SENKRON MAKİNALAR

Doç. Dr. Orhan KAPLAN

Synchronous Machine

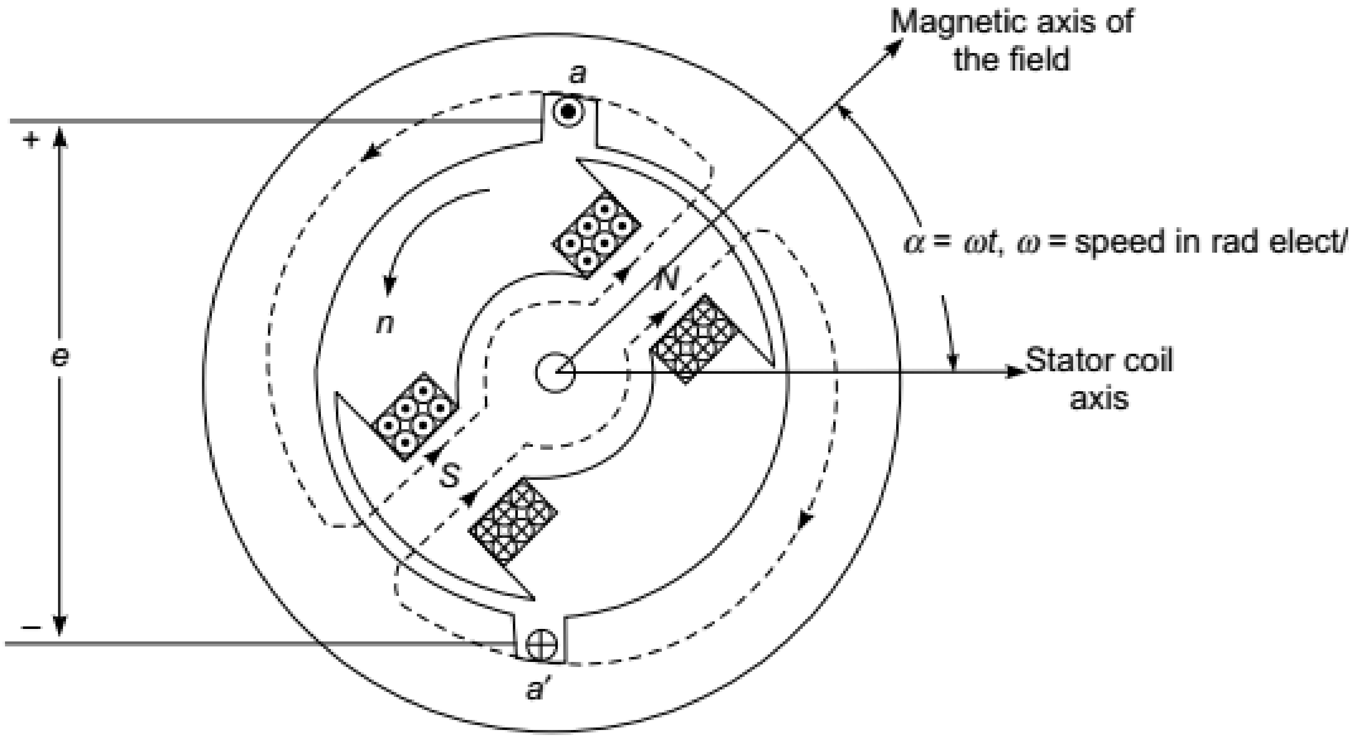


Fig. 5.2 *Elementary synchronous generator—salient-pole 2-pole rotor*

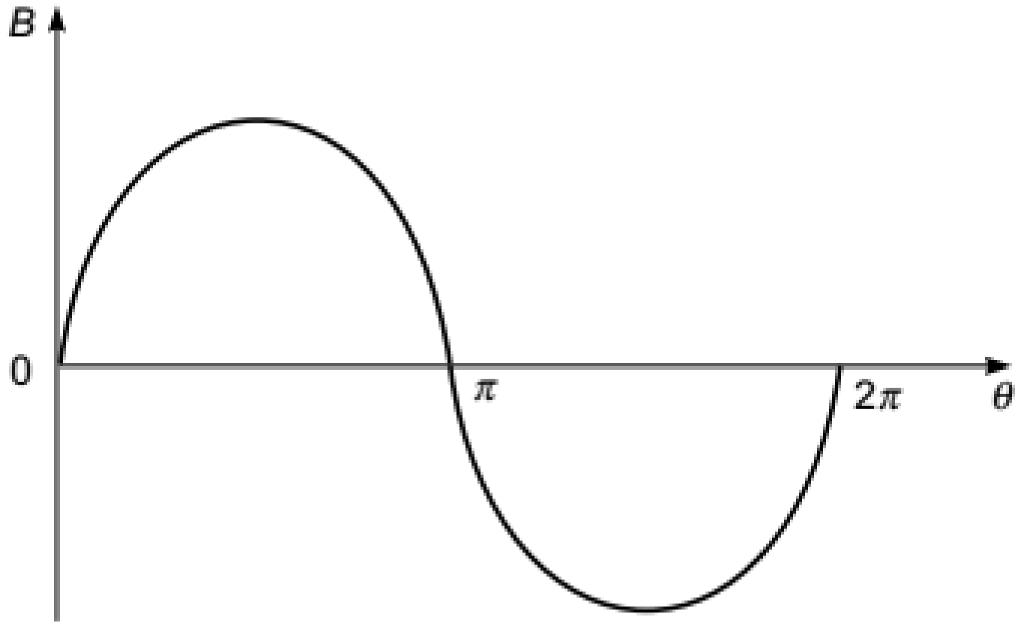


Fig. 5.3 *Flat-topped flux density wave*

SYNCHRONOUS SPEED

$$n_s = \frac{120f}{p} \text{ rpm}$$

where

ω_s is the angular speed of the magnetic field (which is equal to the *angular rotor speed of the synchronous machine*)

ω is the angular frequency of the electrical system

f is the electrical frequency, Hz

p is the number of poles

$$\omega = \left(\frac{p}{2}\right)\omega_m$$

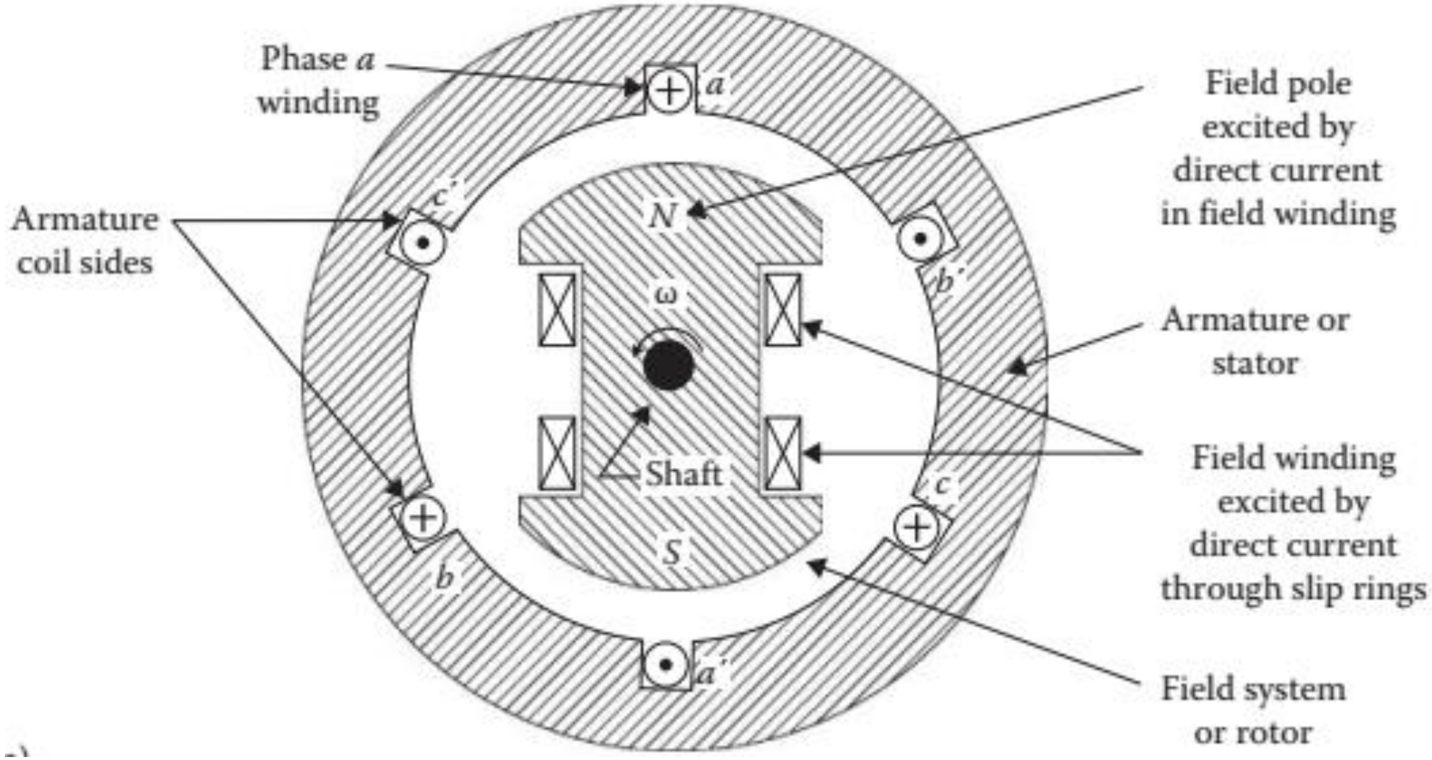
$$\theta = \left(\frac{p}{2}\right)\theta_m$$

