. This induced rotor magnetic field also rotates at the same synchronous speed, n_s. In other words, the stator magnetic field and the rotor magnetic field are stationary with respect to each other. As a result, the rotor develops a torque according to the principle of alignment of magnetic fields.

Thus, the rotor starts to rotate in the direction of the rotating field of the stator, due to Lenz's law. Here, the stator magnetic field can be considered as dragging the rotor magnetic field. The torque is maintained as long as the rotating magnetic field and the induced rotor currents exist. Also, the voltage induced in the rotor windings depends on the speed of the rotor *relative* to the magnetic fields. At steady-state operation, the rotor's shaft speed† n_m is less than the synchronous speed n_s at which the stator rotating field rotates in the air gap. The synchronous speed is determined by the applied stator frequency‡ f_1, in hertz, and the number of poles, p, of the stator winding. Therefore,

$$n_s = \frac{120 f_1}{p} \text{ rev/min} \tag{6.33}$$

Of course, at $n_m = n_s$, there would be no induced voltages or currents in the rotor windings and, therefore, no torque. Thus, *the shaft speed of the rotor can never be equal to the synchronous speed*, but has to be at some value below that speed.

The **slip speed** (also called the slip rpm) is defined as the difference between synchronous speed and rotor speed and indicates how much the rotor slips[§] behind the synchronous speed. Hence,

$$n_{slip} = n_s - n_m \tag{6.34}$$

where

n_{slip} is the slip speed of motor in rpm
n_s is the synchronous speed (i.e., speed of magnetic fields) in rpm
n_m is the mechanical shaft speed of rotor in rpm

Therefore, the term **slip** describes this relative motion in per unit or in percent. Thus, the slip in per unit is

$$s = \frac{n_s - n_m}{n_s} \tag{6.35}$$

and the slip in percent is

$$s = \frac{n_s - n_m}{n_s} \times 100 \tag{6.36}$$

Alternatively, the slip can be defined in terms of angular velocity *w* (rad/s) as

$$s = \frac{\omega_s - \omega_m}{\omega_s} \times 100 \tag{6.37}$$

By closely inspecting Equation 6.35 and Figure 6.4 and simply applying *deductive reasoning*,* one can observe the following:

1. If $s=0$, it means that $n_m=n_s$, that is, the rotor turns at synchronous speed. (In practice, it can only occur if the direct current is injected into the rotor winding.)
2. If $s=1$, it indicates that $n_m=0$, that is, the rotor is stationary. In other words, the rotor is at standstill.
3. If $1>s>0$, it signals that the rotor turns at a speed somewhere between standstill and synchronous speed. In other words, the motor runs at an asynchronous speed as it should, as illustrated in Figure 6.13a.
4. If $s>1$, it signifies that the rotor rotates in a direction opposite of the stator rotating field, as shown in Figure 6.13c. Therefore, in addition to electrical power, mechanical power (i.e., shaft power) must be provided.

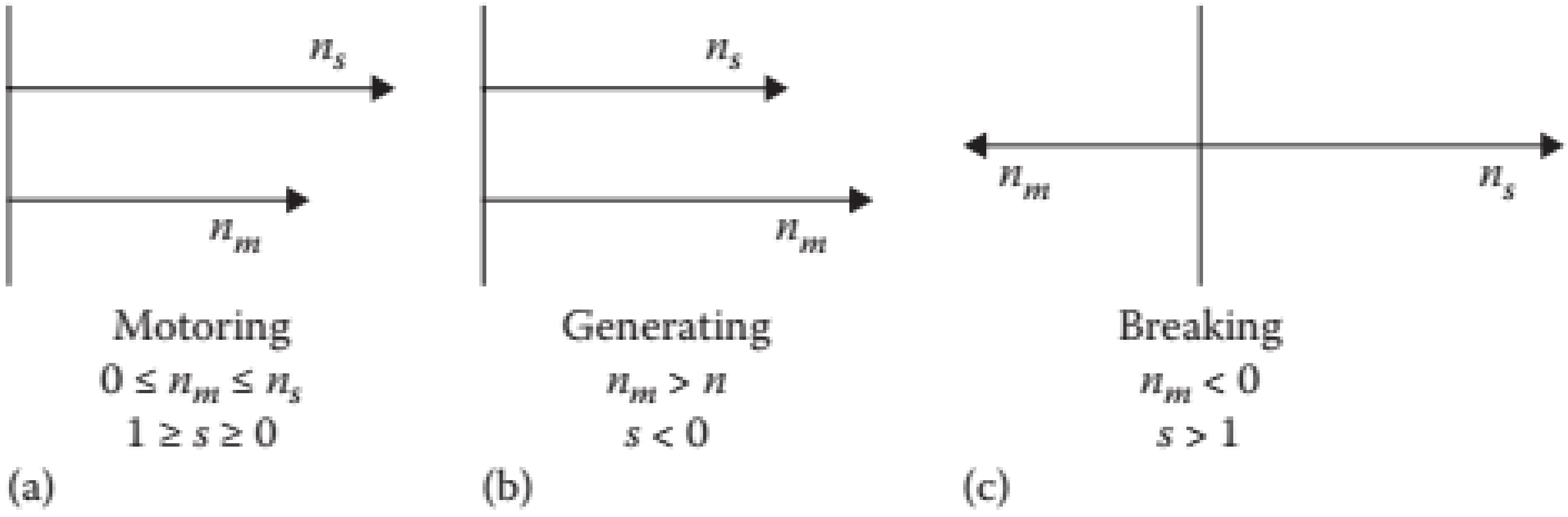
   Since power comes in from both sides, the copper losses of the rotor increase tremendously. The rotor develops a braking torque that forces the motor to stop. This mode of induction machine operation is called **braking** (or *plugging*) **mode**.

5. If $s<0$, it means that the machine operates as a generator with a shaft speed that is greater than the synchronous speed, as shown in Figure 6.13b. This mode of operation is called *generating mode*.

Also note that the mechanical shaft speed of the rotor can be obtained from the following two equations, which involve only slip and synchronous speed:

$$n_m = (1-s)n_s \text{ rpm} \tag{6.38}$$

$$\omega_m = (1-s)\omega_s \text{ rad/s} \tag{6.39}$$



Example 6.1 Determine the number of poles, the slip, and the frequency of the rotor currents at rated load for three-phase 5-hp induction motors rated at:

(a) 220 V, 50 Hz, 1440 rpm.
(b) 120 V, 400 Hz, 3800 rpm.

Solution Using n_r, the rotor speed given, we use Eq. (6.18) to obtain P.
(a) We have

$$P = \frac{120 \times 50}{1440} = 4.17$$

But P should be an even number. Therefore, take $P = 4$. Hence the synchronous speed is

$$n_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

The slip is thus given by

$$s = \frac{n_s - n_r}{n_s} = \frac{1500 - 1440}{1500} = 0.04$$

The rotor frequency is calculated as

$$f_r = sf_s = 0.04 \times 50 = 2 \text{ Hz}$$

(b) We have

$$P = \frac{120 \times 400}{3800} = 12.63$$

Take $P = 12$, to obtain

$$n_s = \frac{120 \times 400}{12} = 4000 \text{ rpm}$$

$$s = \frac{4000 - 3800}{4000} = 0.05$$

$$f_r = 0.05 \times 400 = 20 \text{ Hz}$$

EFFECTS OF SLIP ON THE FREQUENCY AND MAGNITUDE OF INDUCED VOLTAGE OF THE ROTOR

$$f_r = s \times f_1$$

$$E_r = sE_2$$

Example 6.1

A three-phase, 60 Hz, 25 hp, wye-connected induction motor operates at a shaft speed of almost 1800 rpm at no load and 1650 rpm at full load. Determine the following:

(a) The number of poles of the motor
(b) The per-unit and percent slip at full load
(c) The slip frequency of the motor
(d) The speed of the rotor field with respect to the rotor itself
(e) The speed of the rotor field with respect to the stator
(f) The speed of the rotor field with respect to the stator field
(g) The output torque of the motor at the full load

Solution

(a) From Equation 6.33,

$$n_s = \frac{120 f_1}{p}$$

from which

$$p = \frac{120 f_1}{n_s}$$

$$= \frac{120 \times 60}{1800} = 4\,\text{poles}$$

(b) Since

$$n_m = n_s(1 - s)$$

Then

$$s = \frac{n_s - n_m}{n_s}$$

$$= \frac{1800 - 1650}{1800}$$

$$= 0.08333\,\text{pu or } 8.33\%$$

(c) The slip frequency is

$$f_2 = s f_1 = 0.0833 \times 60 = 5\,\text{Hz}$$

(d) The speed of the rotor field with respect to the rotor itself can be determined from

$$n_2 = \frac{120 f_2}{p}$$

$$= \frac{120 \times 5}{4}$$

$$= 150\ \text{rpm}$$

or

$$n_2 = s \times n_s$$

$$= 0.08333 \times 1800$$

$$= 150\ \text{rpm}$$

(e) The speed of the rotor field with respect to the stator can be found from

$$n_m + n_2 = 1650 + 150 = 1800\ \text{rpm}$$

or

$$n_m + n_2 = n_s = 1800\ \text{rpm}$$

(f) The speed of the rotor field with respect to the stator field can be determined from

$$(n_m + n_2) - n_s = 1800 - 1800 = 0$$

or since

$$n_m + n_2 = n_s$$

then

$$n_s - n_s = 0$$

(g) The output torque of the motor at the full load can be determined from

$$\begin{aligned} T_{out} = T_{shaft} &= \frac{P_{out}}{\omega_m} \\ &= \frac{(25\text{ hp})(746\text{ W/hp})}{(1650\text{ rev/min})(2\pi\text{ rad/rev})(1\text{ min}/60\text{ s})} \\ &= 108\text{ N}\cdot\text{m} \end{aligned}$$

or in English units,

$$\begin{aligned} T_{out} = T_{shaft} &= \frac{5252P}{n} \\ &= \frac{5252(25\text{ hp})}{1650\text{ rev/min}} \\ &= 79.6\text{ lb}\cdot\text{ft} \end{aligned}$$

Example 7–1. A 208-V, 10-hp, four-pole, 60-Hz, Y-connected induction motor has a full-load slip of 5 percent.

(a) What is the synchronous speed of this motor?
(b) What is the rotor speed of this motor at the rated load?
(c) What is the rotor frequency of this motor at the rated load?
(d) What is the shaft torque of this motor at the rated load?

Solution

(a) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_e}{P}$$
$$= \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

(c) The rotor frequency of this motor is given by

$$f_r = sf_e = (0.05)(60\text{ Hz}) = 3\text{ Hz}$$

Alternatively, the frequency can be found from Equation (7–9

$$f_r = \frac{P}{120}(n_{\text{sync}} - n_{\text{m}})$$
$$= \frac{4}{120}(1800\text{ r/min} - 1710\text{ r/min}) = 3\text{ Hz}$$

(b) The rotor speed of the motor is given by

$$n_m = (1 - s)n_{\text{sync}}$$
$$= (1 - 0.95)(1800\text{ r/min}) = 1710\text{ r/min}$$

(d) The shaft load torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$
$$= \frac{(10\text{ hp})(746\text{ W/hp})}{(1710\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})} = 41.7\text{ N}\bullet\text{m}$$

The shaft load torque in English units is given by Equation (1–17):

$$\tau_{\text{load}} = \frac{5252P}{n}$$

where τ is in pound-feet, P is in horsepower, and n_m is in revolutions per minute. Therefore,

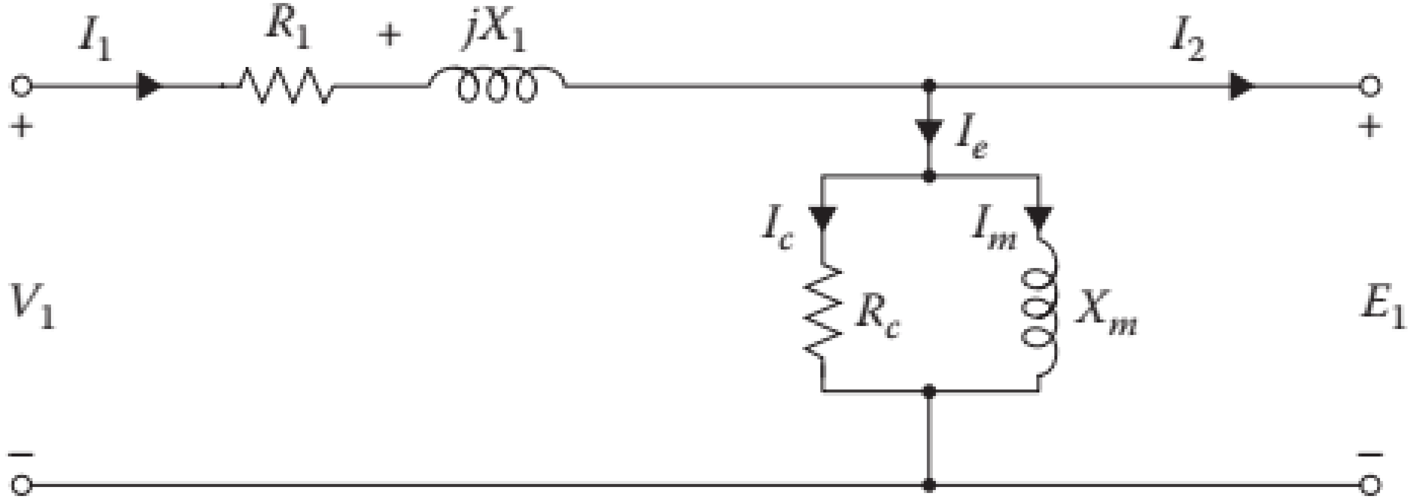
$$\tau_{\text{load}} = \frac{5252(10\text{ hp})}{1710\text{ r/min}} = 30.7\text{ lb}\bullet\text{ft}$$

EQUIVALENT CIRCUIT OF AN INDUCTION MOTOR

Stator Circuit Model

Figure 6.14a shows the equivalent circuit of the stator. The stator terminal voltage differs from the induced voltage (i.e., the counter-emf) in the stator winding because of the voltage drop in the stator leakage impedance. Therefore,

$$\boldsymbol{V}_1 = \boldsymbol{E}_1 + \boldsymbol{I}_1(R_1 + jX_1) \tag{6.46}$$



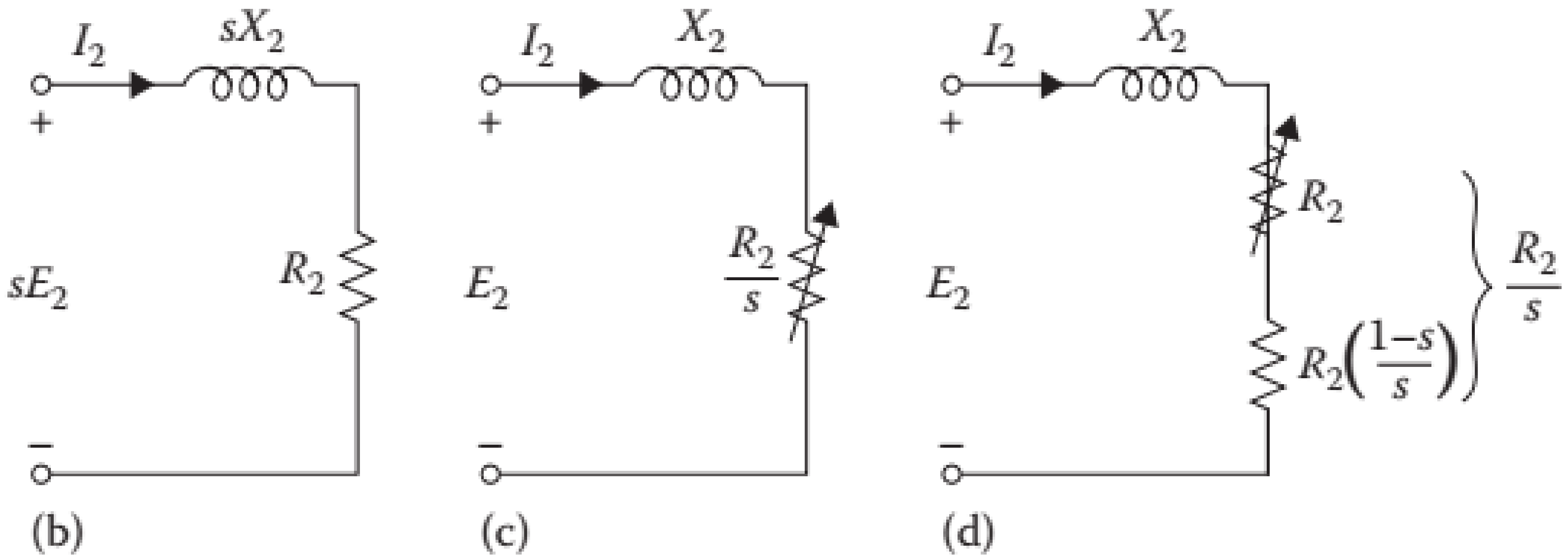
where

$\boldsymbol{V}_1$ is the per-phase stator terminal voltage
$\boldsymbol{E}_1$ is the per-phase induced voltage (counter-emf) in the stator winding
$\boldsymbol{I}_1$ is the stator current
R_1 is the per-phase stator winding resistance
X_1 is the per-phase stator leakage reactance

ROTOR-CIRCUIT MODEL

Figure 6.14b shows the actual rotor circuit of an induction motor operating under load at a slip s. The rotor current per phase can be expressed as

$$\boldsymbol{I}_2 = \frac{sE_2}{R_2 + jsX_2} \tag{6.47}$$



where

E_2 is the per-phase induced voltage in the rotor at standstill (i.e., at stator frequency f_1)
R_2 is the per-phase rotor-circuit resistance
X_2 is the per-phase rotor leakage inductive reactance

The figure illustrates that $\boldsymbol{I}_2$ is a slip-frequency current produced by the slip frequency–induced emf sE_2 acting in a rotor circuit with an impedance per phase of R_2+jsX_2. Therefore, the total rotor copper loss can be expressed as

$$\boldsymbol{I}_2 = \frac{sE_2}{R_2 + jsX_2} \qquad P_{2,cu} = 3I_2^2 R_2$$

which represents the amount of real power involved in the rotor circuit. Equation 6.47 can be rewritten by dividing both the numerator and the denominator by the slip s so that

$$\boldsymbol{I}_2 = \frac{E_2}{(R_2/s) + jX_2}$$

This equation suggests the rotor-equivalent circuit shown in Figure 6.14c. Of course, the magnitude and phase angle of $\boldsymbol{I}_2$ remain unchanged by this process, but there is a significant difference between these two equations and the circuits they represent. The current $\boldsymbol{I}_2$ given by Equation 6.47 is at slip frequency f_2, whereas $\boldsymbol{I}_2$ given by Equation 6.49 is at line frequency f_1.