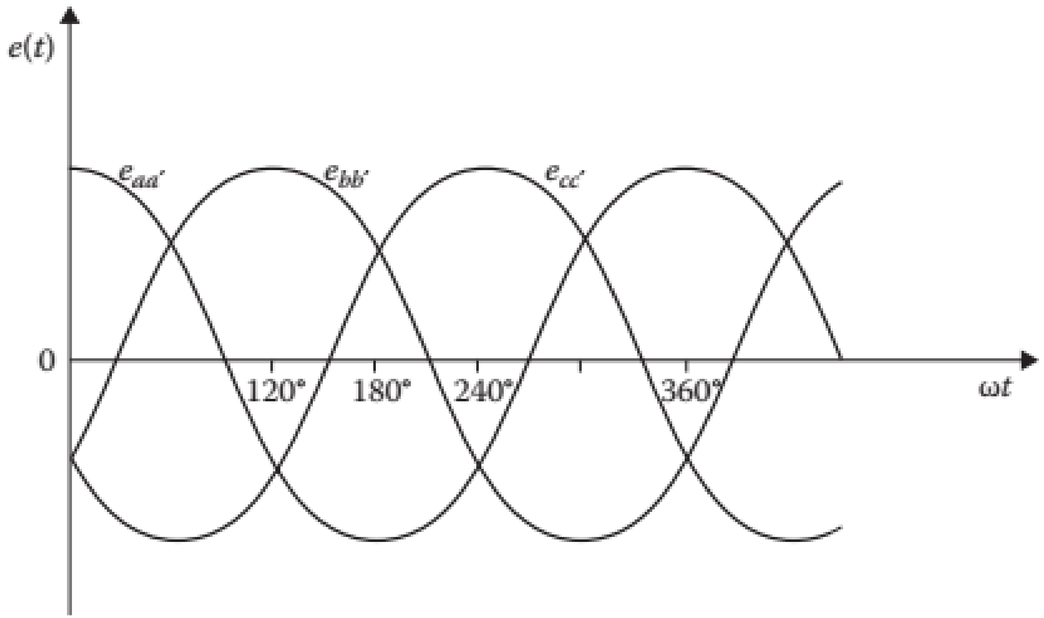
where

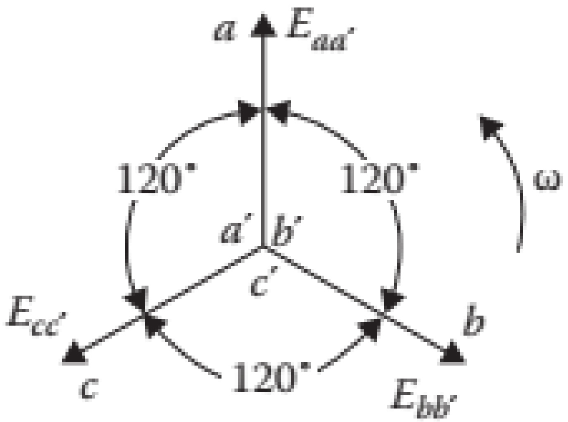
θ is in electrical measure

θ_m is in mechanical measure

SYNCHRONOUS GENERATOR OPERATION







$$e_{aa'}(t) = E_{max}\sin\omega t$$

$$e_{bb'}(t) = E_{max}\sin(\omega t - 120°)$$

$$e_{cc'}(t) = E_{max}\sin(\omega t - 240°)$$

The peak voltage in any phase of a three-phase stator is

$$E_{max} = \omega \times N \times \Phi$$

However, if the winding is distributed over several slots, the induced voltage is less and is given as

$$E_{max} = \omega \times N \times \Phi \times k_w \tag{7.8}$$

since $\omega = 2\pi f$, then

$$E_{max} = 2\pi \times f \times N \times \Phi \times k_w$$

where

N is the number of turns in each phase winding

Φ is the flux per pole due to the excitation current I_f

k_w is the winding factor*

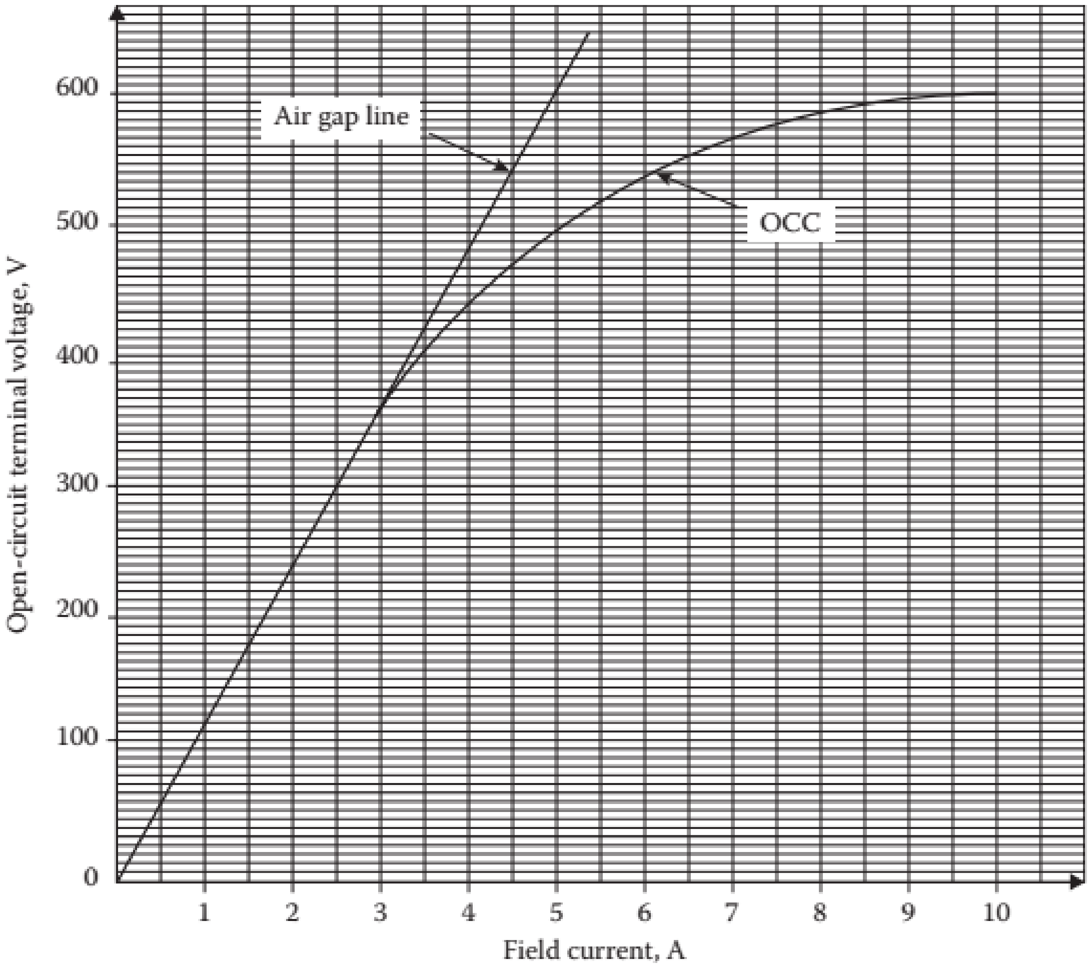
Thus, the rms voltage of any phase of this three-phase stator is