Also in Equation 6.47, the rotor leakage reactance sX_2 changes with speed, but the resistance R_2 remains unchanged; whereas in Equation 6.49, the resistance R_2/s changes with speed, but the leakage reactance X_2 remains unchanged. The total rotor copper loss associated with the equivalent rotor circuit shown in Figure 6.14c is

$$P = 3I_2^2\left(\frac{R_2}{s}\right) \qquad = \frac{P_{2,cu}}{s}$$

Since induction machines are run at low slips, the power associated with Figure 6.14c is substantially greater. The equivalent circuit given in Figure 6.14c is at the stator frequency and therefore is the rotor-equivalent circuit as seen from the stator. Thus, the power determined by using Equation 6.50 is the power transferred across the air gap (i.e., P_g) from the stator to the rotor which includes the rotor copper loss as well as the developed mechanical power. Here, the equation can be expressed in a manner that stresses this fact. Therefore,

$$P = P_g = 3I_2^2 \frac{R_2}{s} = 3I^2 \left[R_2 + \frac{R_2}{s}(1-s) \right]$$

$$P_d = P_{mech} = 3I_2^2 \frac{R_2}{s}(1-s)$$

$$P_d = P_{mech} = (1-s)P_g$$

$$P_d = P_{mech} = \frac{1-s}{s} P_{2,cu}$$

$$P_{2,cu} = 3I_2^2 R_2 = sP_g$$

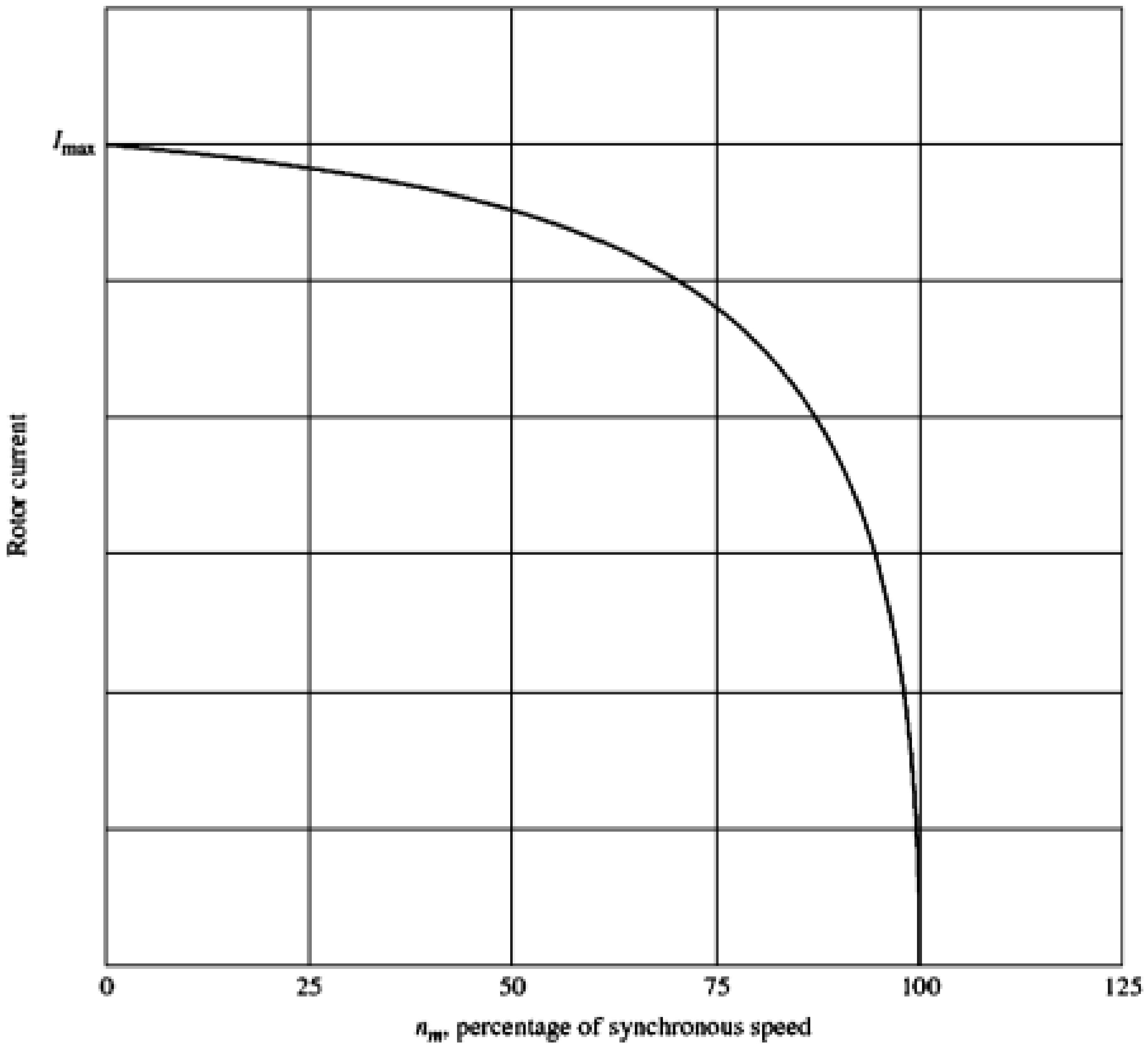
$$P_{rotor\ giriş} = 3I_2^2 \frac{R_2}{s} = P_{rot.cu} + P_{mech}$$

$$P_{rotor\ giriş} = 3I_2^2 R_2 + 3I_2^2 \frac{R_2(1-s)}{s}$$

$$P_{rotor\ giriş} = \frac{P_{rot.cu}}{s}$$

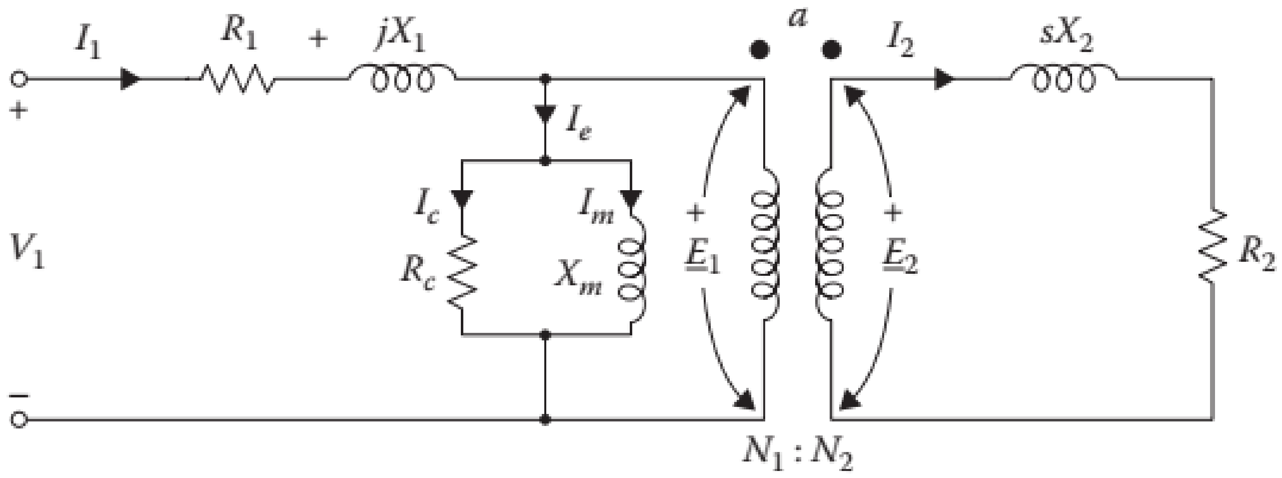
$$P_{rotor\ giriş}(1-s) = P_{mech}$$

$$P_{mech} = \frac{P_{rot.cu}(1-s)}{s}$$

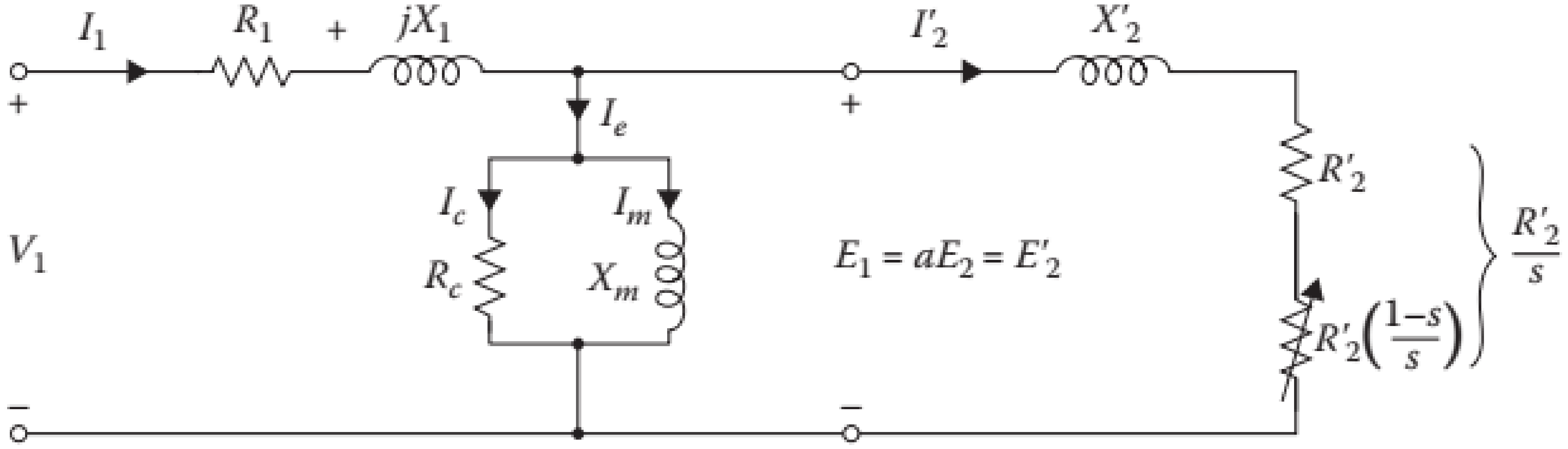


Rotor current as a function of rotor speed.

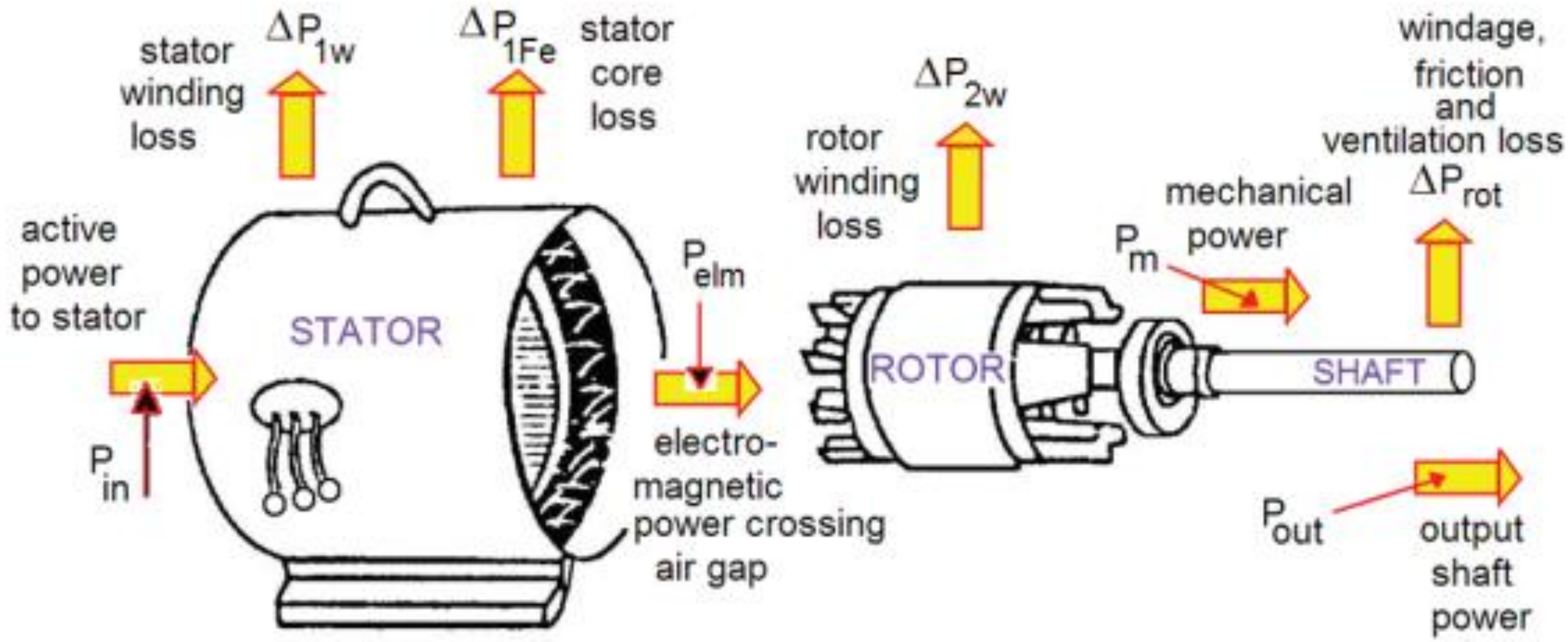
Complete Equivalent Circuit



transformer model of an induction motor



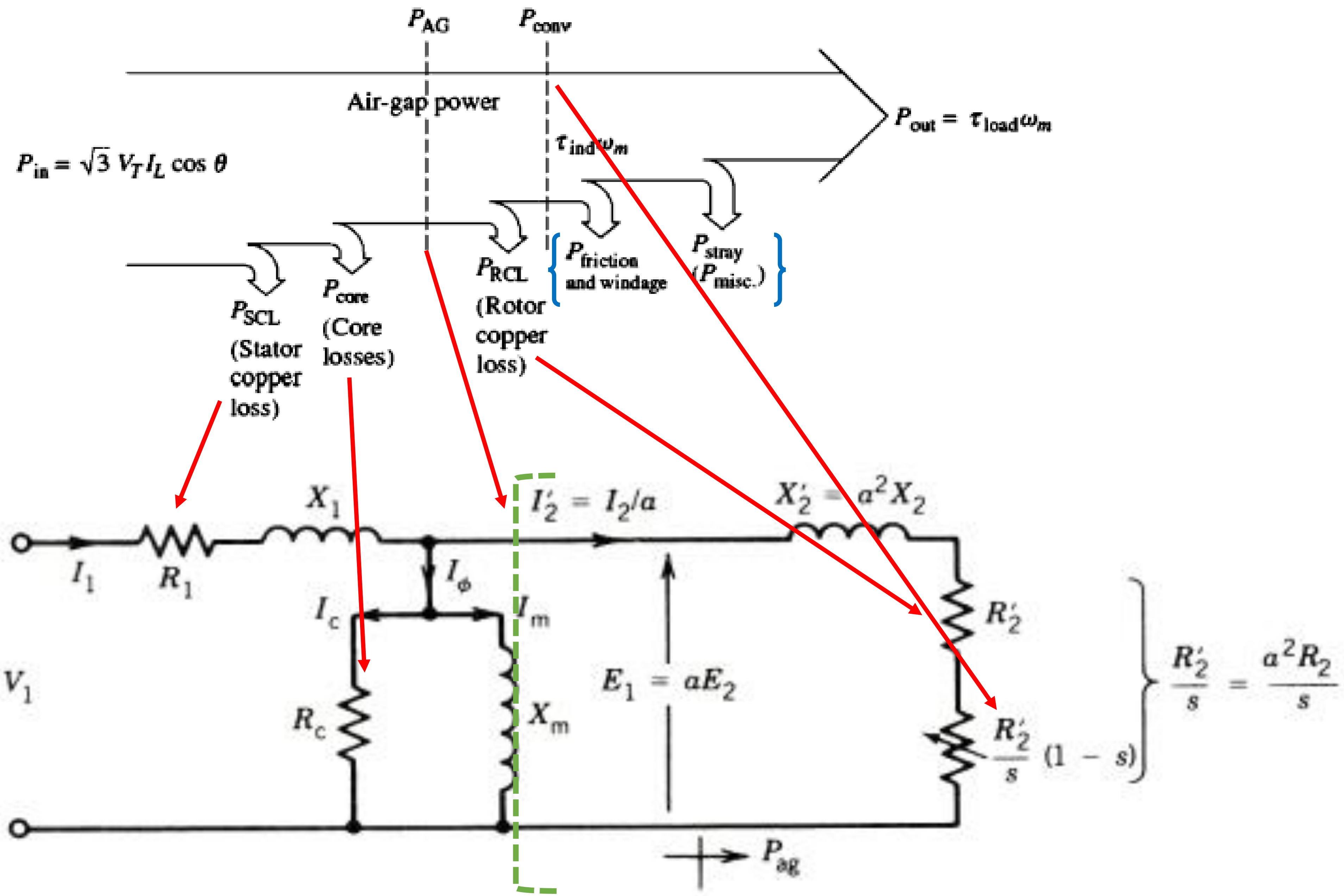
exact equivalent circuit.



$$P_{out} = P_{in} - \Delta P_{1w} - \Delta P_{1Fe} - \Delta P_{2w} - \Delta P_{rot} - \Delta P_{str} = P_m - \Delta P_{rot} - \Delta P_{str}$$

where ΔP_{1w} are the stator winding losses, ΔP_{1Fe} are the stator core losses, ΔP_{2w} are the rotor winding losses, ΔP_{rot} are the rotational (mechanical) losses and ΔP_{str} are the stray load losses. The mechanical power

$$P_m = P_{in} - \Delta P_{1w} - \Delta P_{1Fe} - \Delta P_{2w}$$



P_{AG}
P_{conv}
Air-gap power
$P_{out} = \tau_{load}\omega_m$
$\tau_{ind}\omega_m$
$P_{in} = \sqrt{3}\, V_T I_L \cos\theta$
P_{SCL} (Stator copper loss)
P_{core} (Core losses)
P_{RCL} (Rotor copper loss)
$P_{friction\ and\ windage}$
P_{stray} ($P_{misc.}$)
$I_2' = I_2/a$
$X_2' = a^2X_2$
X_1
I_1
R_1
I_ϕ
I_c
I_m
V_1
R_c
X_m
$E_1 = aE_2$
R_2'
$\frac{R_2'}{s}(1-s)$
$\frac{R_2'}{s} = \frac{a^2R_2}{s}$
P_{ag}

EXAMPLE 5.2

A 3ϕ, 15 hp, 460 V, four-pole, 60 Hz, 1728 rpm induction motor delivers full output power to a load connected to its shaft. The windage and friction loss of the motor is 750 W. Determine the

(a) Mechanical power developed.

(b) Air gap power.

(c) Rotor copper loss.

Solution

(a) Full-load shaft power $= 15 \times 746 = 11{,}190$ W

$$\text{Mechanical power developed} = \text{shaft power} + \text{windage and friction loss}$$
$$= 11{,}190 + 750 = 11{,}940 \text{ W}$$

(b)

$$\text{Synchronous speed} \quad n_s = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

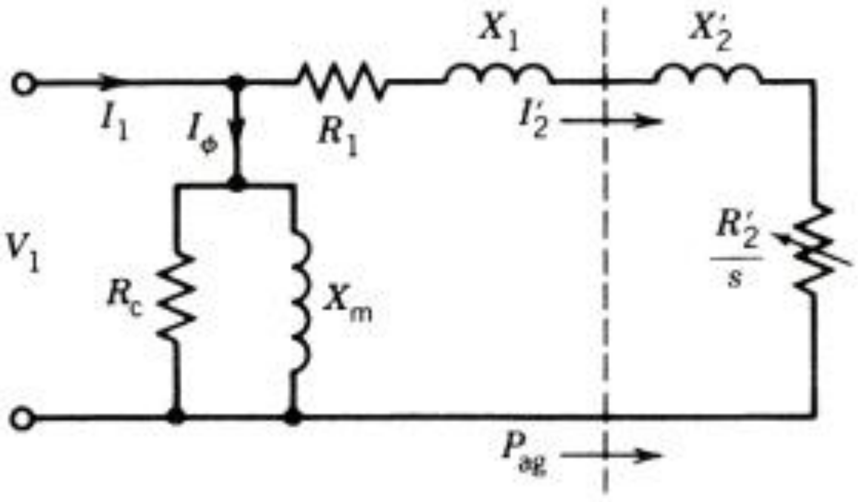
$$\text{Slip} \quad s = \frac{1800 - 1728}{1800} = 0.04$$

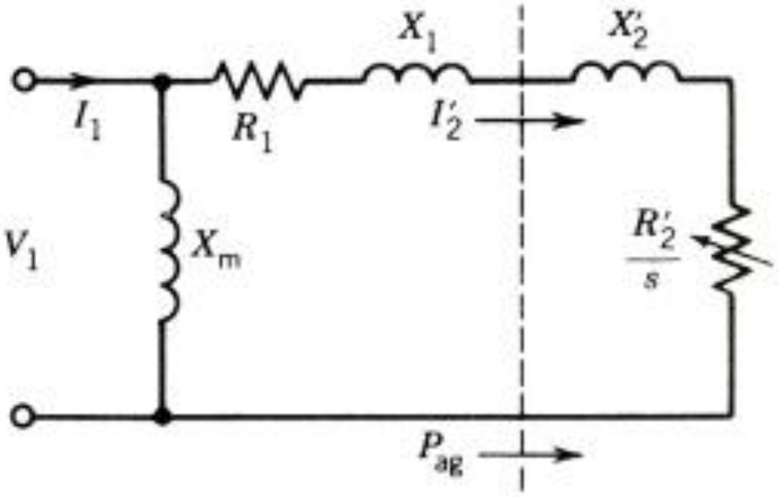
$$\text{Air gap power} \quad P_{ag} = \frac{11{,}940}{1 - 0.04} = 12{,}437.5 \text{ W}$$

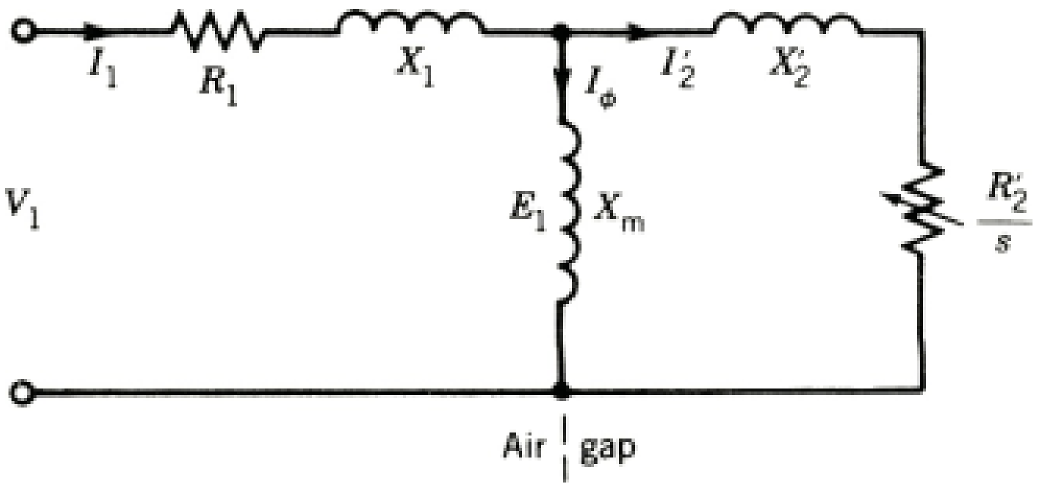
(c)

$$\text{Rotor copper loss} \quad P_2 = 0.04 \times 12{,}437.5$$
$$= 497.5\ W \quad \blacksquare$$