$$E_{max} = \frac{2\pi}{\sqrt{2}} f \times N \times \Phi \times k_w$$

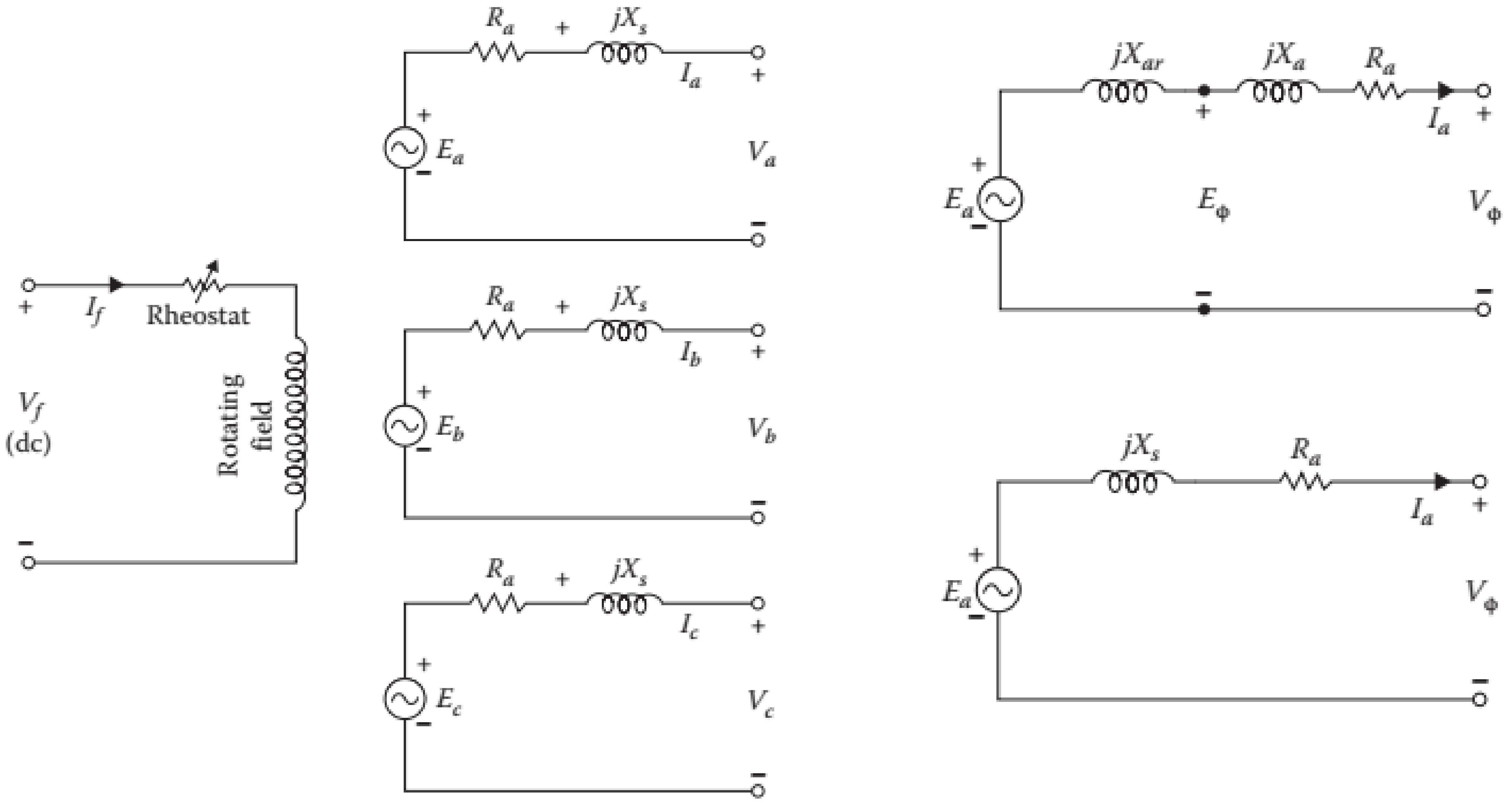
$$E_a = K \times \Phi \times \omega$$

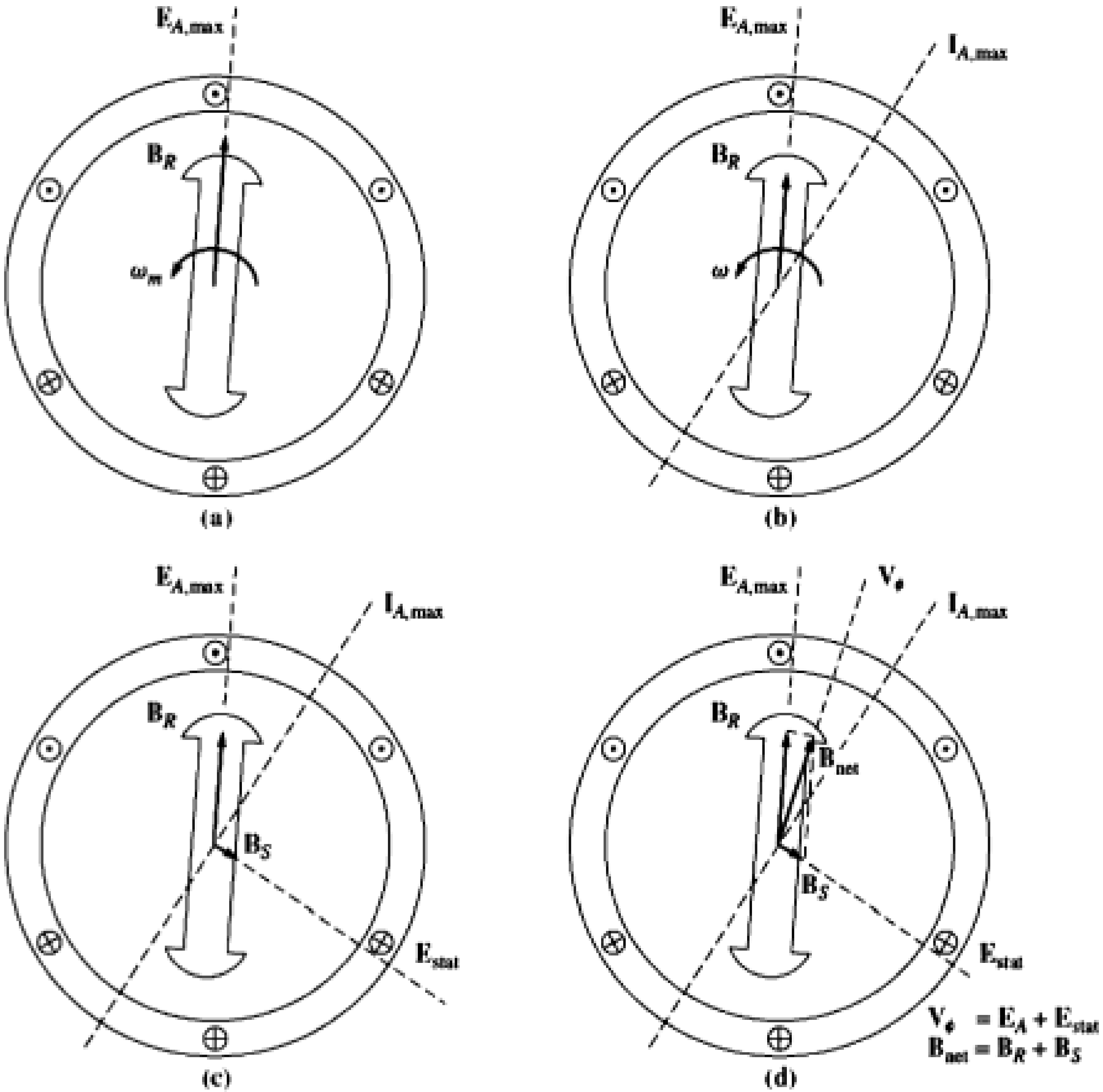
$$E_a = 4.44 f \times N \times \Phi \times k_w$$

$$K = \frac{N \times k_w}{\sqrt{2}}$$



EQUIVALENT CIRCUITS

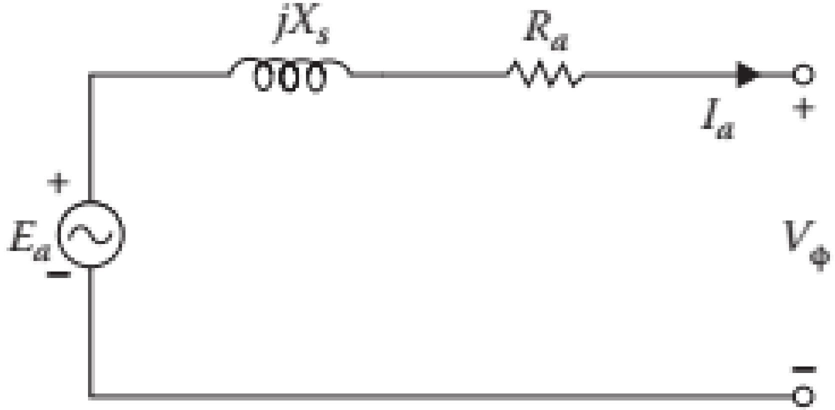




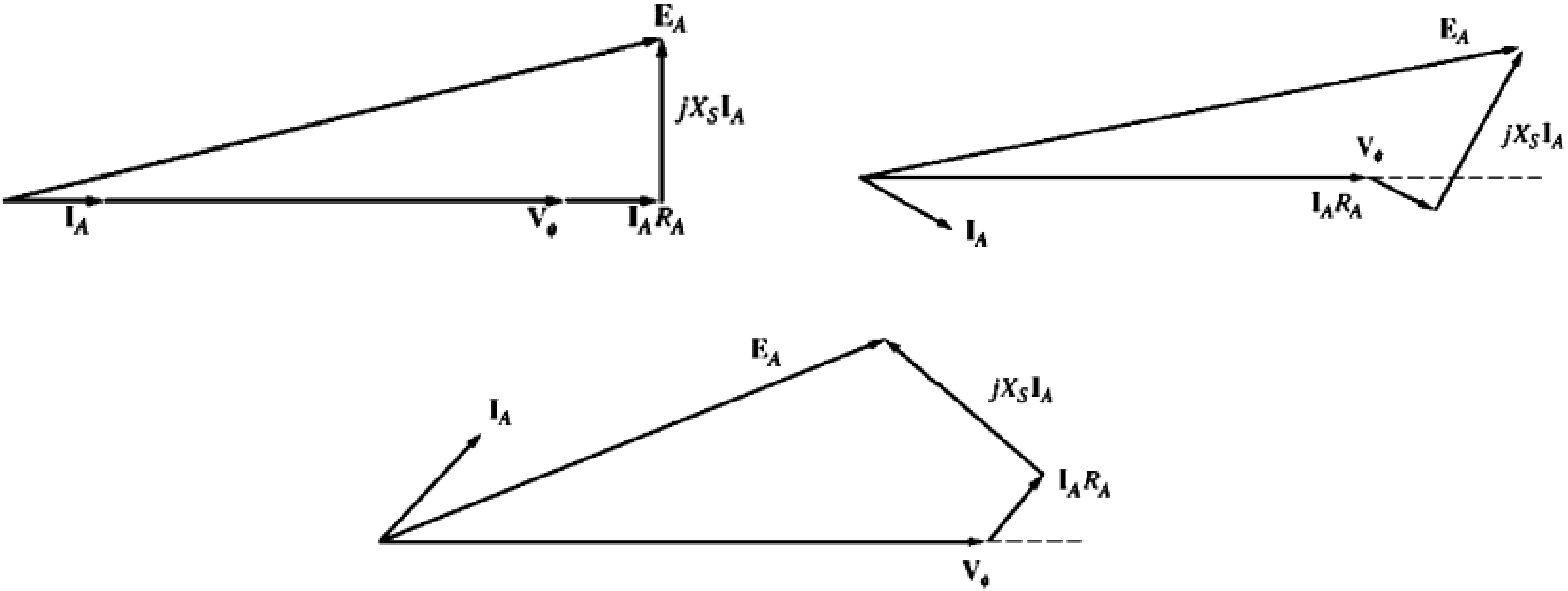
$$\mathbf{V}_\phi = \mathbf{E}_A - jX\mathbf{I}_A - jX_A\mathbf{I}_A - R_A\mathbf{I}_A$$

$$X_S = X + X_A$$

$$\boxed{\mathbf{V}_\phi = \mathbf{E}_A - jX_S\mathbf{I}_A - R_A\mathbf{I}_A}$$

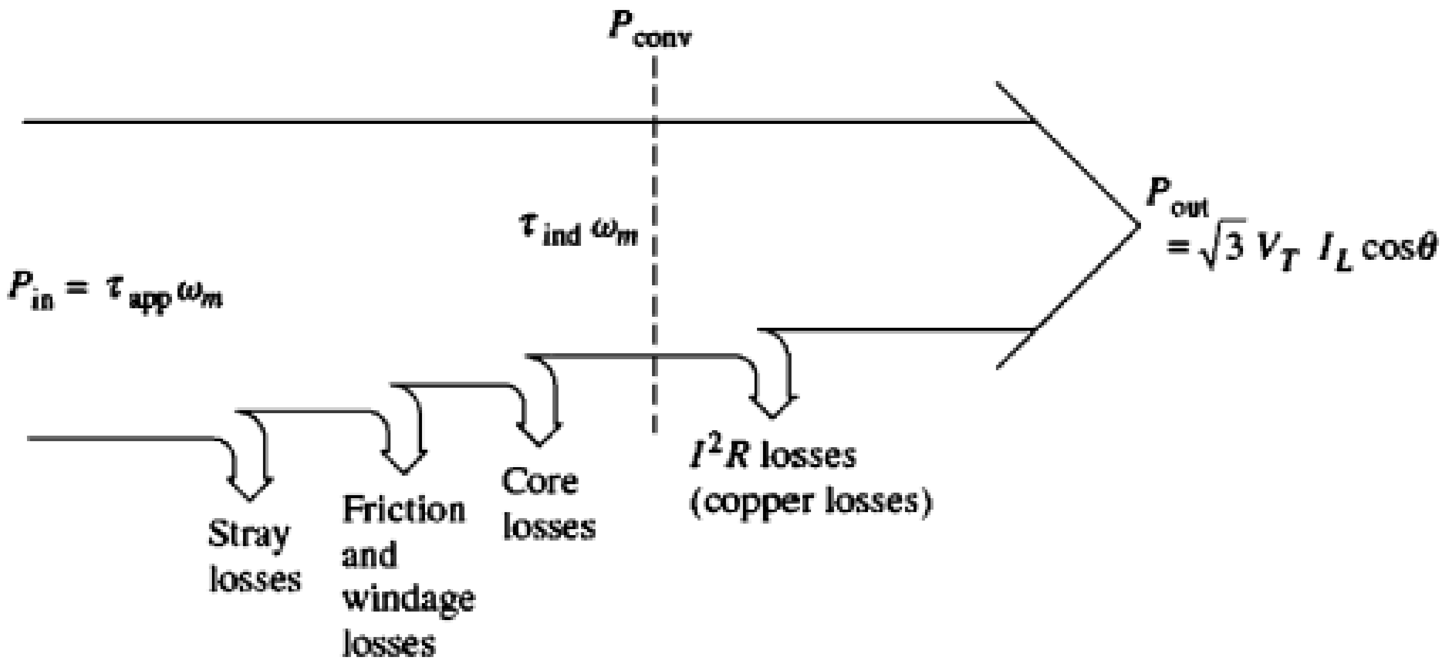


5.5 THE PHASOR DIAGRAM OF A SYNCHRONOUS GENERATOR



Alternatively, *for a given field current and magnitude of load current, the terminal voltage is lower for lagging loads and higher for leading loads.*