VARIOUS EQUIVALENT CIRCUIT CONFIGURATIONS







IEEE-recommended equivalent circuit

Example 7–2. A 480-V, 60-Hz, 50-hp, three-phase induction motor is drawing 60 A at 0.85 PF lagging. The stator copper losses are 2 kW, and the rotor copper losses are 700 W. The friction and windage losses are 600 W, the core losses are 1800 W, and the stray losses are negligible. Find the following quantities:

(a) The air-gap power P_{AG}
(b) The power converted P_{conv}
(c) The output power P_{out}
(d) The efficiency of the motor

(a) The air-gap power is just the input power minus the stator I^2R losses. The input power is given by

$$P_{in} = \sqrt{3}V_T I_L \cos\theta$$
$$= \sqrt{3}(480\text{ V})(60\text{ A})(0.85) = 42.4\text{ kW}$$

From the power-flow diagram, the air-gap power is given by

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$
$$= 42.4\text{ kW} - 2\text{ kW} - 1.8\text{ kW} = 38.6\text{ kW}$$

(b) From the power-flow diagram, the power converted from electrical to mechanical form is

$$P_{conv} = P_{AG} - P_{RCL}$$
$$= 38.6\text{ kW} - 700\text{ W} = 37.9\text{ kW}$$

(c) From the power-flow diagram, the output power is given by

$$P_{out} = P_{conv} - P_{F\&W} - P_{misc}$$
$$= 37.9\text{ kW} - 600\text{ W} - 0\text{ W} = 37.3\text{ kW}$$

or, in horsepower,

$$P_{out} = (37.3\text{ kW})\frac{1\text{ hp}}{0.746\text{ kW}} = 50\text{ hp}$$

(d) Therefore, the induction motor's efficiency is

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$
$$= \frac{37.3\text{ kW}}{42.4\text{ kW}} \times 100\% = 88\%$$

Torque in an Induction Motor

The torque T developed by the motor is related to P_r by

$$T = \frac{P_r}{\omega_r}$$

with ω_r being the angular speed of the rotor. Thus

$$\omega_r = \omega_s(1 - s)$$

The angular synchronous speed ω_s is given by

$$\omega_s = \frac{2\pi n_s}{60}$$

As a result, the torque is given by

$$T = \frac{3I_r^2 R_2}{s\omega_s}$$

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m}$$

$$\tau_{\text{ind}} = \frac{(1 - s)P_{\text{AG}}}{(1 - s)\omega_{\text{sync}}}$$

$$\boxed{\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}}$$

Example 7–3. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \qquad X_2 = 0.464\ \Omega \qquad X_M = 26.3\ \Omega$$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

(a) Speed
(b) Stator current
(c) Power factor
(d) P_{conv} and P_{out}
(e) τ_{ind} and τ_{load}
(f) Efficiency

(a) The synchronous speed is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

or $$\omega_{\text{sync}} = (1800\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 188.5\text{ rad/s}$$

The rotor's mechanical shaft speed is

$$\begin{aligned} n_m &= (1 - s)n_{\text{sync}} \\ &= (1 - 0.022)(1800\text{ r/min}) = 1760\text{ r/min} \end{aligned}$$

or $$\begin{aligned} \omega_m &= (1 - s)\omega_{\text{sync}} \\ &= (1 - 0.022)(188.5\text{ rad/s}) = 184.4\text{ rad/s} \end{aligned}$$

The referred rotor impedance is

$$Z_2 = \frac{R_2}{s} + jX_2$$
$$= \frac{0.332}{0.022} + j0.464$$
$$= 15.09 + j0.464\ \Omega = 15.10\angle 1.76^\circ\ \Omega$$

Therefore, the total impedance is

$$Z_{\text{tot}} = Z_{\text{stat}} + Z_f$$
$$= 0.641 + j1.106 + 12.94\angle 31.1^\circ\ \Omega$$
$$= 11.72 + j7.79 = 14.07\angle 33.6^\circ\ \Omega$$

The resulting stator current is

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{\text{tot}}}$$
$$= \frac{266\angle 0^\circ\ \text{V}}{14.07\angle 33.6^\circ\ \Omega} = 18.88\angle -33.6^\circ\ \text{A}$$

The combined magnetization plus rotor impedance is given by

$$Z_f = \frac{1}{1/jX_M + 1/Z_2}$$
$$= \frac{1}{-j0.038 + 0.0662\angle -1.76^\circ}$$
$$= \frac{1}{0.0773\angle -31.1^\circ} = 12.94\angle 31.1^\circ\ \Omega$$

(c) The power motor power factor is

$$\text{PF} = \cos 33.6^\circ = 0.833 \qquad \text{lagging}$$

(d) The input power to this motor is

$$P_{\text{in}} = \sqrt{3}V_T I_L \cos\theta$$
$$= \sqrt{3}(460\ \text{V})(18.88\ \text{A})(0.833) = 12{,}530\ \text{W}$$

The stator copper losses in this machine are

$$P_{\text{SCL}} = 3I_1^2 R_1$$
$$= 3(18.88\ \text{A})^2(0.641\ \Omega) = 685\ \text{W}$$

The air-gap power is given by

$$P_{\text{AG}} = P_{\text{in}} - P_{\text{SCL}} = 12{,}530\ \text{W} - 685\ \text{W} = 11{,}845\ \text{W}$$

Therefore, the power converted is

$$P_{\text{conv}} = (1 - s)P_{\text{AG}} = (1 - 0.022)(11{,}845\ \text{W}) = 11{,}585\ \text{W}$$

The power P_{out} is given by

$$P_{\text{out}} = P_{\text{conv}} - P_{\text{rot}} = 11{,}585\ \text{W} - 1100\ \text{W} = 10{,}485\ \text{W}$$
$$= 10{,}485\ \text{W}\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = 14.1\ \text{hp}$$

(e) The induced torque is given by

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}$$

$$= \frac{11{,}845 \text{ W}}{188.5 \text{ rad/s}} = 62.8 \text{ N} \bullet \text{m}$$

and the output torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= \frac{10{,}485 \text{ W}}{184.4 \text{ rad/s}} = 56.9 \text{ N} \bullet \text{m}$$

(In English units, these torques are 46.3 and 41.9 lb-ft, respectively.)

(f) The motor's efficiency at this operating condition is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

$$= \frac{10{,}485 \text{ W}}{12{,}530 \text{ W}} \times 100\% = 83.7\%$$

Example 6.4 The rotor resistance and reactance at standstill of a three-phase four-pole 110-V induction motor are 0.18 Ω and 0.75 Ω, respectively. Find the rotor current at starting as well as when the speed is 1720 rpm. Assume that the frequency is 60 Hz. Calculate the equivalent resistance of the load.

Solution

$$R_2 = 0.18\,\Omega \qquad X_2 = 0.75\,\Omega$$

$$n_s = \frac{120f}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$s = \frac{1800 - 1720}{1800} = 0.0444$$

At standstill, $s = 1$,

$$I_r = \frac{V}{\sqrt{R_2^2 + X_2^2}} = \frac{110/\sqrt{3}}{\sqrt{(0.18)^2 + (0.75)^2}} = 82.34 \text{ A}$$

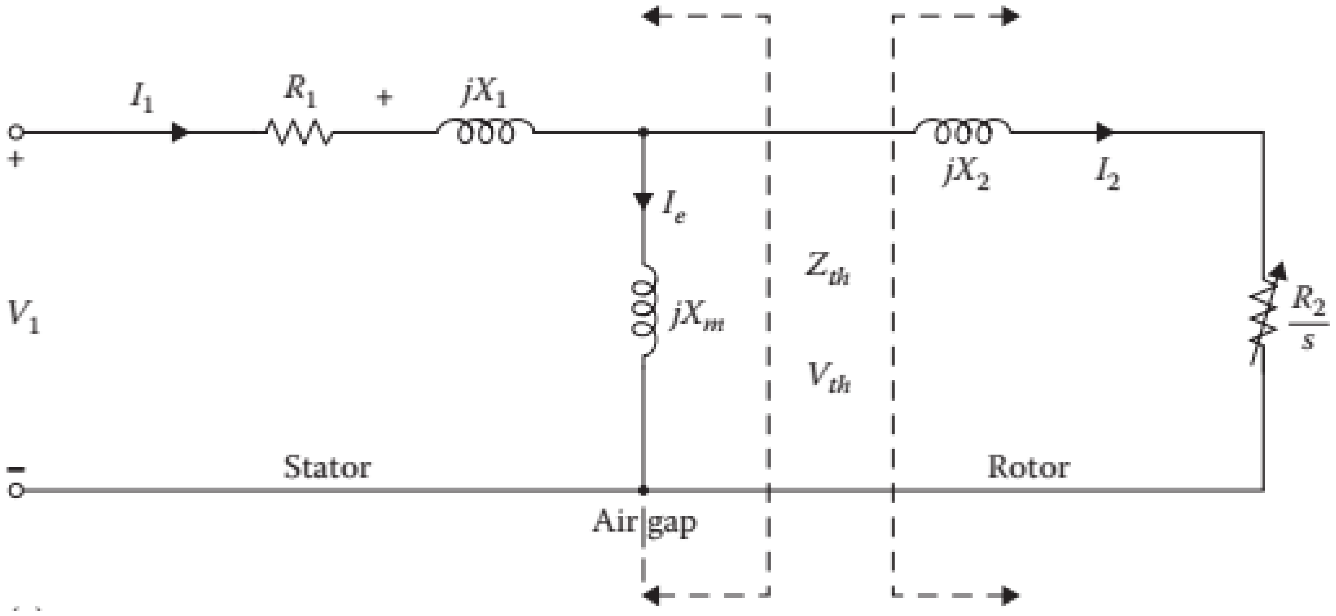
For a speed of 1720 rpm, we get

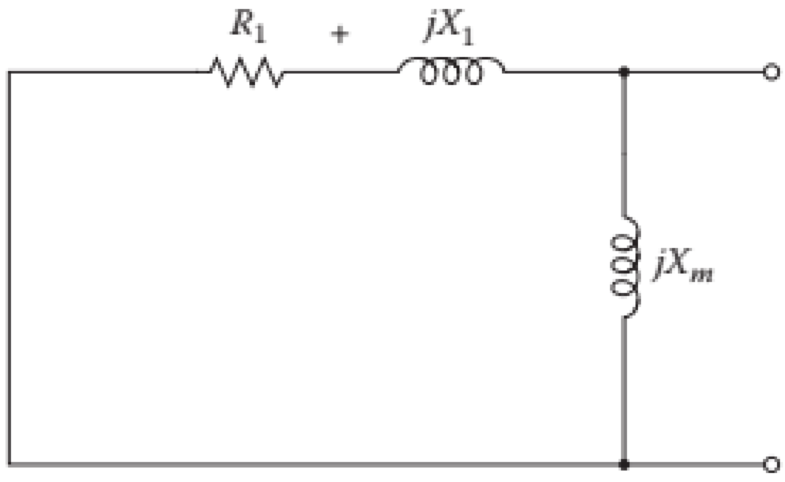
$$I_r = \frac{V}{\sqrt{(R_2/s)^2 + X_2^2}} = 15.419 \text{ A}$$

The equivalent resistance of the load is

$$R_L = R_2 \frac{1-s}{s}$$
$$= 3.87\,\Omega$$

DETERMINATION OF POWER AND TORQUE BY USE OF THÉVENIN'S EQUIVALENT CIRCUIT





(b)

$$\mathbf{V}_{th} = \mathbf{V}_1\left(\frac{jX_m}{R_1 + jX_1 + jX_m}\right)$$

The magnitude of the Thévenin voltage is

$$\mathbf{V}_{th} = \mathbf{V}_1\left(\frac{X_m}{[R_1^2 + (X_1 + jX_m)^2]^{1/2}}\right)$$

However, since $R_1^2 << (X_1 + X_m)^2$, the voltage is approximately

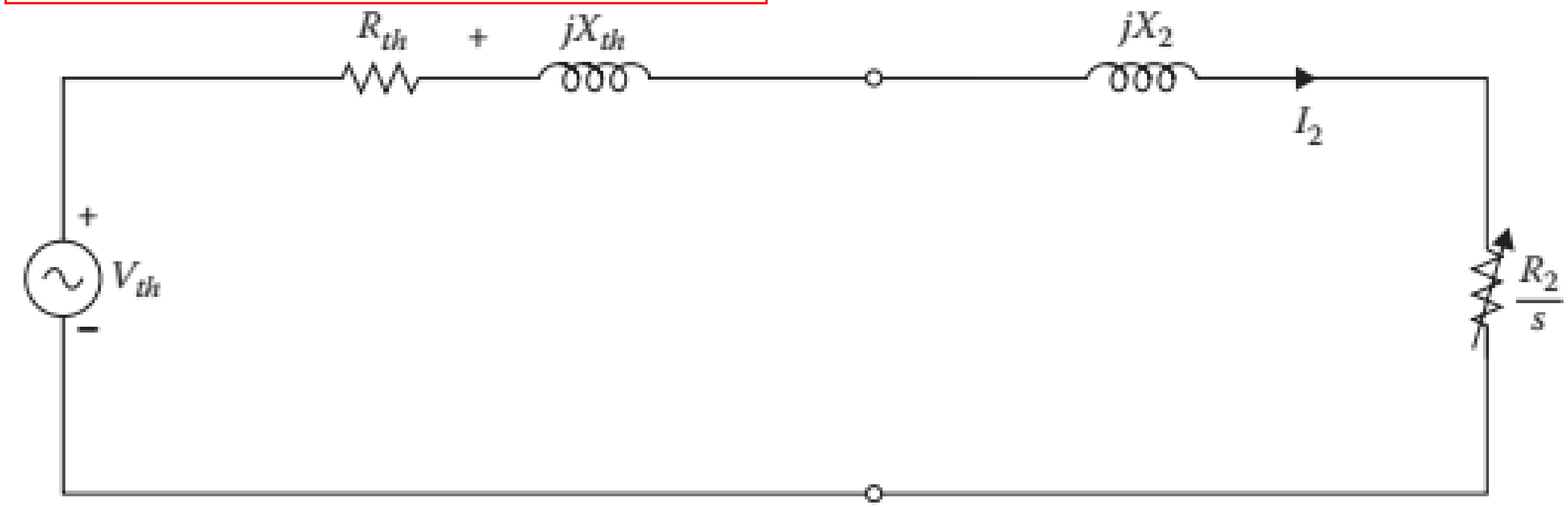
$$\mathbf{V}_{th} = \mathbf{V}_1\left(\frac{X_m}{X_1 + X_m}\right)$$

$$\mathbf{Z}_{th} = R_{th} + jX_{th}$$

$$= \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

Since $X_1 << X_m$ and $R_1^2 << \left(X_1 + X_m^2\right)$, the Thévenin resistance and reactance are approximately

$$R_{th} \cong R_1\left(\frac{X_m}{X_1 + X_m}\right)^2 \qquad X_{th} \cong X_1$$



$$\boldsymbol{I}_2 = \frac{\boldsymbol{V}_{th}}{\boldsymbol{Z}_{th} + \boldsymbol{Z}_2}$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{V}_{th}}{R_{th} + R_2 / s + j(X_{th} + jX_2)}$$

The magnitude of the rotor current is

$$I_2 = \frac{V_{th}}{\left[(R_{th} + R_2 / s)^2 + (X_{th} + X_2)^2\right]^{1/2}}$$

Thus, the corresponding air-gap power is

$$P_g = 3I_2^2\left(\frac{R_2}{s}\right)$$

$$P_g = \frac{3V_{th}(R_2 / s)}{\left[(R_{th} + R_2 / s)^2 + (X_{th} + X_2)^2\right]}$$

Therefore, the developed torque is

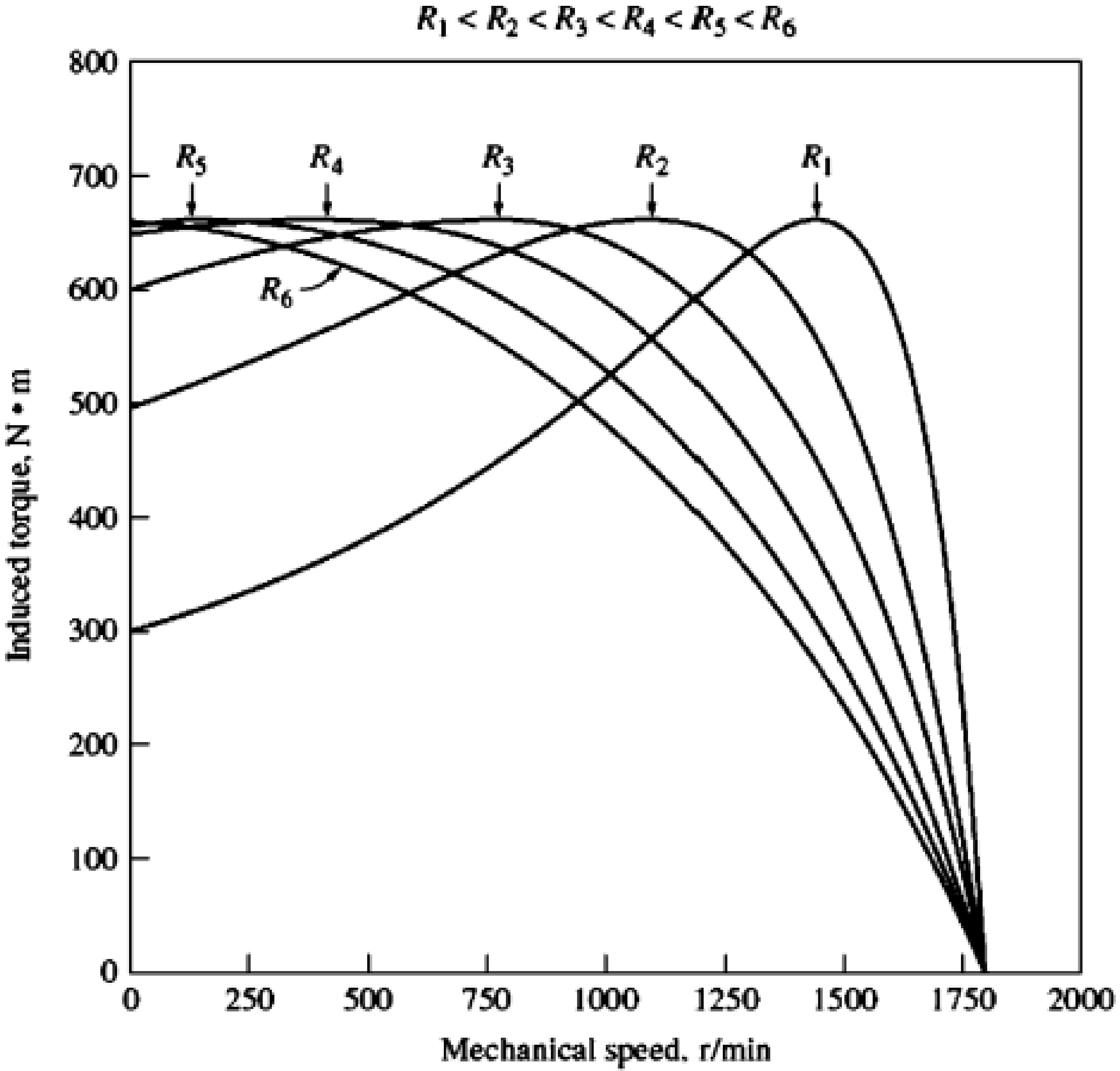
$$T_d = \frac{P_g}{\omega_s}$$

$$= \frac{3I_2^2(R_2 / s)}{\omega_s}$$

$$T_d = \frac{3V_{th}^2(R_2 / s)}{\omega_s\left[(R_{th} + R_2 / s)^2 + (X_{th} + X_2)^2\right]}$$

Since at start-up the slip is unity, the developed starting torque is

$$T_{start} = \frac{3V_{th}^2 R_2}{\omega_s\left[(R_{th} + R_2)^2 + (X_{th} + X_2)^2\right]}$$



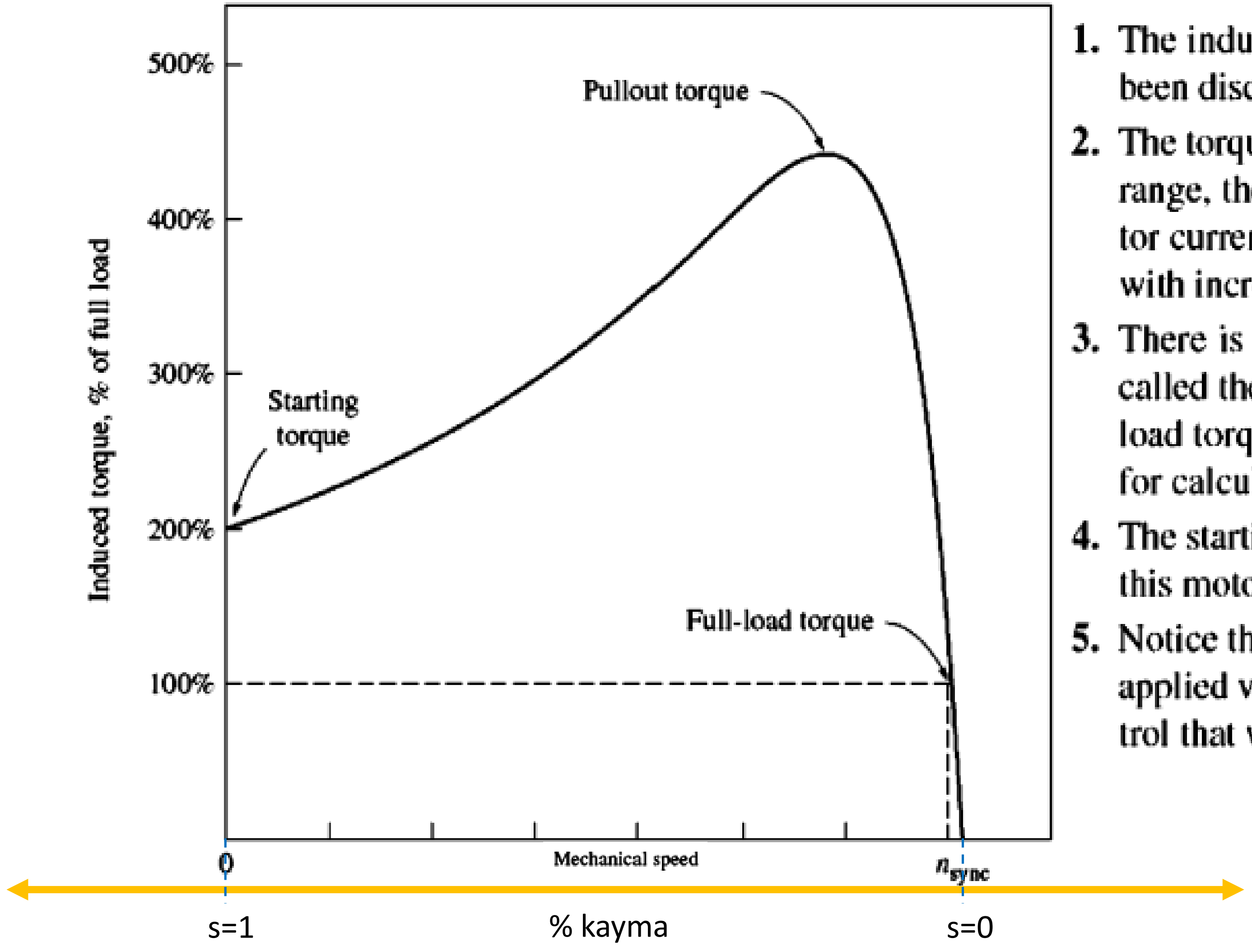
$$Z_{\text{source}} = R_{\text{TH}} + jX_{\text{TH}} + jX_2$$

$$\frac{R_2}{s} = \sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}$$

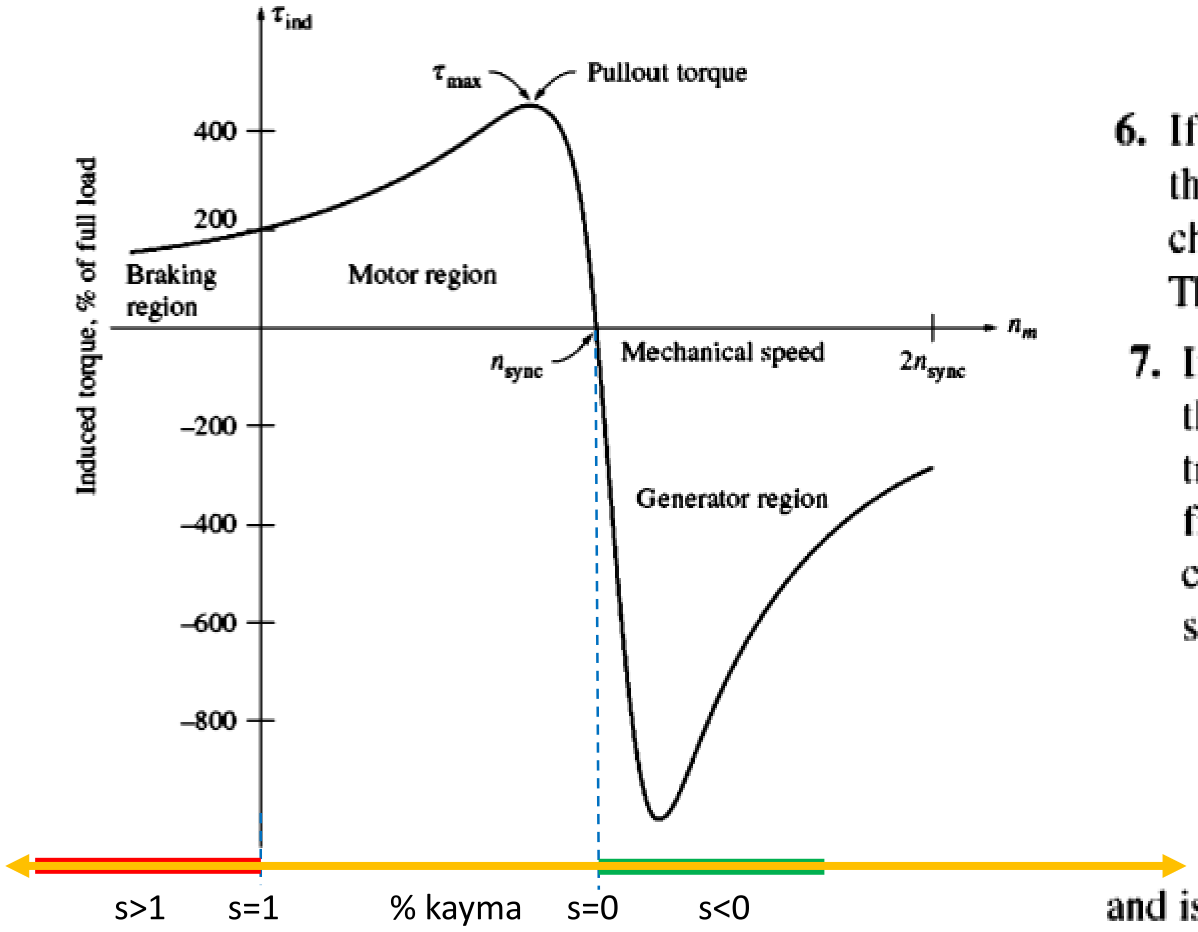
$$\boxed{s_{\max} = \frac{R_2}{\sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}}}$$

$$\boxed{\tau_{\max} = \frac{3V_{\text{TH}}^2}{2\omega_{\text{sync}}\left[R_{\text{TH}} + \sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}\right]}}$$

Comments on the Induction Motor Torque–Speed Curve



1. The induced torque of the motor is zero at synchronous speed. This fact has been discussed previously.
2. The torque–speed curve is nearly linear between no load and full load. In this range, the rotor resistance is much larger than the rotor reactance, so the rotor current, the rotor magnetic field, and the induced torque increase linearly with increasing slip.
3. There is a maximum possible torque that cannot be exceeded. This torque, called the *pullout torque* or *breakdown torque*, is 2 to 3 times the rated full-load torque of the motor. The next section of this chapter contains a method for calculating pullout torque.
4. The starting torque on the motor is slightly larger than its full-load torque, so this motor will start carrying any load that it can supply at full power.
5. Notice that the torque on the motor for a given slip varies as the square of the applied voltage. This fact is useful in one form of induction motor speed control that will be described later.



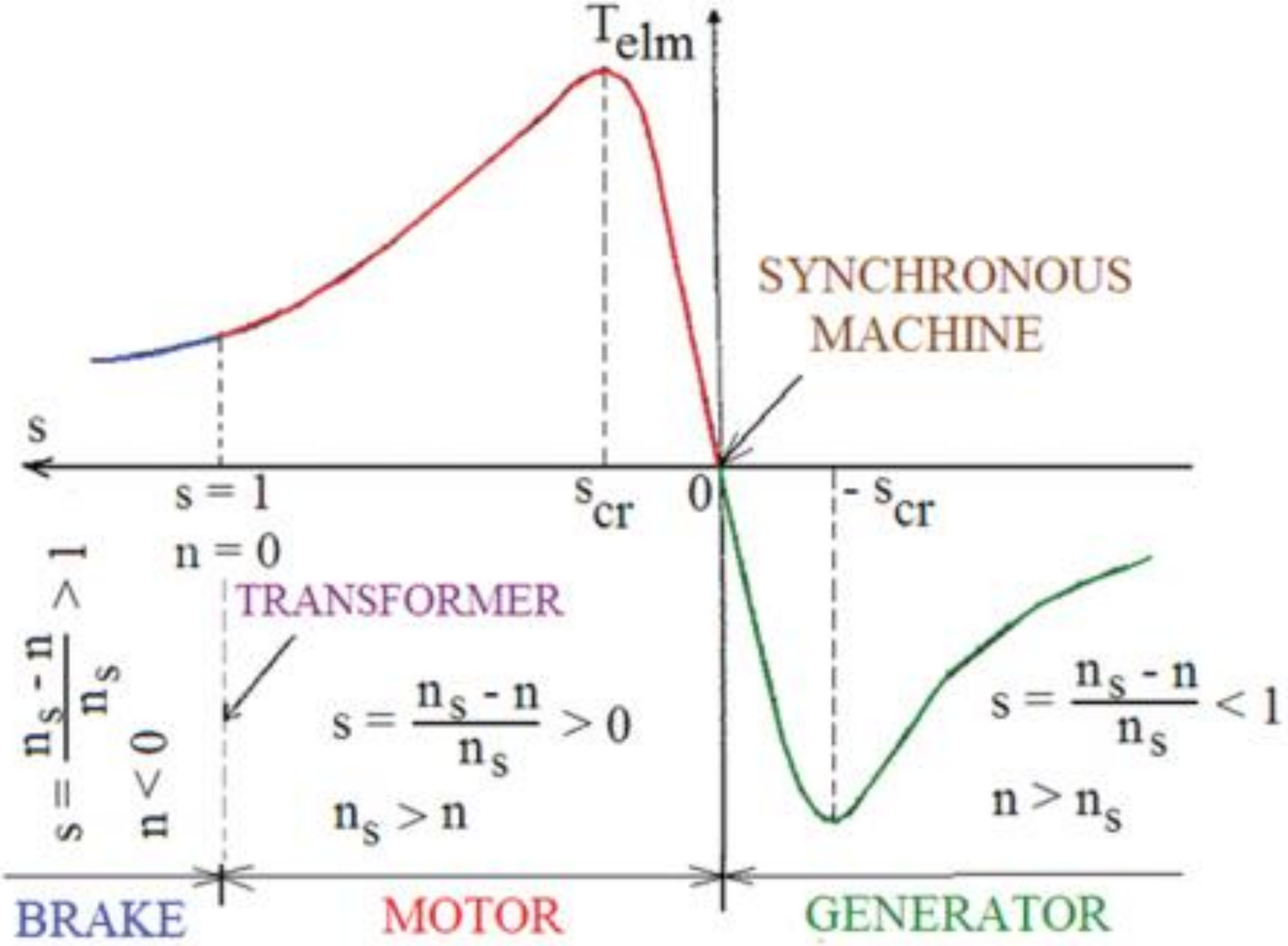
6. If the rotor of the induction motor is driven faster than synchronous speed, then the direction of the induced torque in the machine reverses and the machine becomes a *generator*, converting mechanical power to electric power. The use of induction machines as generators will be described later.

7. If the motor is turning backward relative to the direction of the magnetic fields, the induced torque in the machine will stop the machine very rapidly and will try to rotate it in the other direction. Since reversing the direction of magnetic field rotation is simply a matter of switching any two stator phases, this fact can be used as a way to very rapidly stop an induction motor. The act of switching two phases in order to stop the motor very rapidly is called *plugging*.

The power converted to mechanical form in an induction motor is equal to

$$P_{conv} = \tau_{ind}\,\omega_m$$

and is shown plotted in Figure 7–21. Notice that the peak power supplied by the induction motor occurs at a different speed than the maximum torque; and, of course, no power is converted to mechanical form when the rotor is at zero speed.



T_{elm}
SYNCHRONOUS
MACHINE
s
s = 1
n = 0
s_{cr}
0
$-s_{cr}$
$s = \frac{n_s - n}{n_s} > 1$
n < 0
TRANSFORMER
$s = \frac{n_s - n}{n_s} > 0$
$n_s > n$
$s = \frac{n_s - n}{n_s} < 1$
$n > n_s$
BRAKE
MOTOR
GENERATOR

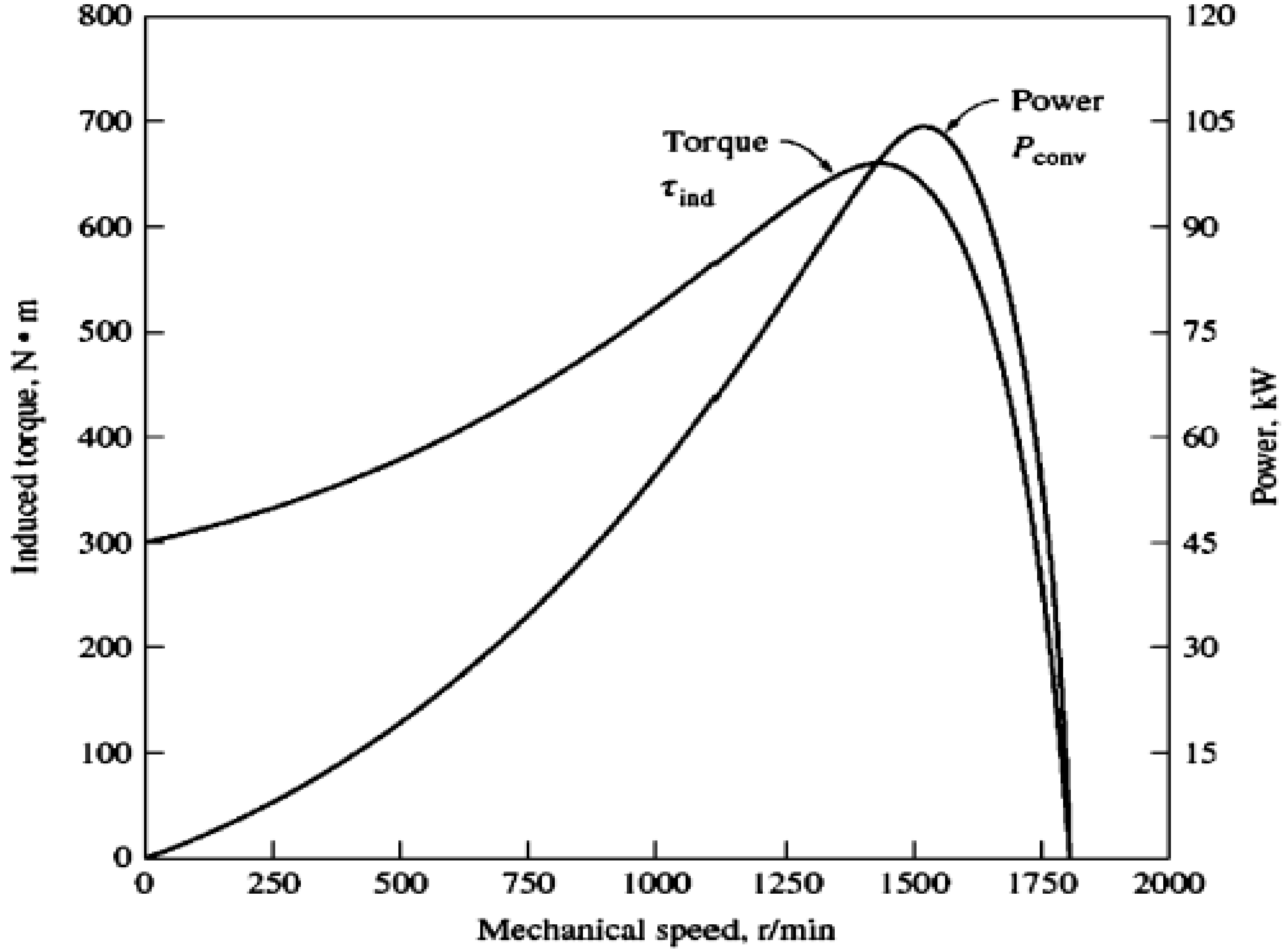


FIGURE 7–21
Induced torque and power converted versus motor speed in revolutions per minute for an example four-pole induction motor.

Example 7–4. A two-pole, 50-Hz induction motor supplies 15 kW to a load at a speed of 2950 r/min.

(*a*) What is the motor's slip?
(*b*) What is the induced torque in the motor in N • m under these conditions?
(*c*) What will the operating speed of the motor be if its torque is doubled?
(*d*) How much power will be supplied by the motor when the torque is doubled?

Solution

(*a*) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{2\text{ poles}} = 3000\text{ r/min}$$

Therefore, the motor's slip is

$$s = \frac{n_{\text{sync}} - n_{\text{m}}}{n_{\text{sync}}}(\times 100\%)$$

$$= \frac{3000\text{ r/min} - 2950\text{ r/min}}{3000\text{ r/min}}(\times 100\%)$$

$$= 0.0167 \text{ or } 1.67\%$$

(*b*) The induced torque in the motor must be assumed equal to the load torque, and P_{conv} must be assumed equal to P_{load}, since no value was given for mechanical losses. The torque is thus

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_{\text{m}}}$$

$$= \frac{15\text{ kW}}{(2950\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})}$$

$$= 48.6\text{ N} \bullet \text{m}$$

(*c*) In the low-slip region, the torque–speed curve is linear, and the induced torque is directly proportional to slip. Therefore, if the torque doubles, then the new slip will be 3.33 percent. The operating speed of the motor is thus

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.0333)(3000\text{ r/min}) = 2900\text{ r/min}$$

(*d*) The power supplied by the motor is given by

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$

$$= (97.2\text{ N} \bullet \text{m})(2900\text{ r/min})(2\pi\text{ rad/r})(1\text{ min}/60\text{ s})$$

$$= 29.5\text{ kW}$$

Example 7–5. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \qquad X_2 = 0.464\ \Omega \qquad X_M = 26.3\ \Omega$$

(*a*) What is the maximum torque of this motor? At what speed and slip does it occur?

(*b*) What is the starting torque of this motor?

(*c*) When the rotor resistance is doubled, what is the speed at which the maximum torque now occurs? What is the new starting torque of the motor?

(*d*) Calculate and plot the torque–speed characteristics of this motor both with the original rotor resistance and with the rotor resistance doubled.

Solution

The Thevenin voltage of this motor is

$$V_{TH} = V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

$$= \frac{(266\text{ V})(26.3\ \Omega)}{\sqrt{(0.641\ \Omega)^2 + (1.106\ \Omega + 26.3\ \Omega)^2}} = 255.2\text{ V}$$

The Thevenin resistance is

$$R_{TH} \approx R_1\left(\frac{X_M}{X_1 + X_M}\right)^2$$

$$\approx (0.641\ \Omega)\left(\frac{26.3\ \Omega}{1.106\ \Omega + 26.3\ \Omega}\right)^2 = 0.590\ \Omega$$

The Thevenin reactance is

$$X_{TH} \approx X_1 = 1.106\ \Omega$$

(*a*) The slip at which maximum torque occurs is given by Equation (7–53):

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \tag{7–53}$$

$$= \frac{0.332\ \Omega}{\sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}} = 0.198$$

This corresponds to a mechanical speed of

$$n_m = (1 - s)n_{sync} = (1 - 0.198)(1800\text{ r/min}) = 1444\text{ r/min}$$

The torque at this speed is

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}]} \tag{7–54}$$

$$= \frac{3(255.2\text{ V})^2}{2(188.5\text{ rad/s})[0.590\ \Omega + \sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}]}$$

$$= 229\text{ N}\cdot\text{m}$$

(*b*) The starting torque of this motor is found by setting $s = 1$ in Equation (7–50):

$$\tau_{\text{start}} = \frac{3V_{\text{TH}}^2 R_2}{\omega_{\text{sync}}[(R_{\text{TH}} + R_2)^2 + (X_{\text{TH}} + X_2)^2]}$$

$$= \frac{3(255.2\text{ V})^2(0.332\ \Omega)}{(188.5\text{ rad/s})[(0.590\ \Omega + 0.332\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 104\text{ N}\bullet\text{m}$$

(*c*) If the rotor resistance is doubled, then the slip at maximum torque doubles, too. Therefore,

$$s_{\max} = 0.396$$

and the speed at maximum torque is

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.396)(1800\text{ r/min}) = 1087\text{ r/min}$$

The maximum torque is still

$$\tau_{\max} = 229\text{ N}\bullet\text{m}$$

The starting torque is now

$$\tau_{\text{start}} = \frac{3(255.2\text{ V})^2(0.664\ \Omega)}{(188.5\text{ rad/s})[(0.590\ \Omega + 0.664\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 170\text{ N}\bullet\text{m}$$

EXAMPLE 5.4

A three-phase, 460 V, 1740 rpm, 60 Hz, four-pole wound-rotor induction motor has the following parameters per phase:

$$R_1 = 0.25 \text{ ohms}, \qquad R_2' = 0.2 \text{ ohms}$$
$$X_1 = X_2' = 0.5 \text{ ohms}, \qquad X_m = 30 \text{ ohms}$$

The rotational losses are 1700 watts. With the rotor terminals short-circuited, find

(a) **(i)** Starting current when started direct on full voltage.
(ii) Starting torque.