5.6 POWER AND TORQUE IN SYNCHRONOUS GENERATORS

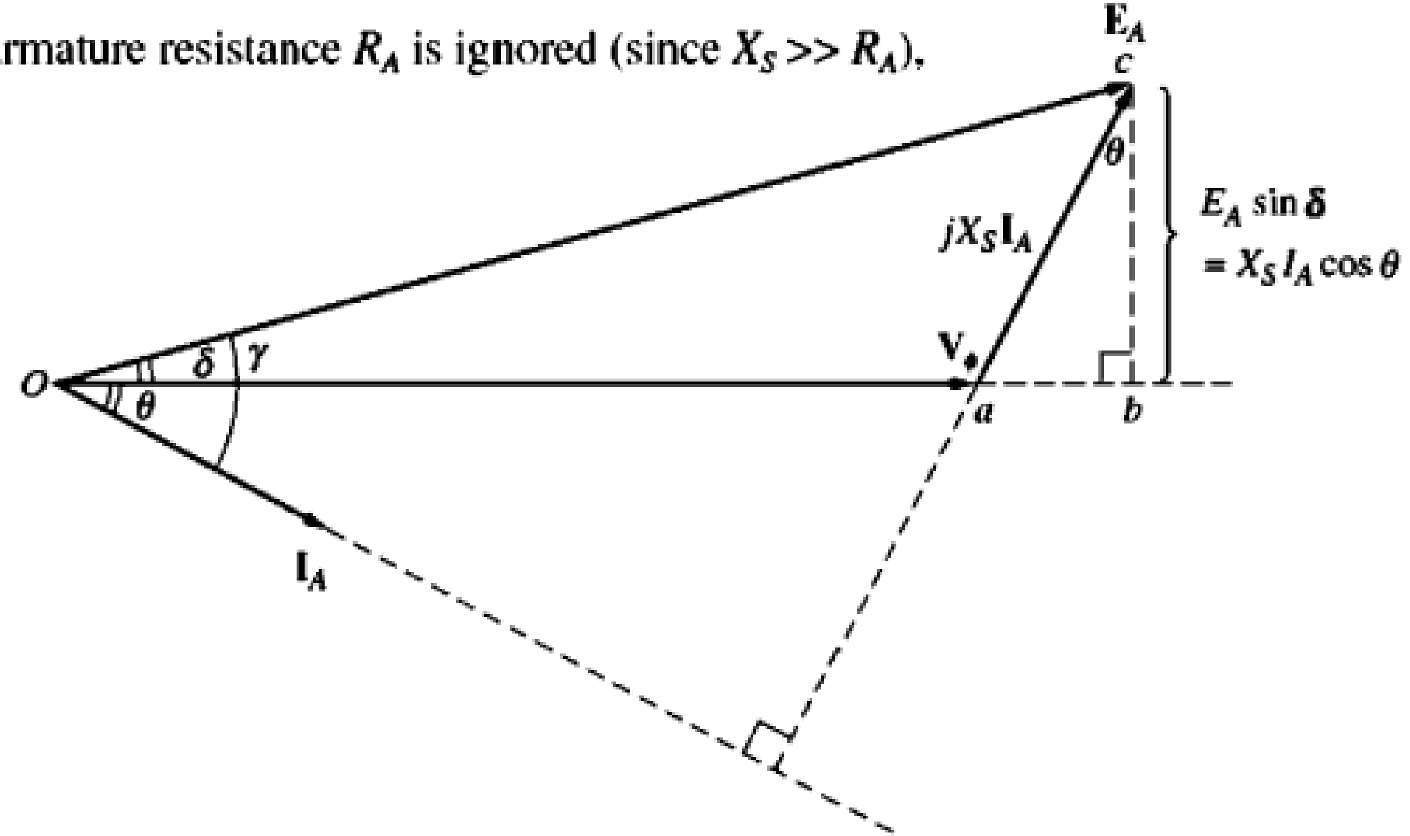


$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$
$$= 3E_A I_A \cos\gamma$$

$$P_{\text{out}} = \sqrt{3} V_T I_L \cos\theta$$

$$P_{\text{out}} = 3V_\phi I_A \cos\theta$$

If the armature resistance R_A is ignored (since $X_S >> R_A$),



$$\boxed{\tau_{\text{ind}} = \frac{3V_\phi E_A \sin\delta}{\omega_m X_S}}$$

$$I_A \cos\theta = \frac{E_A \sin\delta}{X_S}$$

$$\boxed{P = \frac{3V_\phi E_A \sin\delta}{X_S}} \qquad (5\text{–}20)$$

Equation (5–20) shows that the power produced by a synchronous generator depends on the angle δ between $\mathbf{V}_\phi$ and $\mathbf{E}_A$. The angle δ is known as the *torque angle* of the machine. Notice also that the maximum power that the generator can supply occurs when $\delta = 90°$. At $\delta = 90°$, $\sin\delta = 1$, and

$$P_{\max} = \frac{3V_\phi E_A}{X_S} \qquad (5\text{–}21)$$

Example 7.3

A three-phase, 13.2 kV, 60 Hz, 50 MVA, wye-connected cylindrical-rotor synchronous generator has an armature reactance of 2.19 Ω per phase. The leakage reactance is 0.137 times the armature reactance. The armature resistance is small enough to be negligible. Also ignore the saturation. Assume that the generator delivers full-load current at the rated voltage and 0.8 lagging power factor. Determine the following:

(a) The synchronous reactance in ohms per phase
(b) The rated load current
(c) The air gap voltage
(d) The internal generated voltage
(e) The power angle
(f) The voltage regulation

Solution

(a) The leakage reactance per phase is

$$X_a = 0.137X_{ar}$$
$$= 0.137(2.19\ \Omega)$$
$$\cong 0.3\ \Omega$$

Therefore, the synchronous reactance per phase is

$$X_s = X_a + X_{ar}$$
$$= 0.3 + 2.19$$
$$= 2.49\ \Omega$$

(b) The rated load (or full load) current is

$$I_a = \frac{S}{\sqrt{3}V_t}$$
$$= \frac{50\times10^6}{\sqrt{3}(13{,}200)}$$
$$= 2{,}186.93\ \text{A}$$

and when expressed as a phasor,

$$\boldsymbol{I}_a = I_a(\cos\theta - j\sin\theta)$$
$$= 2{,}186.93(0.8 - j0.6)$$
$$= 1{,}749.55 - j1{,}312.16$$
$$= 2{,}186.93\angle -36.87^\circ\ \text{A}$$

(c) The air-gap voltage (also known as the *voltage behind the leakage reactance*) is

$$\boldsymbol{E}_\phi = \boldsymbol{V}_\phi + jX_a\boldsymbol{I}_a$$
$$= 13{,}200\angle 0^\circ + j0.3(2{,}186.93\angle -36.87^\circ)$$
$$= 8{,}031.84\angle 3.75^\circ\ \text{V}$$