(b) **(i)** Full-load slip.
(ii) Full-load current.
(iii) Ratio of starting current to full-load current.
(iv) Full-load power factor.
(v) Full-load torque.
(vi) Internal efficiency and motor efficiency at full load.

(c) **(i)** Slip at which maximum torque is developed.
(ii) Maximum torque developed.

(d) How much external resistance per phase should be connected in the rotor circuit so that maximum torque occurs at start?

Solution

(a)

$$V_1 = \frac{460}{\sqrt{3}} = 265.6 \text{ volts/phase}$$

(i) At start $s = 1$. The input impedance is

$$Z_1 = 0.25 + j0.5 + \frac{j30(0.2 + j0.5)}{0.2 + j30.5}$$

$$= 1.08\angle 66^\circ \ \Omega$$

$$I_{st} = \frac{265.6}{1.08\angle 66^\circ} = 245.9\angle -66^\circ \text{ A}$$

(ii)

$$\omega_{syn} = \frac{1800}{60} \times 2\pi = 188.5 \text{ rad/sec}$$

$$V_{th} = \frac{265.6(j30.0)}{(0.25 + j30.5)} \approx 261.3 \text{ V}$$

$$Z_{th} = \frac{j30(0.25 + j0.5)}{0.25 + j30.5} = 0.55\angle 63.9^\circ$$

$$= 0.24 + j0.49$$

$$R_{th} = 0.24 \ \Omega$$

$$X_{th} = 0.49 \simeq X_1$$

$$T_{st} = \frac{P_{ag}}{\omega_{syn}} = \frac{I_2'^2 R_2'/s}{\omega_{syn}}$$

$$= \frac{3}{188.5} \frac{261.3^2}{(0.24 + 0.2)^2 + (0.49 + 0.5)^2} \times \frac{0.2}{1}$$

$$= \frac{3}{188.5} \times (241.2)^2 \times \frac{0.2}{1}$$

$$s = 185.2 \text{ N} \cdot \text{m}$$

(b) **(i)** $$s = \frac{1800 - 1740}{1800} = 0.0333$$

(ii) $$\frac{R_2'}{s} = \frac{0.2}{0.0333} = 6.01\ \Omega$$

$$Z_1 = (0.25 + j0.5) + \frac{(j30)(6.01 + j0.5)}{6.01 + j30.5}$$

$$= 0.25 + j0.5 + 5.598 + j1.596$$

$$= 6.2123\underline{/19.7^\circ}\ \Omega$$

$$I_{\mathrm{FL}} = \frac{265.6}{6.2123\underline{/19.7}}$$

$$= 42.754\underline{/-19.7^\circ}$$

(iii) $$\frac{I_{\mathrm{st}}}{I_{\mathrm{FL}}} = \frac{245.9}{42.754} = 5.75$$

(iv) $$\mathrm{PF} = \cos(19.7^\circ) = 0.94\ (\text{lagging})$$

(v) $$T = \frac{3}{188.5}\frac{(261.3)^2}{(0.24 + 6.01)^2 + (0.49 + 0.5)^2} \times 6.01$$

$$= \frac{3}{188.5} \times 41.29^2 \times 6.01$$

$$= 163.11\ \mathrm{N \cdot m}$$

(vi) Air gap power:

$$P_{\mathrm{ag}} = T\omega_{\mathrm{syn}} = 163.11 \times 188.5 = 30{,}746.2\ \mathrm{W}$$

Rotor copper loss:

$$P_2 = sP_{\mathrm{ag}} = 0.0333 \times 30{,}746.2 = 1023.9\ \mathrm{W}$$

$$P_{\mathrm{mech}} = (1 - 0.0333)30{,}746.2 = 29{,}722.3\ \mathrm{W}$$

$$P_{\mathrm{out}} = P_{\mathrm{mech}} - P_{\mathrm{rot}} = 29{,}722.3 - 1700 = 28{,}022.3\ \mathrm{W}$$

$$P_{\mathrm{input}} = 3V_1 I_1 \cos\theta$$

$$= 3 \times 265.6 \times 42.754 \times 0.94 = 32{,}022.4\ \mathrm{W}$$

$$\mathrm{Eff}_{\mathrm{motor}} = \frac{28{,}022.3}{32{,}022.4} \times 100 = 87.5\%$$

$$\mathrm{Eff}_{\mathrm{internal}} = (1 - s) = 1 - 0.0333 = 0.967 \rightarrow 96.7\%$$

(c) **(i)** From Eq. 5.58,

$$s_{T_{\max}} = \frac{0.2}{[0.24^2 + (0.49 + 0.5)^2]^{1/2}} = \frac{0.2}{1.0187} = 0.1963$$

(ii) From Eq. 5.59,

$$T_{\max} = \frac{3}{2 \times 188.5}\left[\frac{261.3^2}{0.24 + [0.24^2 + (0.49 + 0.5)^2]^{1/2}}\right]$$

$$= 431.68\ \mathrm{N \cdot m}$$

$$\frac{T_{\max}}{T_{\mathrm{FL}}} = \frac{431.68}{163.11} = 2.65$$

$$\frac{R_2'}{s_{T_{\max}}} = [R_{\mathrm{th}}^2 + (X_{\mathrm{th}} + X_2')^2]^{1/2}$$

$$s_{T_{\max}} = \frac{R_2'}{[R_{\mathrm{th}}^2 + (X_{\mathrm{th}} + X_2')^2]^{1/2}}$$

$$T_{\max} = \frac{1}{2\omega_{\mathrm{syn}}} \frac{V_{\mathrm{th}}^2}{R_{\mathrm{th}} + [R_{\mathrm{th}}^2 + (X_{\mathrm{th}} + X_2')^2]^{1/2}}$$

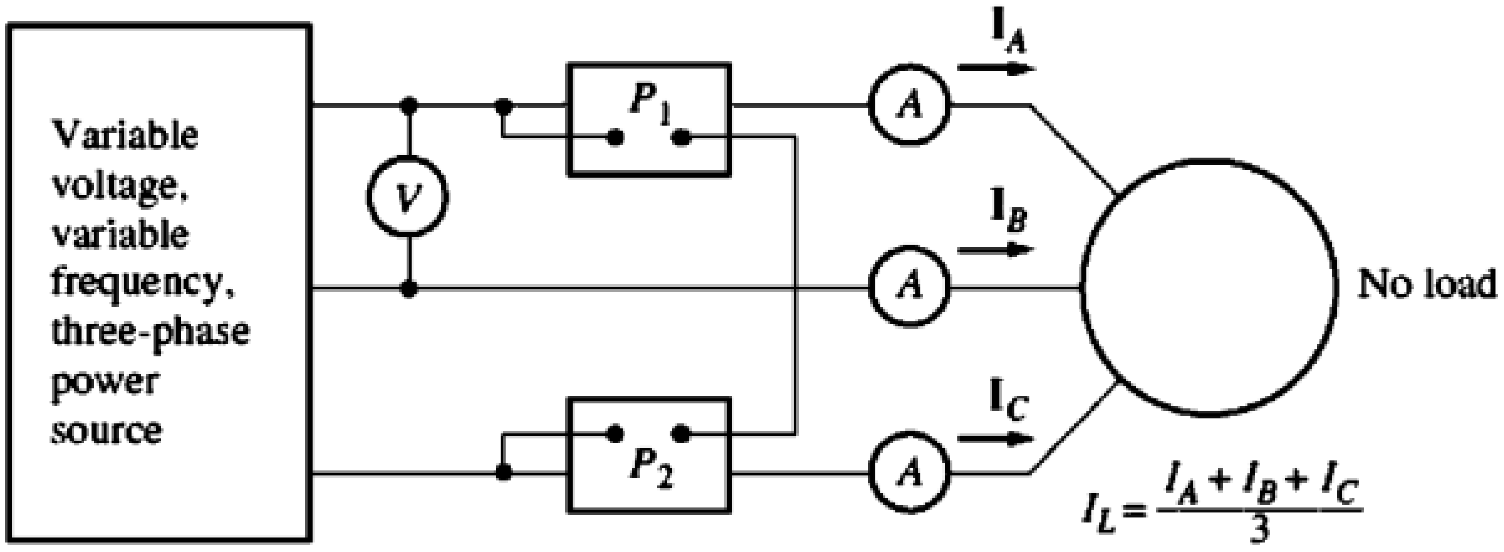
(d)

$$s_{T_{\max}} = \frac{R_2' + R_{\mathrm{ext}}'}{[0.24^2 + (0.49 + 0.5)^2]^{1/2}} = \frac{R_2' + R_{\mathrm{ext}}'}{1.0186}$$

$$R_{\mathrm{ext}}' = 1.0186 - 0.2 = 0.8186\ \Omega/\mathrm{phase}$$

TESTS TO DETERMINE CIRCUIT-MODEL PARAMETERS

The No-Load (NL) Test



In this test the motor is run on no-load at rated voltage and frequency. The applied voltage and current and power input to motor are measured by the metering

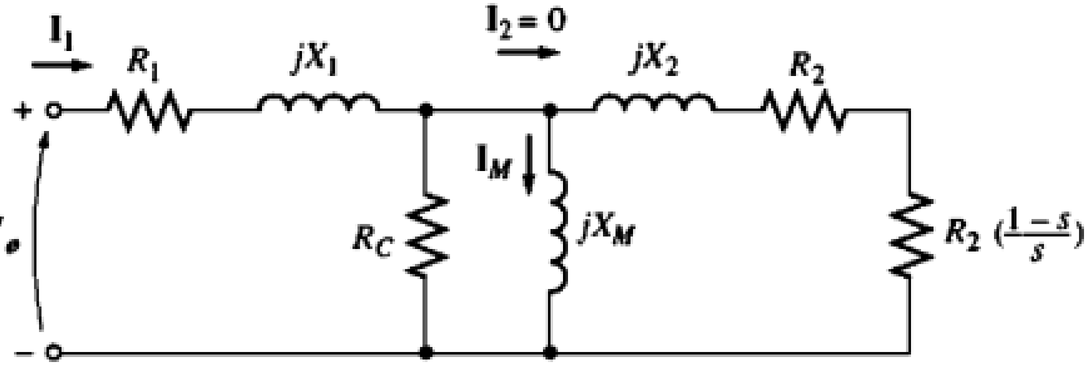
Let the meter readings be

Power input = P_0 (3-phase)

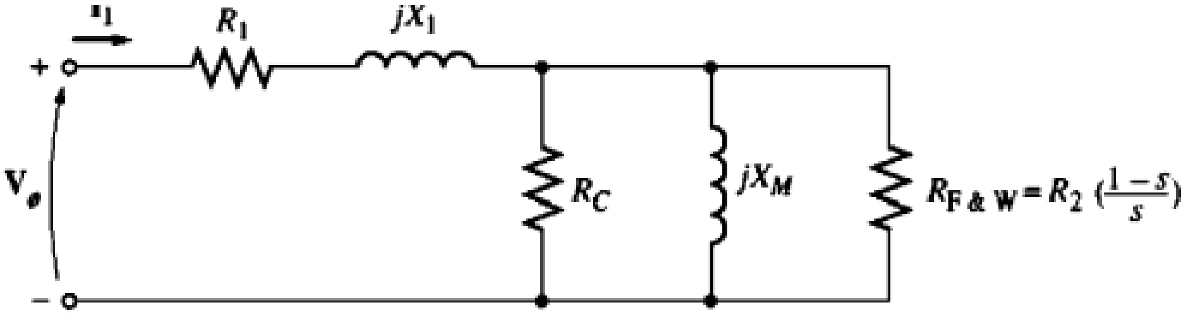
Current = I_0 (average of the three meter readings)

Voltage = V_0 (line-to-line rated voltage)

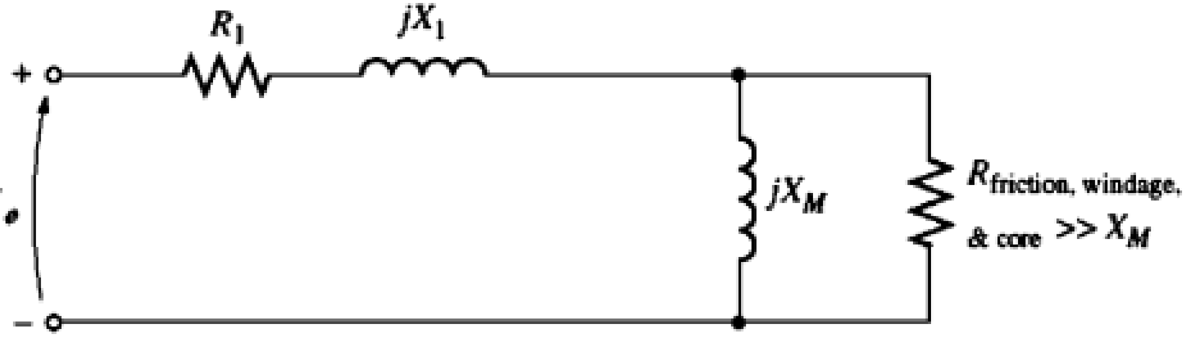
Initial equivalent circuit:



Since $R_2\left(\frac{1-s}{s}\right) >> R_2$ and $R_2\left(\frac{1-s}{s}\right) >> X_2$, this circuit reduces to:



Combining $R_{F\,\&\,W}$ and R_C yields:

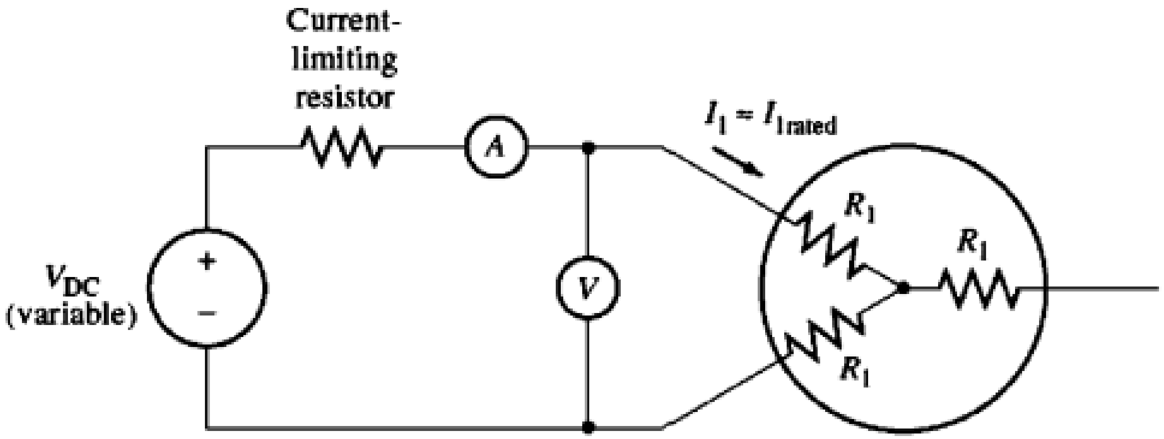


$$\begin{aligned} P_{in} &= P_{SCL} + P_{core} + P_{F\&W} + P_{misc} \\ &= 3I_1^2 R_1 + P_{rot} \end{aligned}$$

$$P_{rot} = P_{core} + P_{F\&W} + P_{misc}$$

$$\boxed{\left|Z_{eq}\right| = \frac{V_\phi}{I_{1,nl}} \approx X_1 + X_M}$$

The DC Test for Stator Resistance



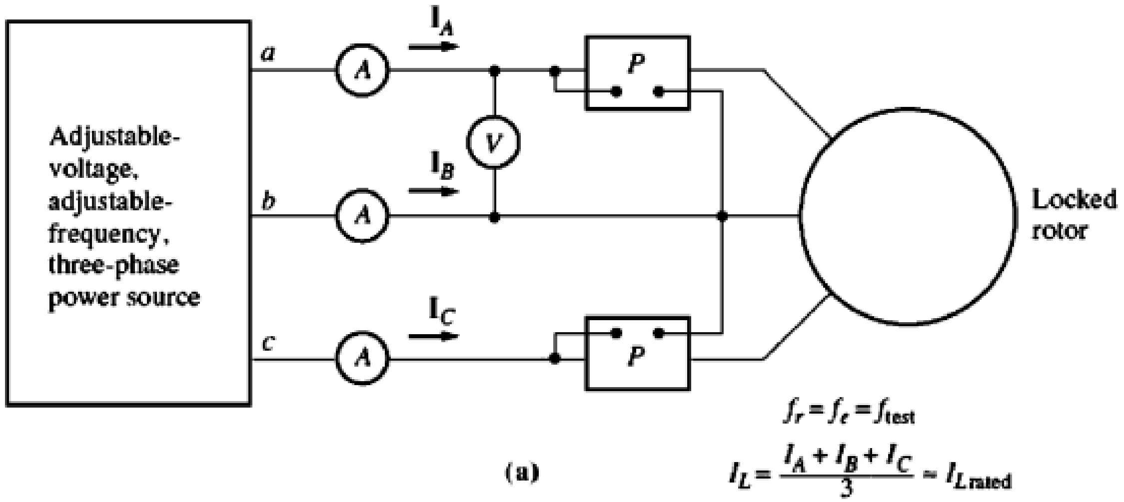
$$2R_1 = \frac{V_{DC}}{I_{DC}}$$

$$\boxed{R_1 = \frac{V_{DC}}{2I_{DC}}}$$

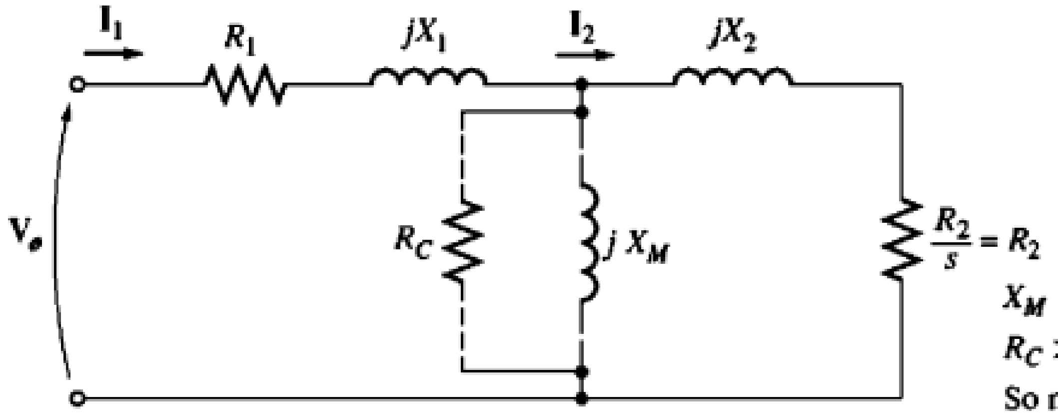
The value of R_1 calculated in this fashion is not completely accurate, since it neglects the skin effect that occurs when an ac voltage is applied to the windings. More details concerning corrections for temperature and skin effect can be found in IEEE Standard 112.

Blocked-Rotor (BR) Test

This test is used to determine the series parameters of the circuit model of an induction motor. The circuit is similar to that of a transformer short-circuit test. Short circuiting the load resistance in the circuit model of Fig. 9.8 corresponds to making $s = 1$ so that $R_2'(1/s - 1) = 0$. This means that the rotor must be stationary during this test, which requires that it be *blocked mechanically* from rotating while the stator is excited with appropriate reduced voltage. The circuit model seen under these conditions is given



(a)



$X_M >> |R_2 + jX_2|$

$R_C >> |R_2 + jX_2|$

So neglect R_C and X_M

$$P = \sqrt{3} V_T I_L \cos\theta$$

so the locked-rotor power factor can be found as

$$\boxed{\mathrm{PF} = \cos\theta = \frac{P_{\text{in}}}{\sqrt{3} V_T I_L}}$$

and the impedance angle θ is just equal to $\cos^{-1}$ PF.

The magnitude of the total impedance in the motor circuit at this time is

$$\boxed{|Z_{\mathrm{LR}}| = \frac{V_\phi}{I_1} = \frac{V_T}{\sqrt{3} I_L}} \tag{7–63}$$

and the angle of the total impedance is θ. Therefore,

$$\begin{aligned} Z_{\mathrm{LR}} &= R_{\mathrm{LR}} + jX'_{\mathrm{LR}} \\ &= |Z_{\mathrm{LR}}|\cos\theta + j|Z_{\mathrm{LR}}|\sin\theta \end{aligned} \tag{7–64}$$

The locked-rotor resistance R_{LR} is equal to

$$\boxed{R_{\mathrm{LR}} = R_1 + R_2} \tag{7–65}$$

while the locked-rotor reactance X'_{LR} is equal to

$$X'_{\mathrm{LR}} = X'_1 + X'_2 \tag{7–66}$$

where X'_1 and X'_2 are the stator and rotor reactances *at the test frequency*, respectively.

The rotor resistance R_2 can now be found as

$$R_2 = R_{LR} - R_1 \tag{7–67}$$

$$\boxed{X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = X_1 + X_2}$$

	X_1 and X_2 as functions of X_{LR}	
Rotor Design	X_1	X_2
Wound rotor	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design A	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design B	$0.4\ X_{LR}$	$0.6\ X_{LR}$
Design C	$0.3\ X_{LR}$	$0.7\ X_{LR}$
Design D	$0.5\ X_{LR}$	$0.5\ X_{LR}$

Example 7–8. The following test data were taken on a 7.5-hp, four-pole, 208-V, 60-Hz, design A, Y-connected induction motor having a rated current of 28 A.

DC test:

$V_{DC} = 13.6\ \text{V}$ $\quad I_{DC} = 28.0\ \text{A}$

No-load test:

$V_T = 208\ \text{V}$ $\quad f = 60\ \text{Hz}$

$I_A = 8.12\ \text{A}$ $\quad P_{in} = 420\ \text{W}$

$I_B = 8.20\ \text{A}$

$I_C = 8.18\ \text{A}$

Locked-rotor test:

$V_T = 25\ \text{V}$ $\quad f = 15\ \text{Hz}$

$I_A = 28.1\ \text{A}$ $\quad P_{in} = 920\ \text{W}$

$I_B = 28.0\ \text{A}$

$I_C = 27.6\ \text{A}$

(*a*) Sketch the per-phase equivalent circuit for this motor.

(*b*) Find the slip at the pullout torque, and find the value of the pullout torque itself.

Solution

(a) From the dc test,

$$R_1 = \frac{V_{DC}}{2I_{DC}} = \frac{13.6\text{ V}}{2(28.0\text{ A})} = 0.243\ \Omega$$

From the no-load test,

$$I_{L,av} = \frac{8.12\text{ A} + 8.20\text{ A} + 8.18\text{ A}}{3} = 8.17\text{ A}$$

$$V_{\phi,nl} = \frac{208\text{ V}}{\sqrt{3}} = 120\text{ V}$$

Therefore,

$$|Z_{nl}| = \frac{120\text{ V}}{8.17\text{ A}} = 14.7\ \Omega = X_1 + X_M$$

When X_1 is known, X_M can be found. The stator copper losses are

$$P_{SCL} = 3I_1^2 R_1 = 3(8.17\text{ A})^2(0.243\ \Omega) = 48.7\text{ W}$$

Therefore, the no-load rotational losses are

$$P_{rot} = P_{in,nl} - P_{SCL,nl}$$
$$= 420\text{ W} - 48.7\text{ W} = 371.3\text{ W}$$

From the locked-rotor test,

$$I_{L,av} = \frac{28.1\text{ A} + 28.0\text{ A} + 27.6\text{ A}}{3} = 27.9\text{ A}$$

The locked-rotor impedance is

$$|Z_{LR}| = \frac{V_\phi}{I_A} = \frac{V_T}{\sqrt{3}I_A} = \frac{25\text{ V}}{\sqrt{3}(27.9\text{ A})} = 0.517\ \Omega$$

and the impedance angle θ is

$$\theta = \cos^{-1}\frac{P_{in}}{\sqrt{3}V_T I_L}$$
$$= \cos^{-1}\frac{920\text{ W}}{\sqrt{3}(25\text{ V})(27.9\text{ A})}$$
$$= \cos^{-1} 0.762 = 40.4°$$

Therefore, $R_{LR} = 0.517 \cos 40.4° = 0.394\ \Omega = R_1 + R_2$. Since $R_1 = 0.243\ \Omega$, R_2 must be $0.151\ \Omega$. The reactance at 15 Hz is

$$X'_{LR} = 0.517 \sin 40.4° = 0.335\ \Omega$$

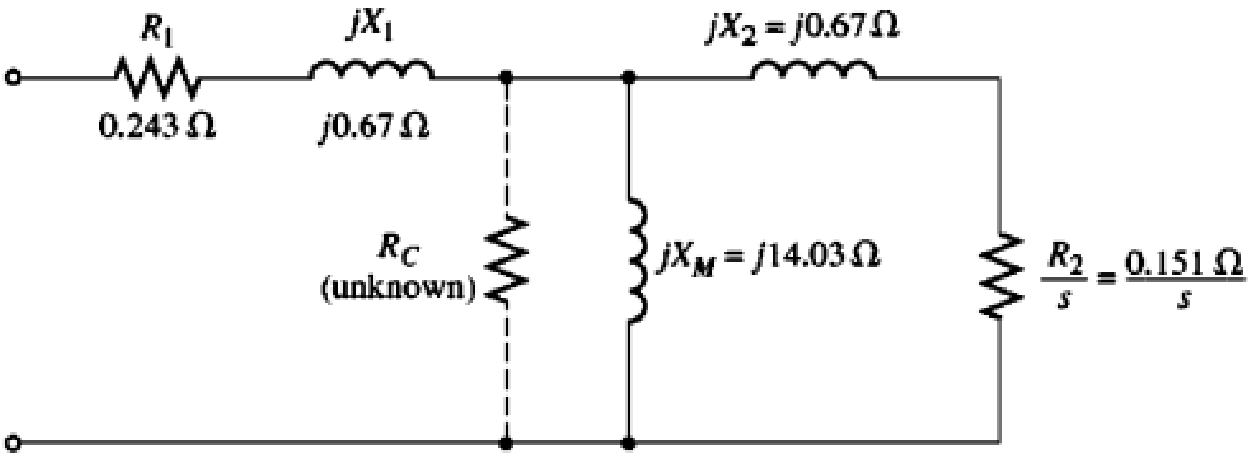
The equivalent reactance at 60 Hz is

$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = \left(\frac{60\text{ Hz}}{15\text{ Hz}}\right) 0.335\ \Omega = 1.34\ \Omega$$

For design class A induction motors, this reactance is assumed to be divided equally between the rotor and stator, so

$$X_1 = X_2 = 0.67\ \Omega$$

$$X_M = |Z_{nl}| - X_1 = 14.7\ \Omega - 0.67\ \Omega = 14.03\ \Omega$$



(b) For this equivalent circuit, the Thevenin equivalents are found from Equations (7–41b), (7–44), and (7–45) to be

$$V_{TH} = 114.6\ \text{V} \qquad R_{TH} = 0.221\ \Omega \qquad X_{TH} = 0.67\ \Omega$$

Therefore, the slip at the pullout torque is given by

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \tag{7–53}$$

$$= \frac{0.151\ \Omega}{\sqrt{(0.243\ \Omega)^2 + (0.67\ \Omega + 0.67\ \Omega)^2}} = 0.111 = 11.1\%$$

The maximum torque of this motor is given by

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X^2)}]} \tag{7–54}$$

$$= \frac{3(114.6\ \text{V})^2}{2(188.5\ \text{rad/s})[0.221\ \Omega + \sqrt{(0.221\ \Omega)^2 + (0.67\ \Omega + 0.67\ \Omega)^2}]}$$

$$= 66.2\ \text{N} \cdot \text{m}$$

$$\mathbf{V}_{th} = \mathbf{V}_1\left(\frac{X_m}{X_1 + X_m}\right) \qquad R_{th} \cong R_1\left(\frac{X_m}{X_1 + X_m}\right)^2$$

$$\mathbf{Z}_{th} = R_{th} + jX_{th}$$

$$= \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} \qquad X_{th} \cong X_1$$

Example 6.1

No-load and locked-rotor tests have been performed on a three-phase, four-pole, 60-Hz, 10-kW, Y-connected, 208-V (line-to-line) cage induction motor, with the following results:

No-load test: input frequency $f = 60$ Hz, input voltage (line-to-line) $V_{10L} = 208$ V, no-load current $I_{10} = 6.49$ A, no-load power $P_{in0} = 332$ W, stator winding resistance per phase $R_1 = 0.25\ \ \Omega$.

Locked-rotor test (s=1): input frequency $f = 60$ Hz, input voltage (line-to-line) $V_{1shL} = 78.5$ V, input current $I_{1n} = 42$ A, input active power $P_{insh} = 2116.8$ W.

7.6 VARIATIONS IN INDUCTION MOTOR TORQUE–SPEED CHARACTERISTICS

Control of Motor Characteristics by Cage Rotor Design

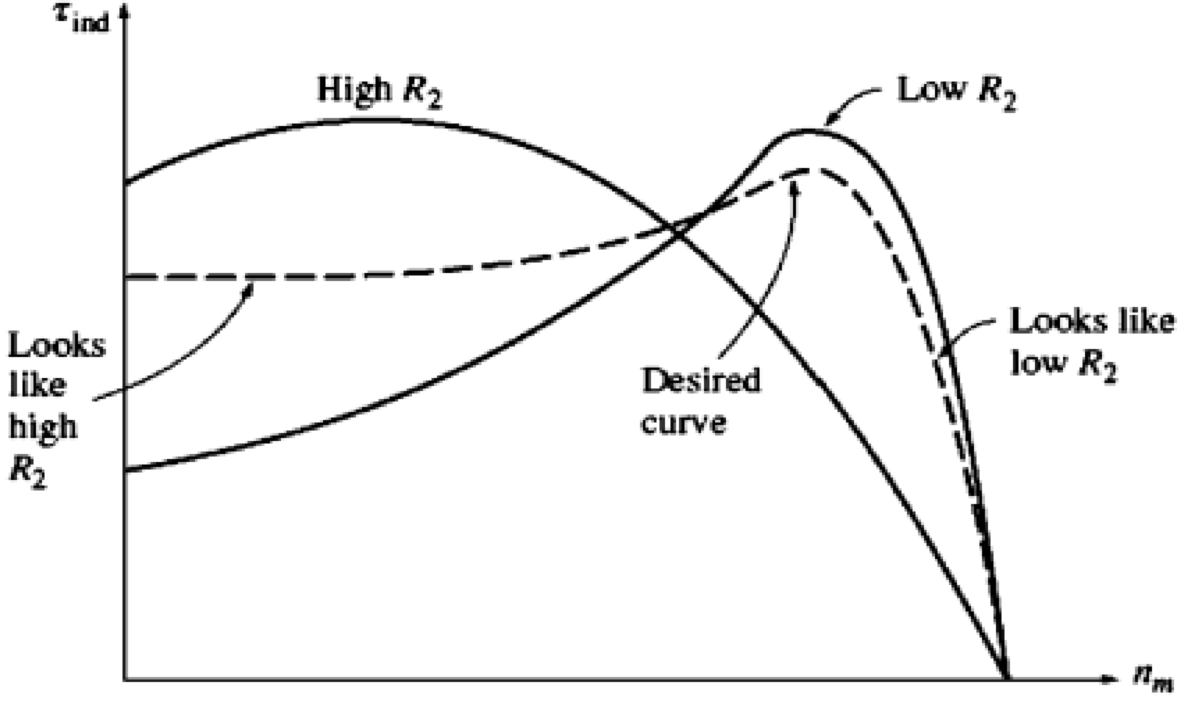
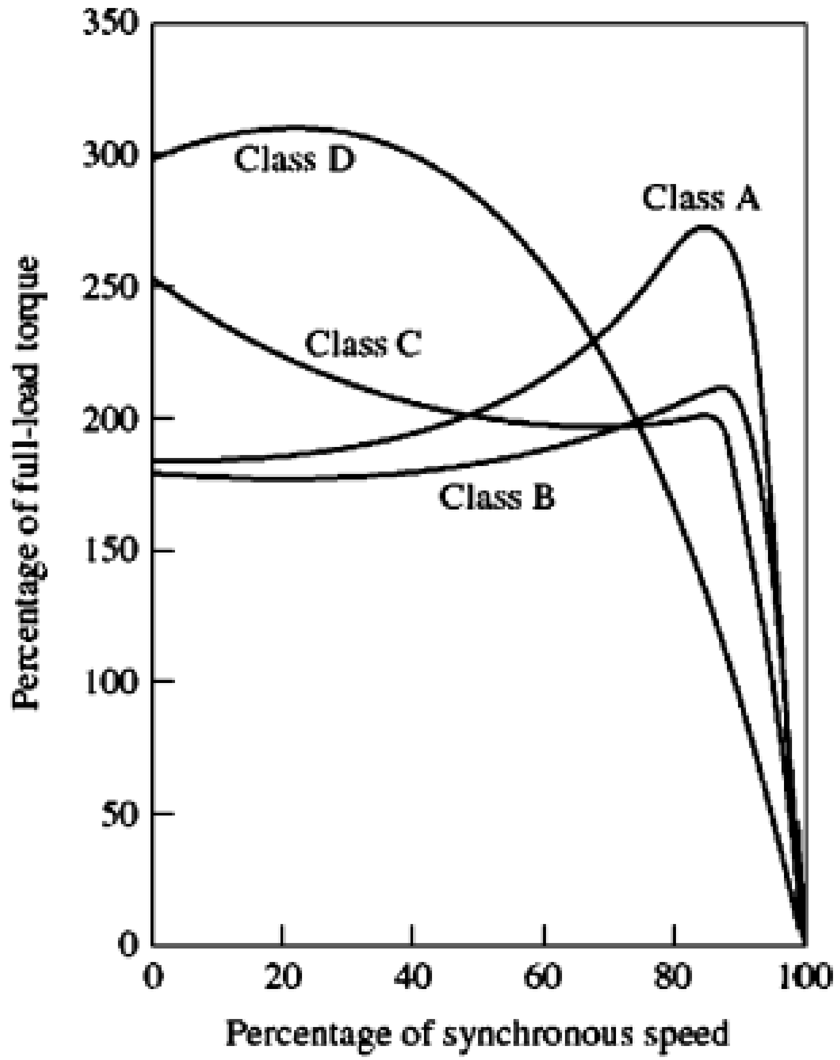


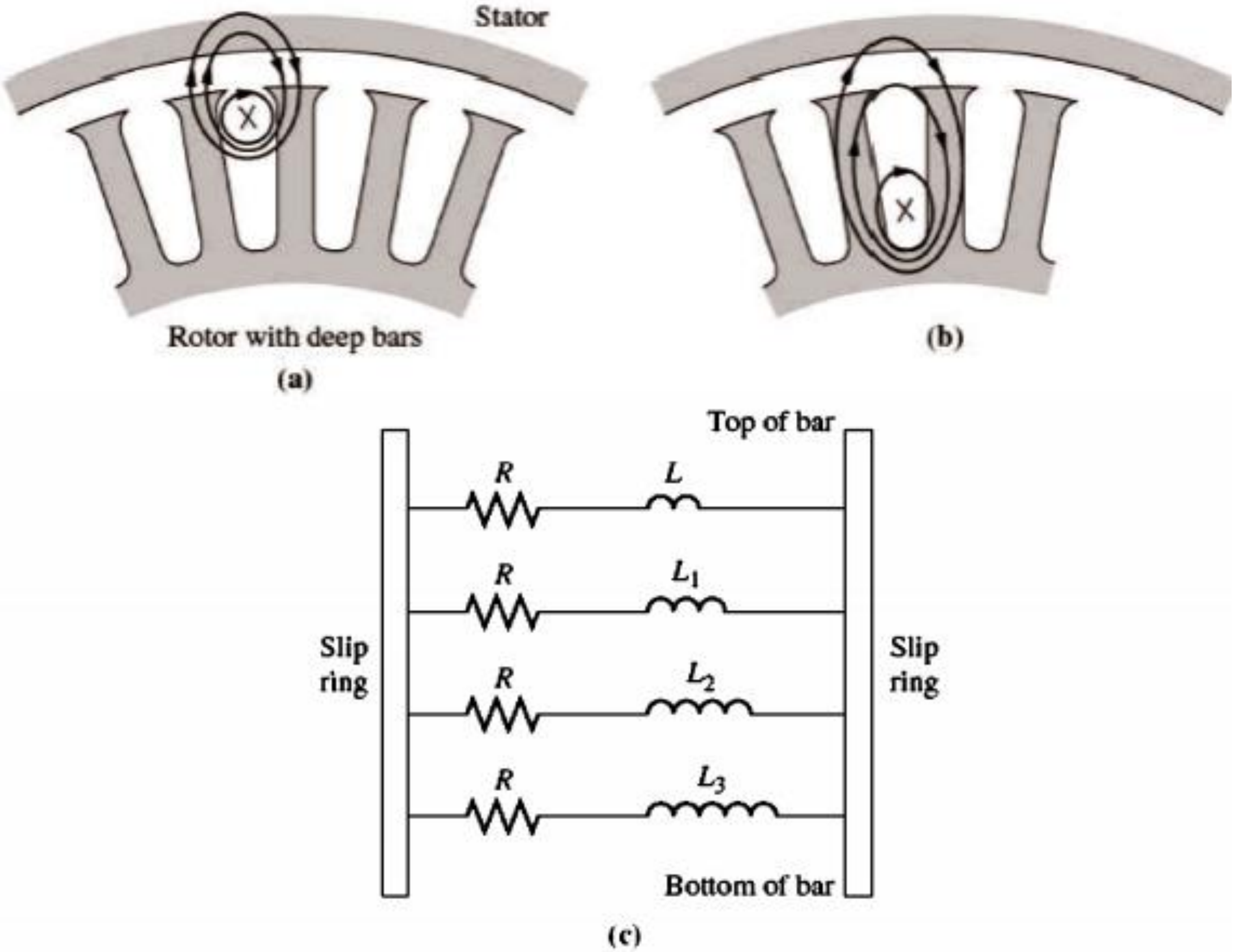
FIGURE 7–24
A torque–speed characteristic curve combining high-resistance effects at low speeds (high slip) with low-resistance effects at high speed (low slip).




Induction Motor Design Classes



Deep-Bar and Double-Cage Rotor Designs

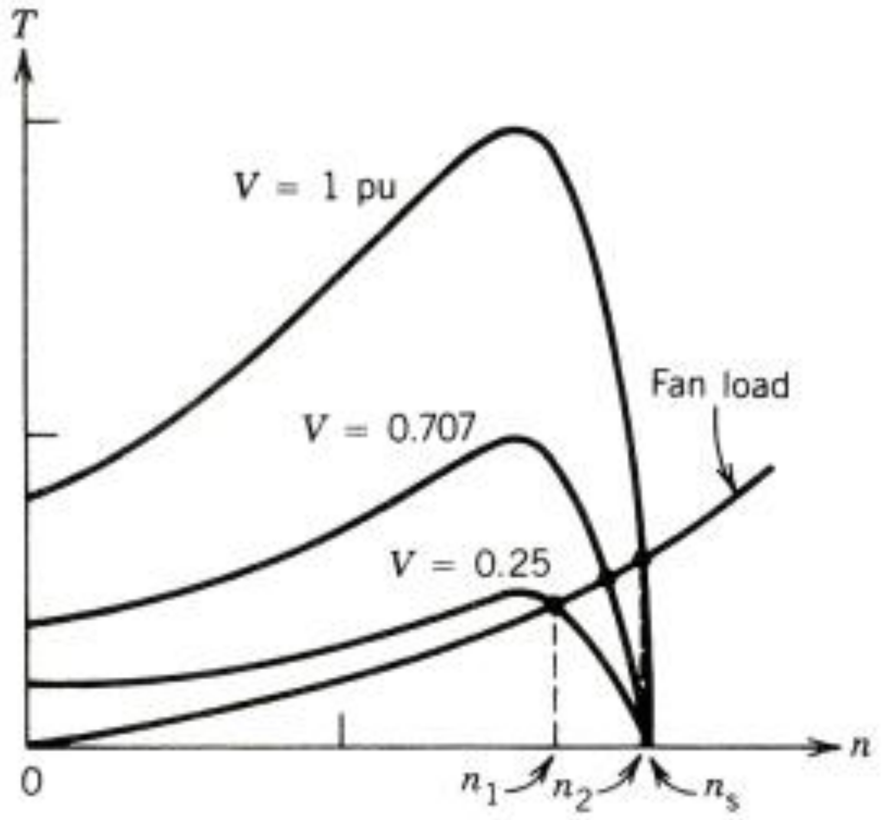


7.9 SPEED CONTROL OF INDUCTION MOTORS

$$n_{\text{sync}} = \frac{120 f_e}{P}$$

1. *Pole-changing method*. In this method, the stator winding of the motor can be designed so that by simple changes in coil connections the number of poles can be changed by the ratio of 2 to 1. In this way, two synchronous speeds can be obtained. This method is not suitable for wound-rotor motors, since the rotor windings would also have to be reconnected to have the same number of poles as the stator.
   However, a squirrel-cage rotor automatically develops a number of magnetic poles equal to those of the air-gap field. With two independent sets of stator windings, each arranged for pole changing, as many as four synchronous speeds can be achieved in a squirrel-cage motor. For example, 600, 1200, 1800, and 3600 rev/min can be attained for a 60 Hz operation. In addition, the motor phases can be connected either in wye or delta, resulting in eight possible combinations.