(d) The internal generated voltage (also known as the *voltage behind the synchronous reactance*) is

$$\boldsymbol{E}_a = \boldsymbol{V}_\phi + jX_s\boldsymbol{I}_a$$
$$= 13{,}200\angle 0^\circ + j2.49(2{,}186.93\angle -36.87^\circ)$$
$$= 11{,}727.44\angle 21.81^\circ\ \text{V}$$

(e) The power angle (also called the *torque angle*) is

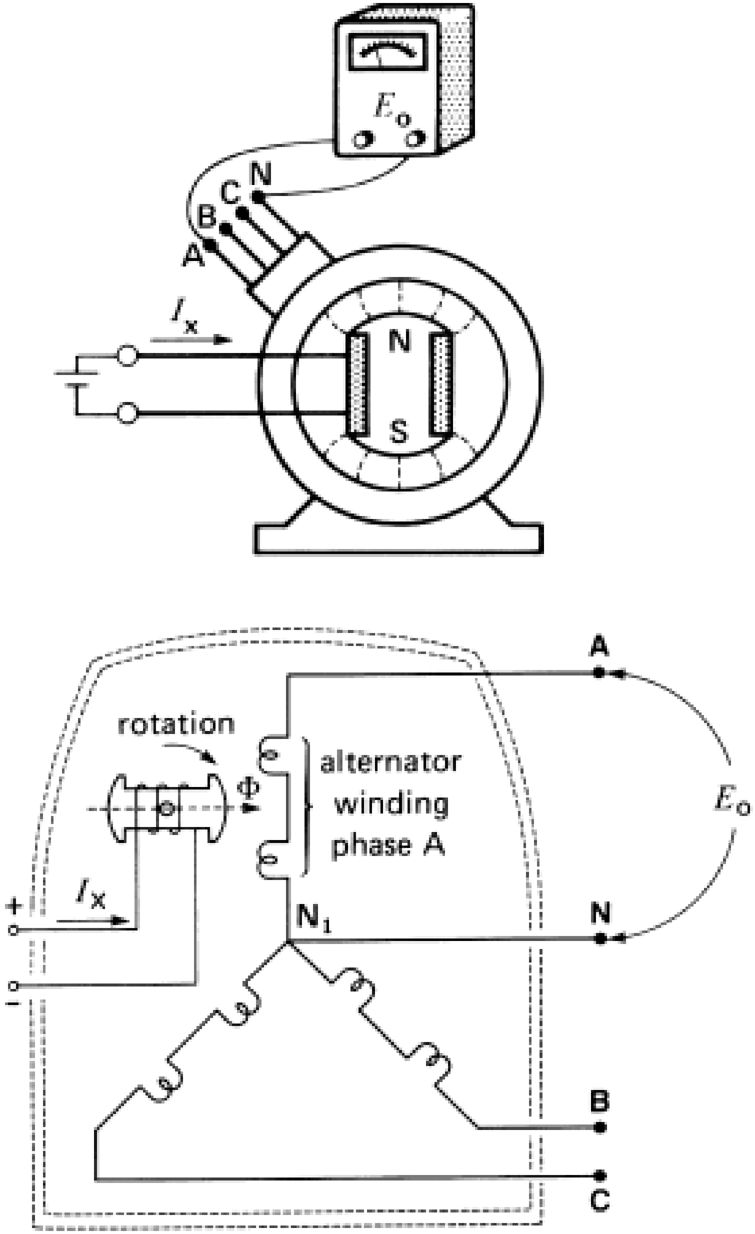
$$\delta = 21.81°$$

(f) The voltage regulation at full load is

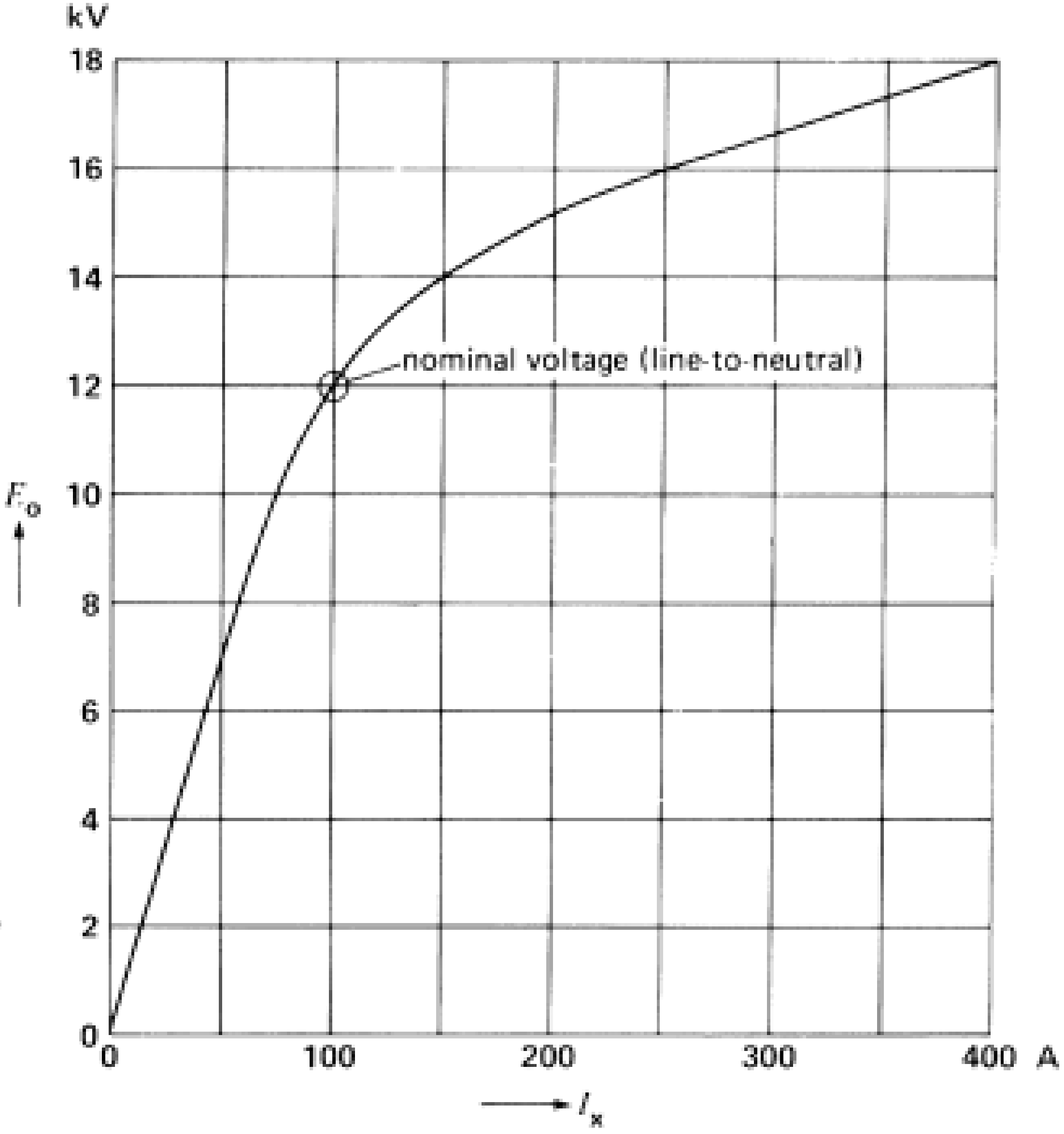
$$V\ Reg = \frac{E_a - V_\phi}{V_\phi} = \frac{11{,}727.44 - 7{,}621.02}{7{,}621.02} \times 100 = 53.88\%$$

5.7 MEASURING SYNCHRONOUS GENERATOR MODEL PARAMETERS

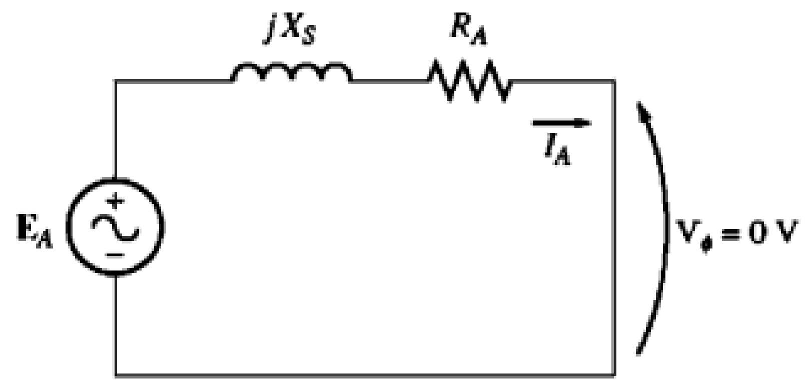
1. The relationship between field current and flux (and therefore between the field current and E_A)
2. The synchronous reactance
3. The armature resistance



(b)

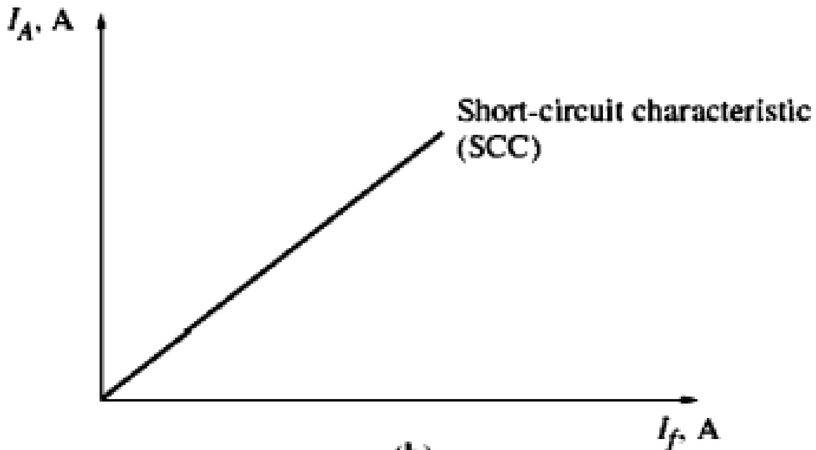


b. No-load saturation curve of a 36 MVA, 21 kV, 3-phase generator.

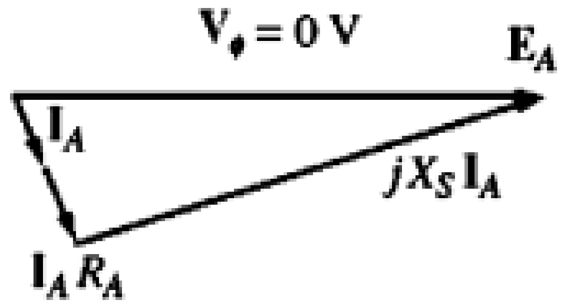


$$I_A = \frac{E_A}{\sqrt{R_A^2 + X_S^2}}$$

$$Z_S = \sqrt{R_A^2 + X_S^2} = \frac{E_A}{I_A} \qquad X_S >> R_A, \qquad \boxed{X_S \approx \frac{E_A}{I_A} = \frac{V_{\phi,oc}}{I_A}}$$



(b)



If E_A and I_A are known for a given situation, then the synchronous reactance X_S can be found.

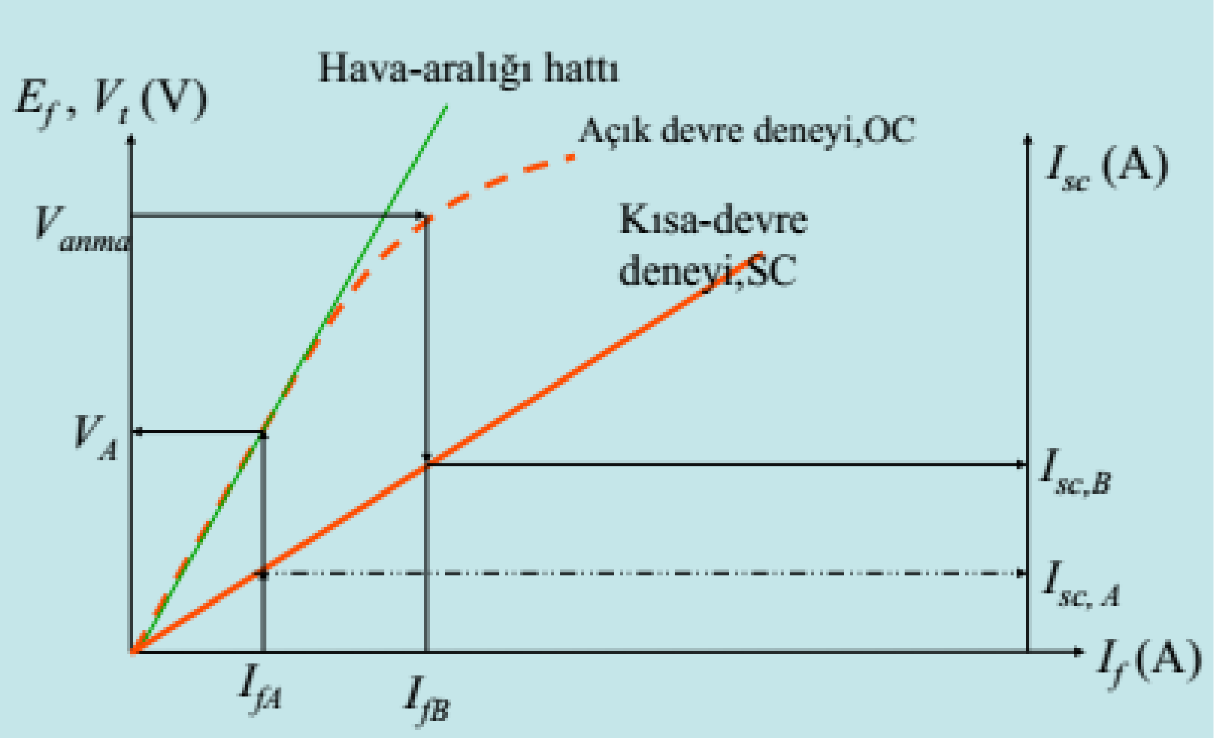
Therefore, an *approximate* method for determining the synchronous reactance X_S at a given field current is

1. Get the internal generated voltage E_A from the OCC at that field current.
2. Get the short-circuit current flow $I_{A,SC}$ at that field current from the SCC.
3. Find X_S by applying Equation (5–26).

Senkron reaktansın X_s bulunması

- Belirli bir alan akımı I_{fA} için iç gerilim E_f (=V_A) açık devre deneyinden bulunabilir, kısa devre akımı $I_{sc,A}$ kısa devre deneyinden bulunabilir.
- Senkron reaktans X_s izleyen denklemler ile elde edilebilir:

$$Z_{s,doymamış} = \sqrt{R_a^2 + X_{s,doymamış}^2} = \frac{V_A(=E_f)}{|I_{scA}|}$$



$$X_{s,doymamış} = \sqrt{Z_{s,doymamış}^2 - R_a^2}$$

R_a değeri DA deneyinden bilinir.