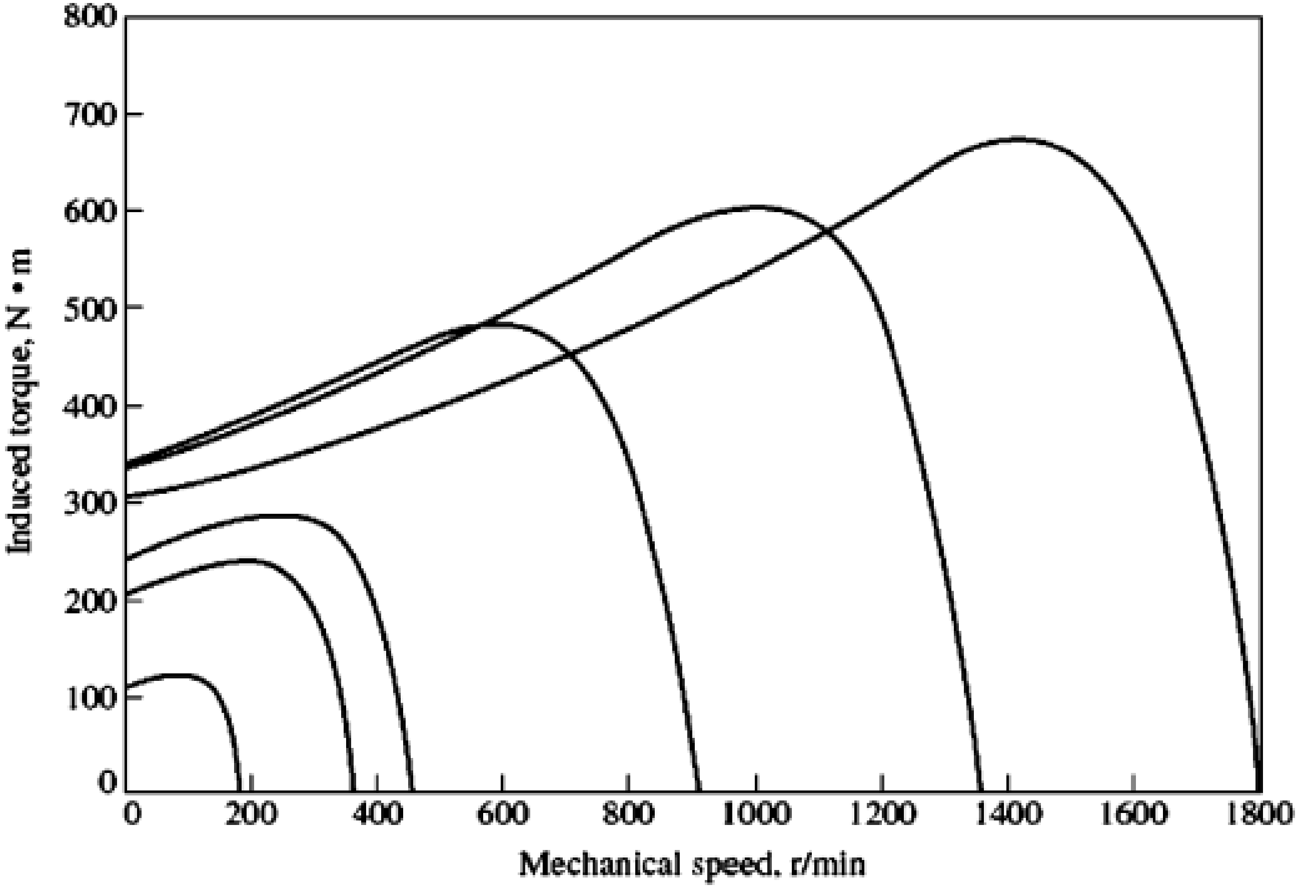
LINE VOLTAGE CONTROL



$$T_d = \frac{3V_{th}^2(R_2/s)}{\omega_s\left[(R_{th}+R_2/s)^2+(X_{th}+X_2)^2\right]}$$

3. *Variable line-voltage method*. The torque developed by an induction motor is proportional to the square of the applied voltage. Therefore, the speed of the motor can be controlled over a limited range by changing the line voltage. If the voltage can be varied continuously from V_1 to V_2, the speed of the motor can also be varied continuously from speeds n_1 to n_2 for a given load. This method is used for small squirrel-cage motors driving fans and pumps.

Speed Control by Changing the Line Frequency



$$n_{\text{sync}} = \frac{120 f_e}{P}$$

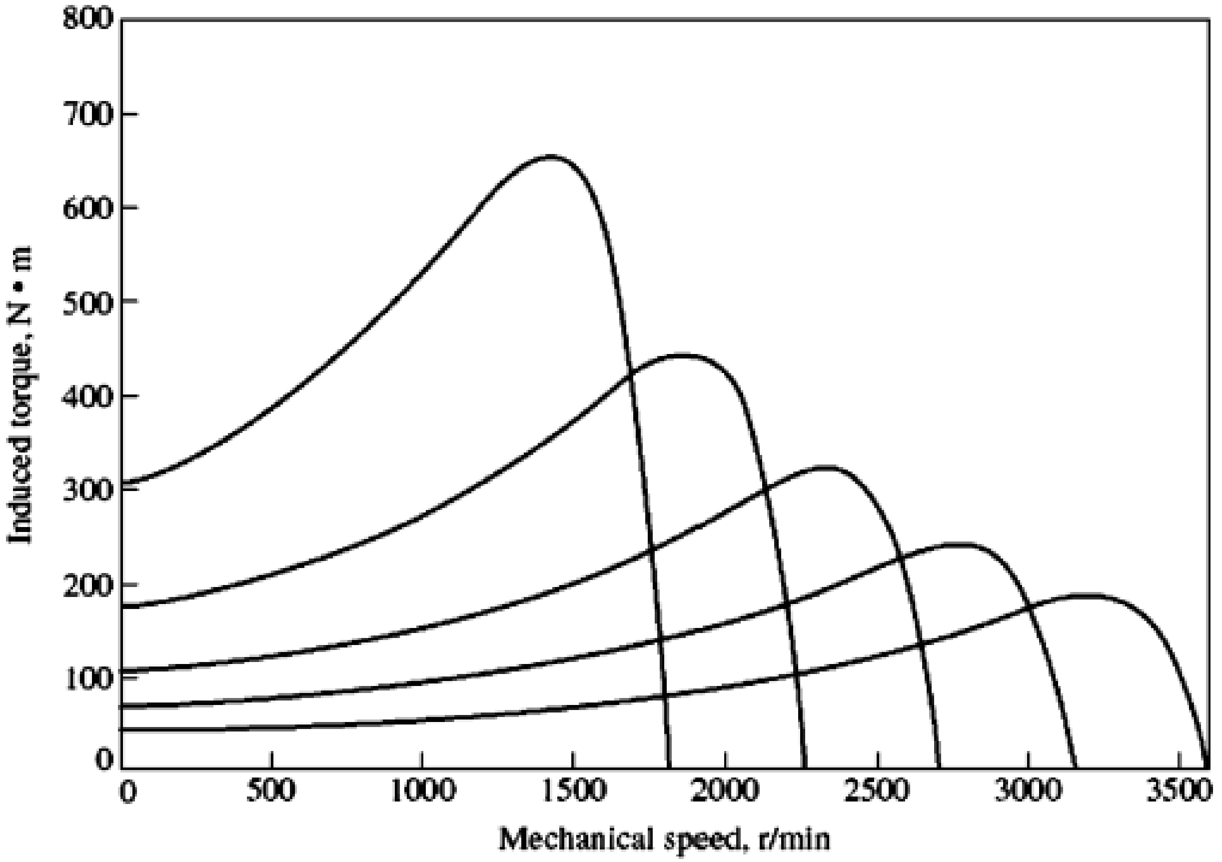
$$v(t) = -N\frac{d\phi}{dt}$$

If a voltage $v(t) = V_M \sin \omega t$ is applied to the core, the resulting flux ϕ is

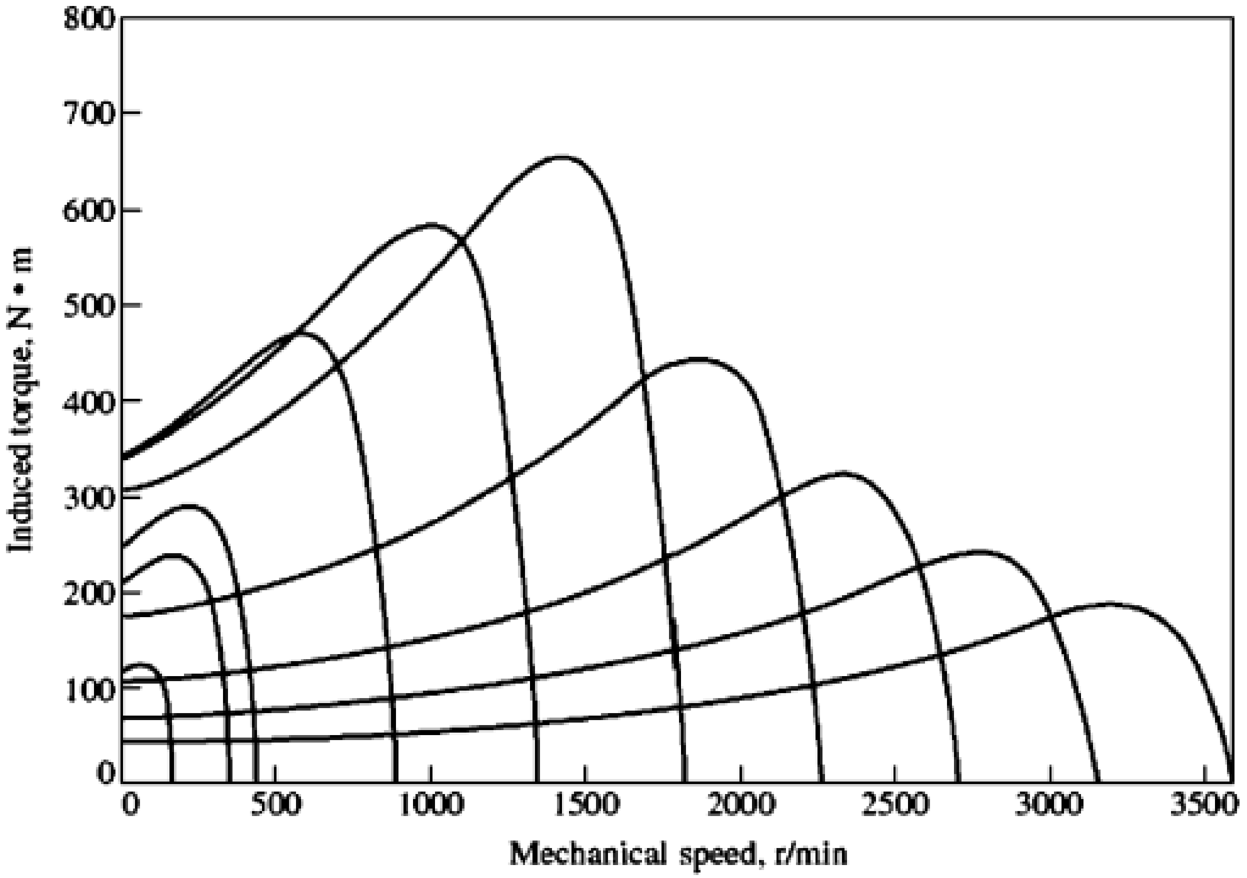
$$\phi(t) = \frac{1}{N_P}\int v(t)\,dt$$

$$= \frac{I}{N_P}\int V_M \sin \omega t\, dt$$

$$\boxed{\phi(t) = -\frac{V_M}{\omega N_P}\cos \omega t}$$

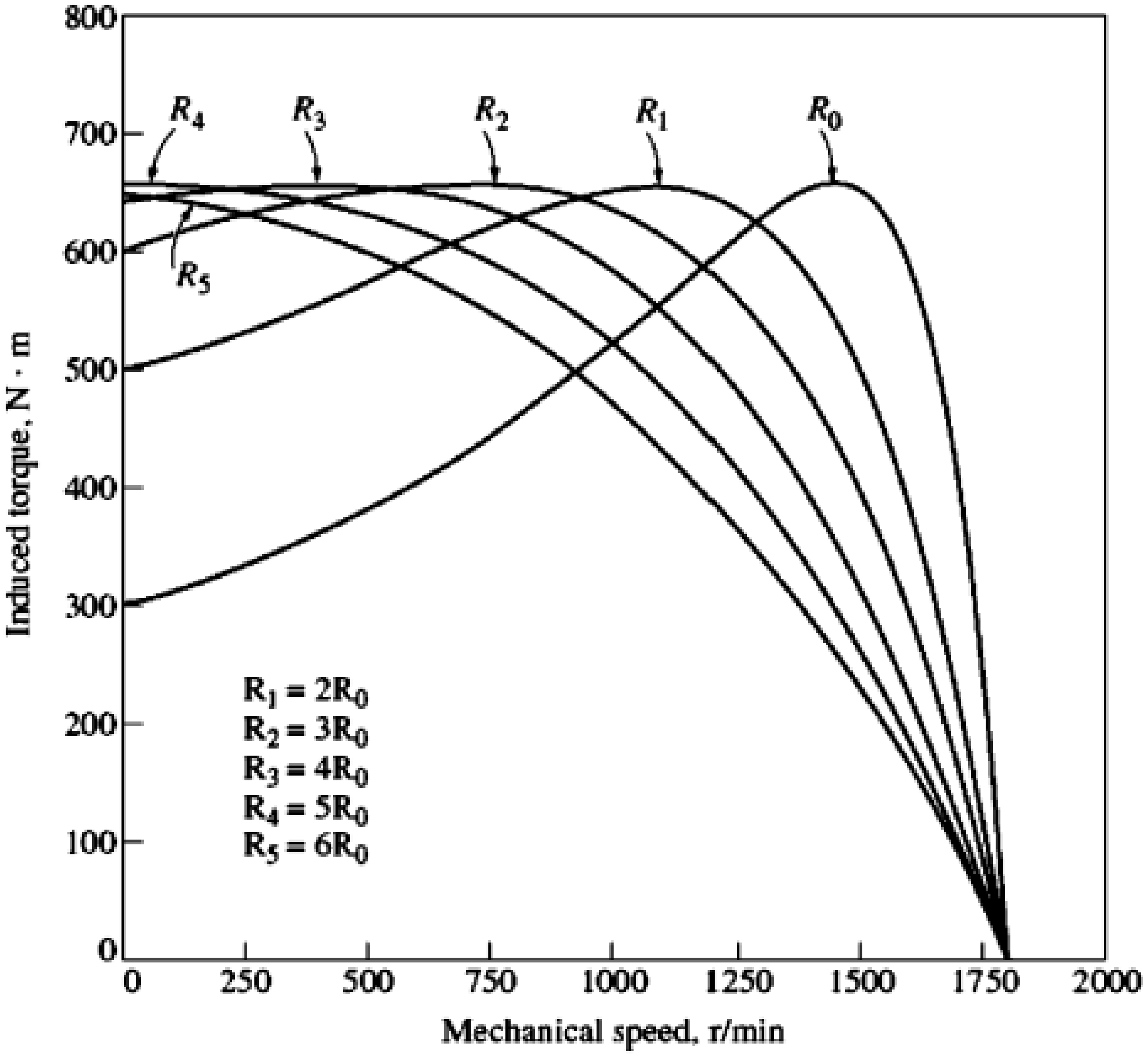


800
700
600
500
400
300
200
100
0
0
500
1000
1500
2000
2500
3000
3500
Induced torque, N • m
Mechanical speed, r/min

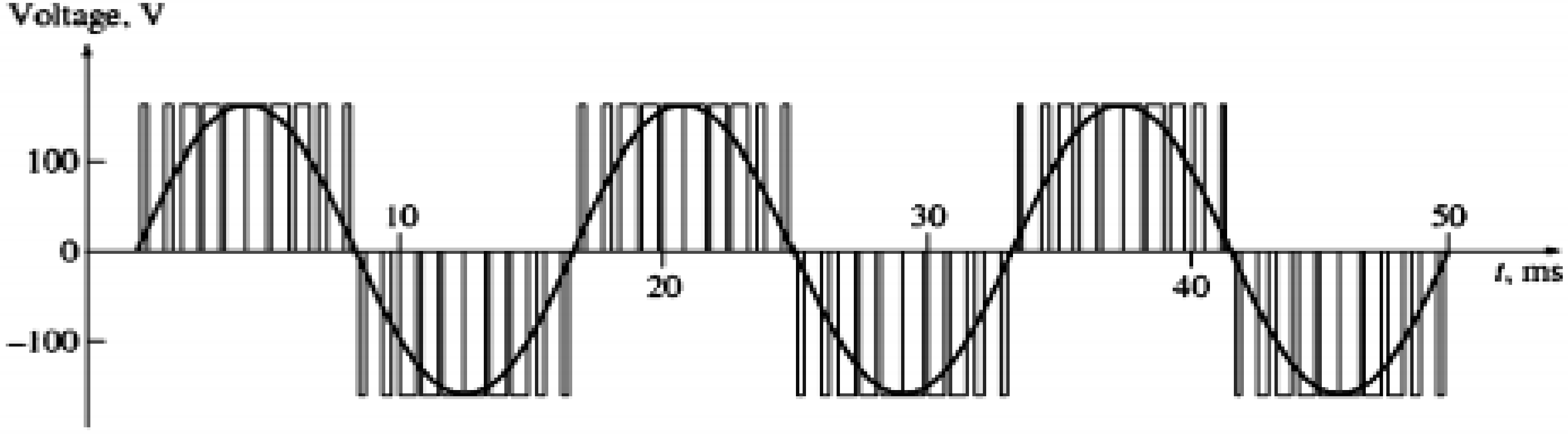


800
700
600
500
400
300
200
100
0
0
500
1000
1500
2000
2500
3000
3500
Induced torque, N • m
Mechanical speed, r/min

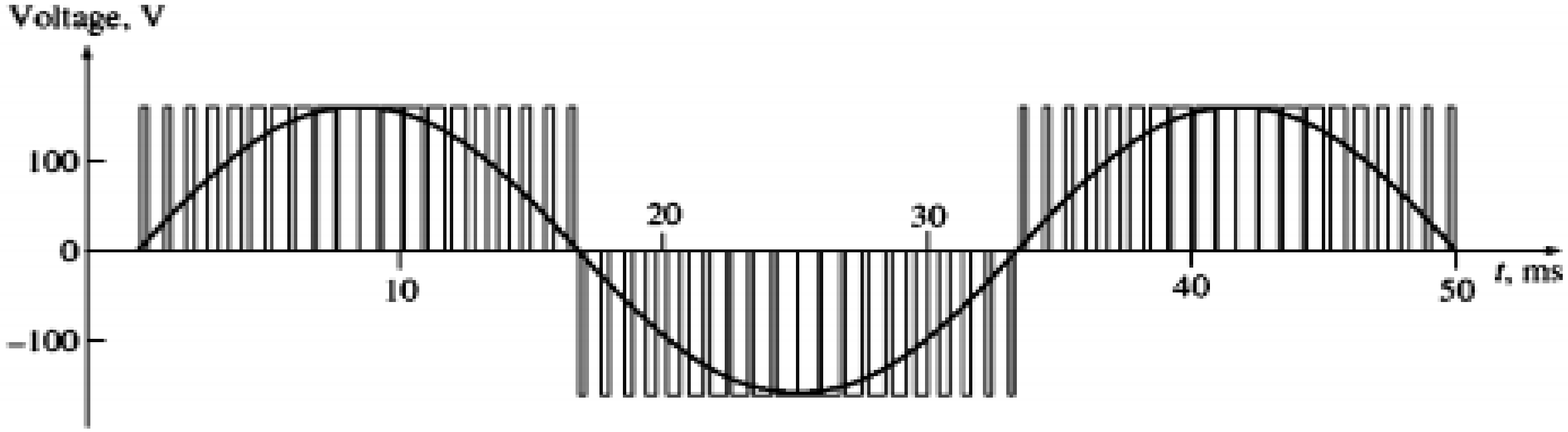
Speed Control by Changing the Rotor Resistance



7.10 SOLID-STATE INDUCTION MOTOR DRIVES



(a)



(b)

FIGURE 7–46
Variable-frequency control with a PWM waveform: (a) 60-Hz, 120-V PWM waveform; (b) 30-Hz, 120-V PWM waveform.

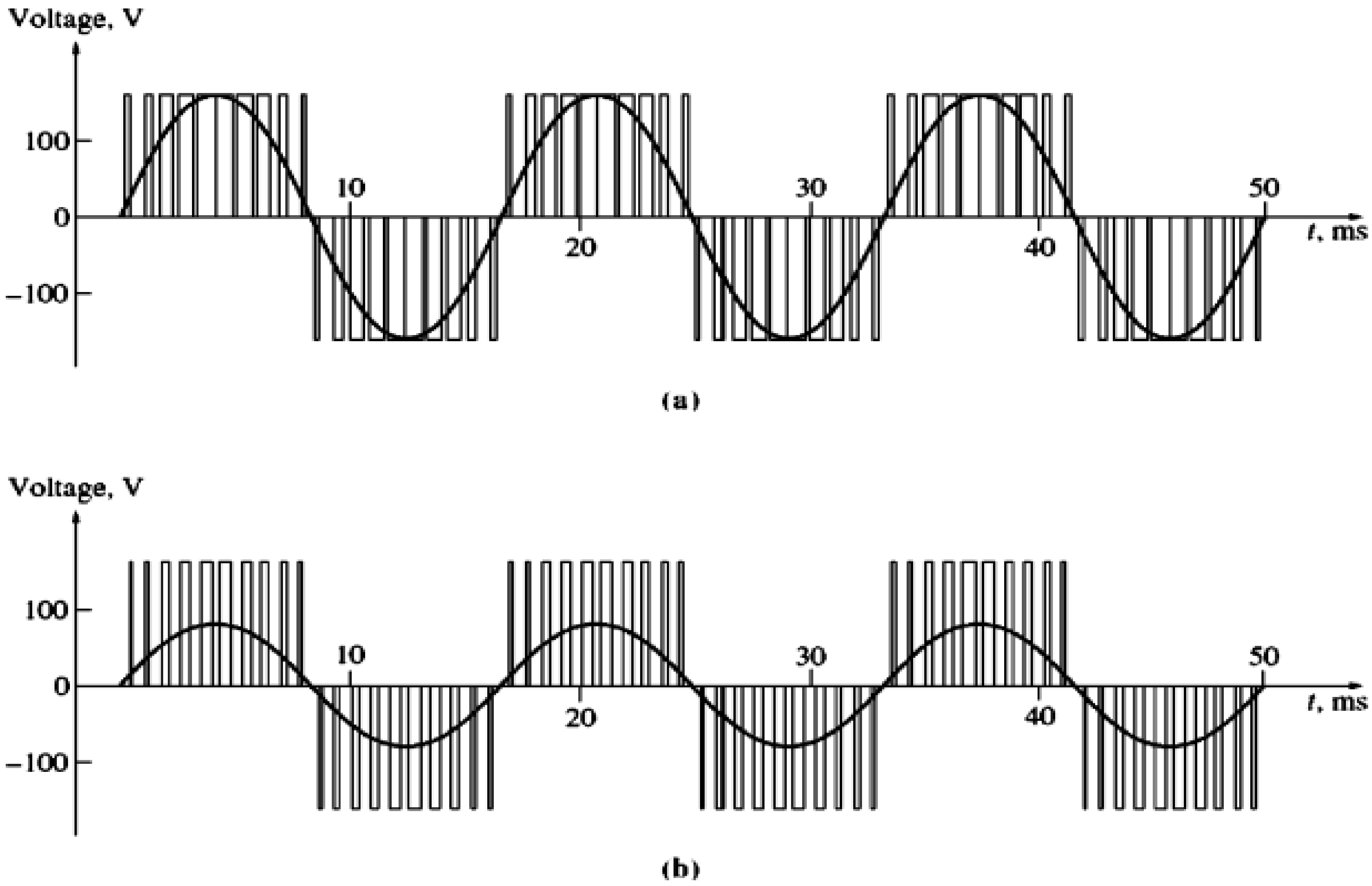
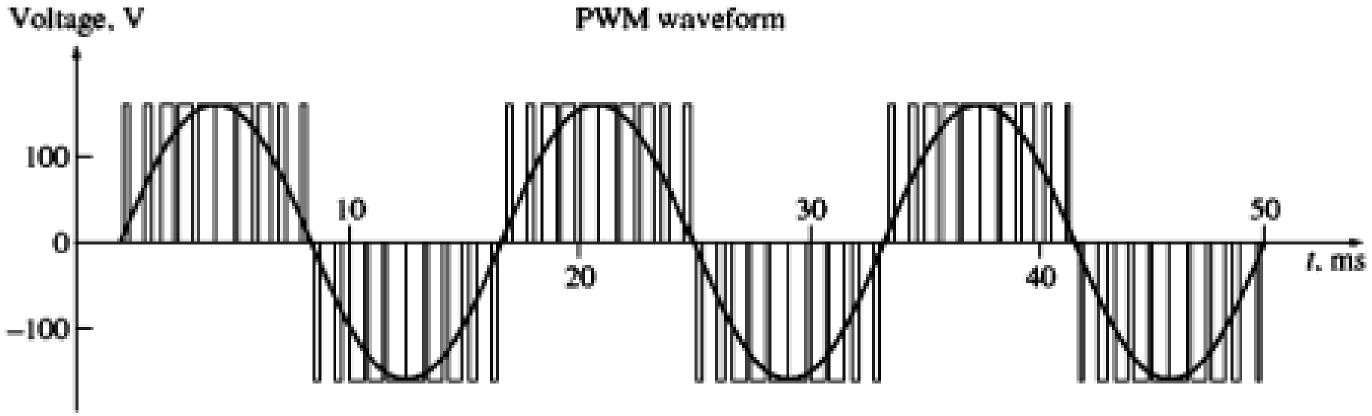
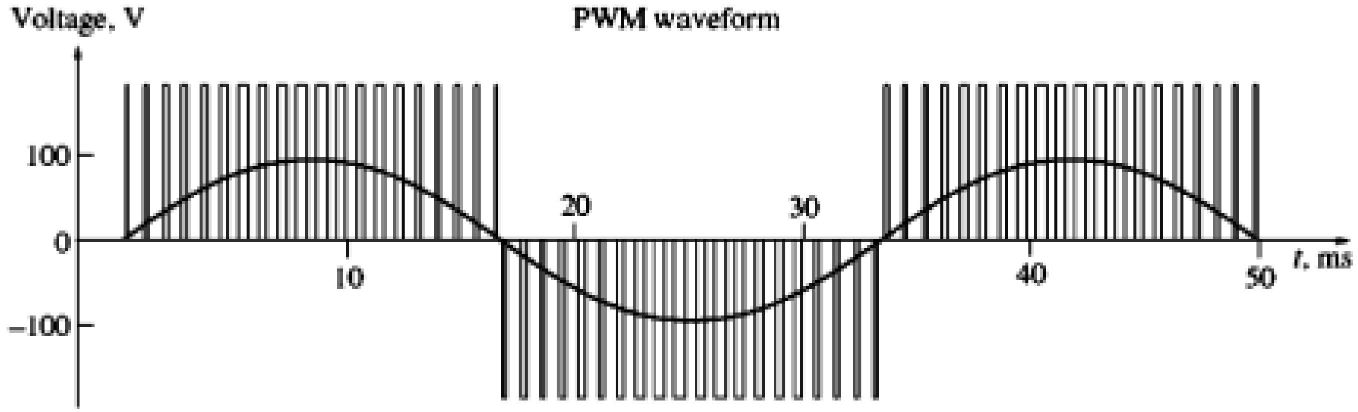


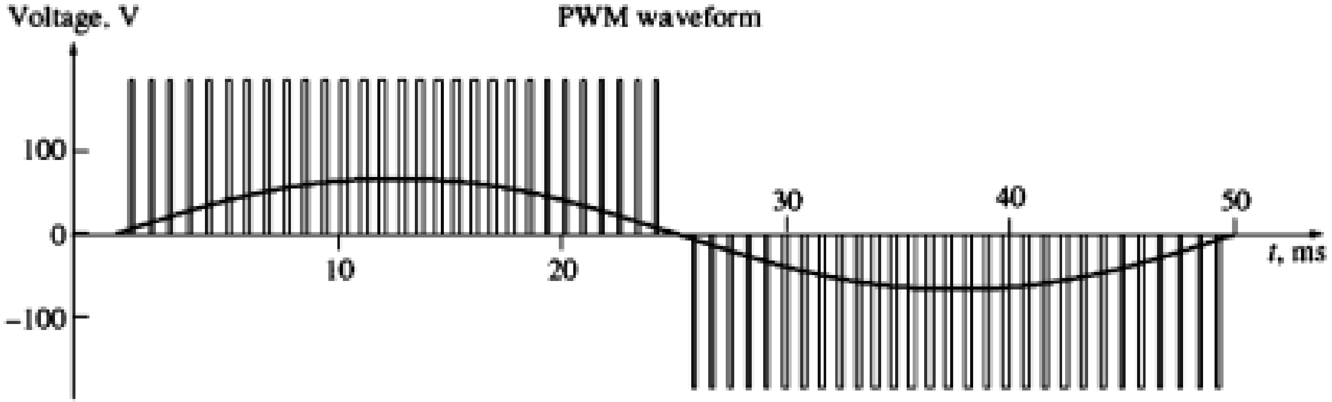
FIGURE 7–47
Variable voltage control with a PWM waveform: (a) 60-Hz, 120-V PWM waveform; (b) 60-Hz, 60-V PWM waveform.



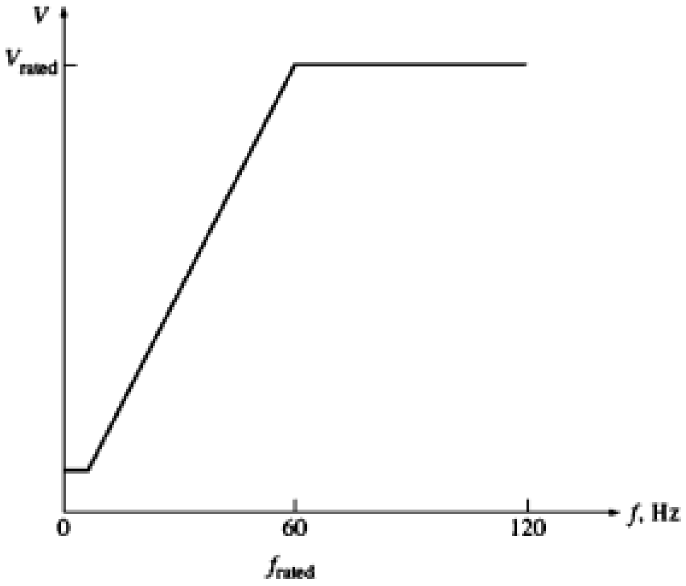
(a) 60-Hz, 120-V PWM waveform



(b) 30-Hz, 60-V PWM waveform

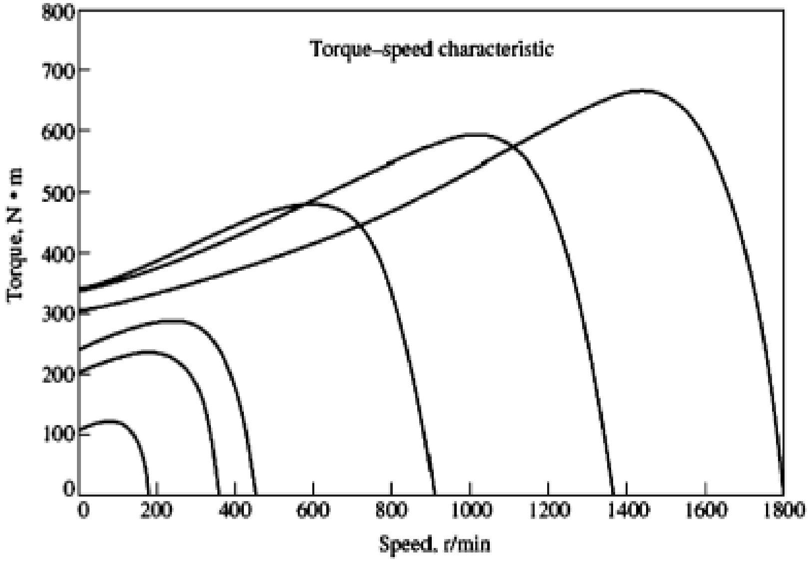


(c) 20-Hz, 40-V PWM waveform.



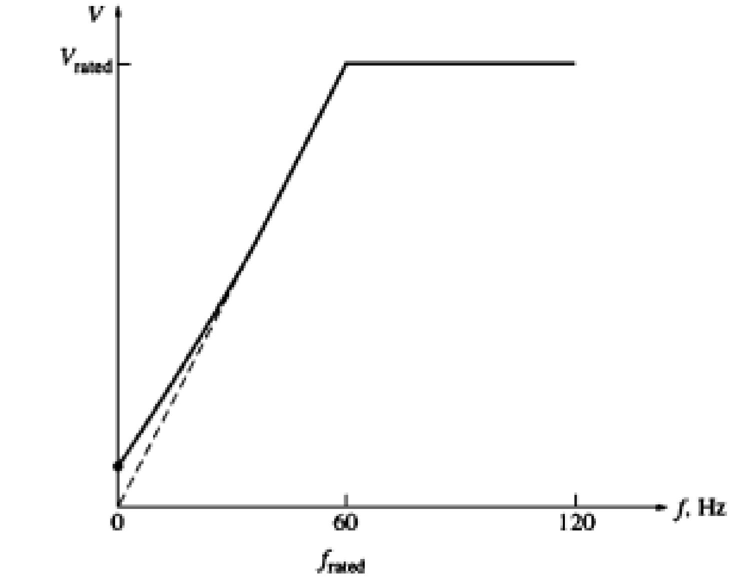
V
V_{rated}
0
60
120
f, Hz
f_{rated}

(a)



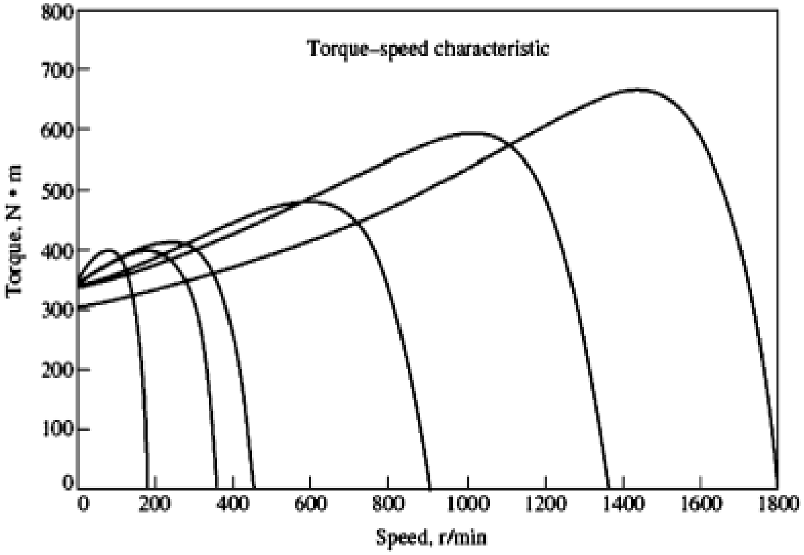
Torque–speed characteristic
Torque, N • m
Speed, r/min

(b)



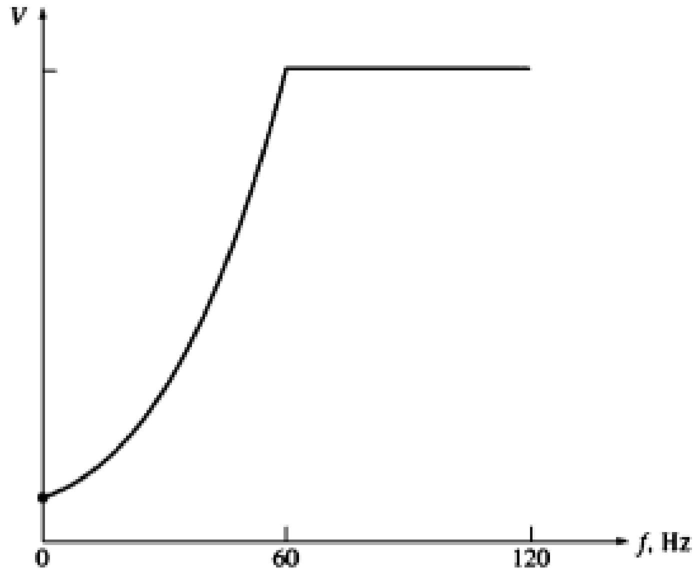
V
V_{rated}
0
60
120
f, Hz
f_{rated}

(a)



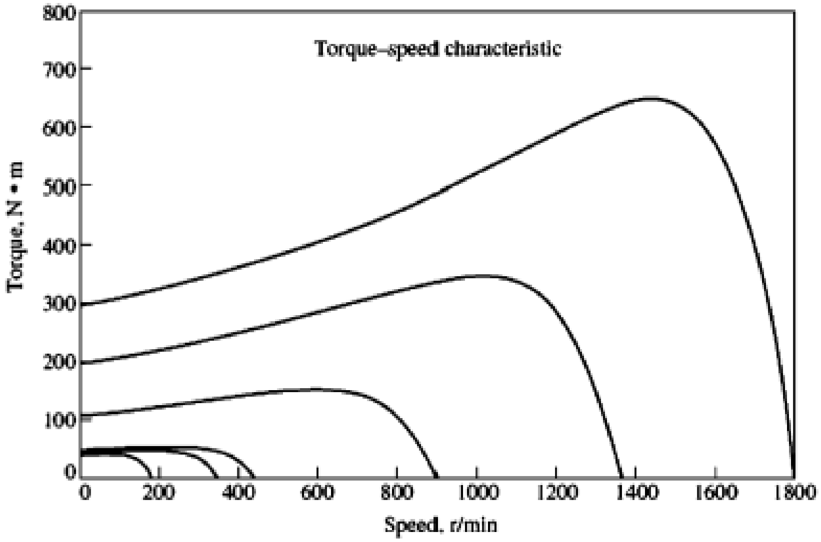
Torque–speed characteristic
Torque, N • m
Speed, r/min

(b)



V
0
60
120
f, Hz

(a)



Torque–speed characteristic
Torque, N • m
Speed, r/min

(b)

			Sınav	Türü	(X)Ara Sınav, ( )Final, ()Bütünleme, ()Mazeret		
	Teknoloji Fakültesi Elektrik Elektronik Mühendisliği Bölümü **SINAV KAĞIDI**			Tarihi	28.03.2018	Saati	15.00
				Süresi	70 dakika		
Ders	Kodu, Adı	**EE-314 ELEKTRİK MAKİNALARI 2**	Öğrenci	No			
	Öğretim Üyeleri	Prof. Dr. Güngör BAL Dr. Öğr. Üy. Orhan KAPLAN		Adı Soyadı			
				İmzası			

Konu No	6,10-14	6, 8-10	6, 8-10	15, 16			**Sınav Notu**
Soru No (Puanı)	1(40p)	2(30p)	3(30p)	4(30p)			
Öğrenci Puanı						*BAŞARILAR…*	

SORULAR

1) 440 V, 100 hp, 60 Hz, 4 kutuplu, 3 fazlı, Y bağlı asenkron motorun statora aktarılmış faz başına parametre değerleri tabloda verilmiştir. Döner kayıpları ve nüve kayıpları ihmal edilmiştir. Motor şebekeden 40,5 A akım ve 21,8 kW güç çekerken;

Rs= 0,46 Ω	Xs=1,05 Ω	Xm=20,4 Ω
Rr=0,32 Ω	Xr=0,45 Ω	

a) Motorun hızı nr=?

b) Bu şartlar altında motorda endüklenen tork $Tind$=?

c) Motorda maksimum torkun meydana geleceği kayma değerini,

d) Motorun maksimum tork ile kalkınması için rotor devresine ilave edilmesi gereken direnç değerini hesaplayınız.

2) 75 kW, 60 Hz, 480 V, 1176 d/d, 3-faz asenkron motor tam yükte ve 0,8 geri güç katsayısında, %90 verimle çalışmaktadır. Bu anma çalışma şartlarında aşağıda istenenleri hesaplayınız.

a) Motorun çektiği akımı,
b) Motorun çektiği gerçek (aktif), görünür ve reaktif güçleri
c) Tam yükteki çıkış momentini
d) Kaymayı

3) 3 faz, 15 hp, 4 kutuplu,60 Hz asenkron motor 1728 d/d hızda dönerken milinde bağlı bulunan yüke anma çıkış gücünü sağlamaktadır. Motorun rüzgar-sürtünme ve kaçak yük kayıpları750 W'tır.

a) Üretilen mekanik gücü,
b) Hava aralığı gücünü,
c) Rotor bakı kayıplarını hesaplayınız.

4) 208 V, 60 Hz, 4 kutuplu, 3 fazlı, Y bağlı, B sınıfı bir asenkron motorun anma rotor hızı 1710 d/d'dır. DA deneyde iki terminal arası ölçülen direnç değeri Rda=2,4 Ω dur. Motorun boş çalışma ve kilitli rotor deneylerinden alınan değerler tabloda verilmiştir. Bu değerlere göre motorun bir faz eşdeğer devresini oluşturan değerleri hesaplayıp eşdeğer devreyi çiziniz.

Boş çalışma deneyi	Kilitli rotor deneyi
Vb= 208 V	Vk=30 V
Ib= 1,86 A	Ik= 6,4 A
Pb= 214W	Pk= 185 W

Not: *2. ve 3. sorudan sadece bir tanesi seçilip cevaplandırılacaktır, bu iki soru birlikte cevaplandırılamaz. Birlikte cevaplandırılması durumunda sorulardan sadece biri puanlandırmaya dahil edilir.*

Seçim hakkı sadece bu iki soru arasındadır. 1. ve 4. soru herkes tarafından cevaplanması gereken seçimli olmayan sorulardır.