$X_{s,doymamış} >> R_a$ olduğundan

$$X_{s,doymamış} \approx \frac{E_f}{I_{scA}} = \frac{V_{t,oc}}{I_{scA}}$$

Doyum durumunda X_s

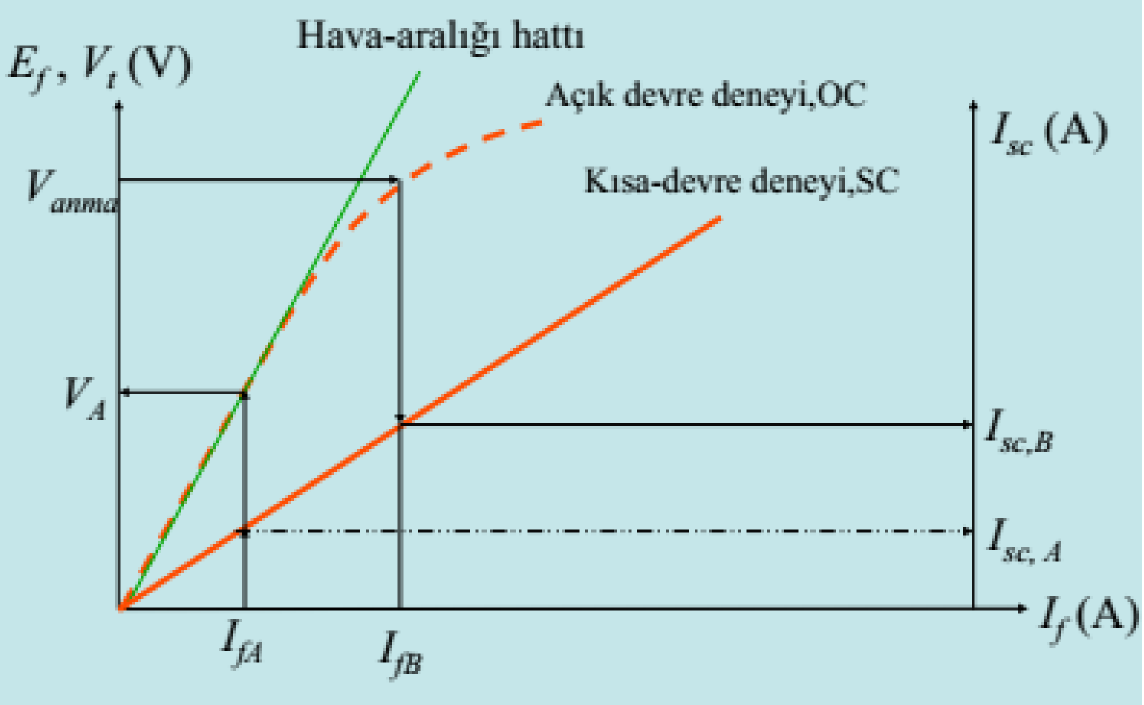
$V = V_{anma}$ iken

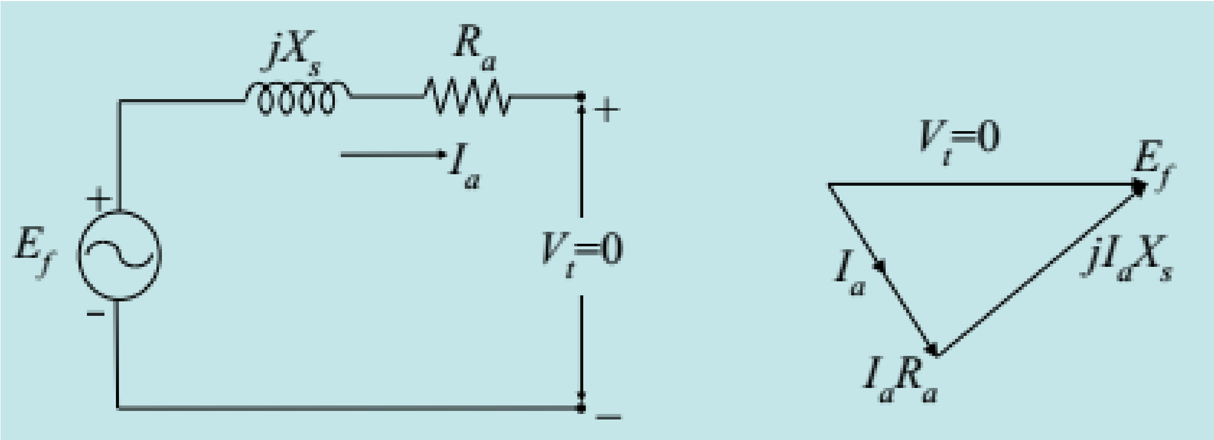
$$Z_{s,doymuş} = \sqrt{R_a^2 + X_{s,doymuş}^2}$$

$$Z_{s,doymuş} = \frac{V_{anma}\left(= E_f\right)}{\left|I_{scB}\right|}$$

$$X_{s,doymuş} = \sqrt{Z_{s,doymuş}^2 - R_a^2}$$

R_a değeri DA deneyinden bilinir





Doyum durumundaki eşdeğer devre ve fazör diyagramı

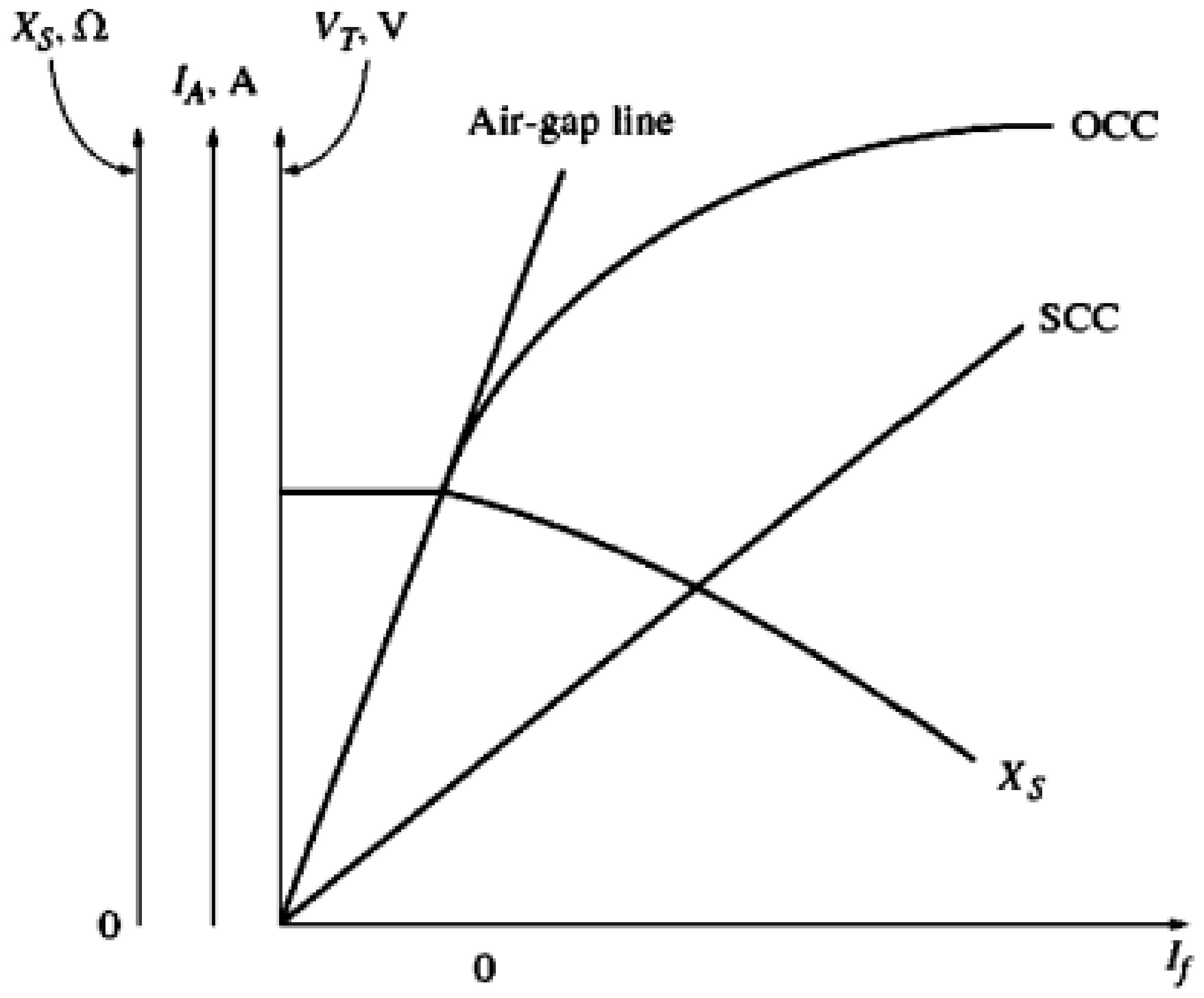
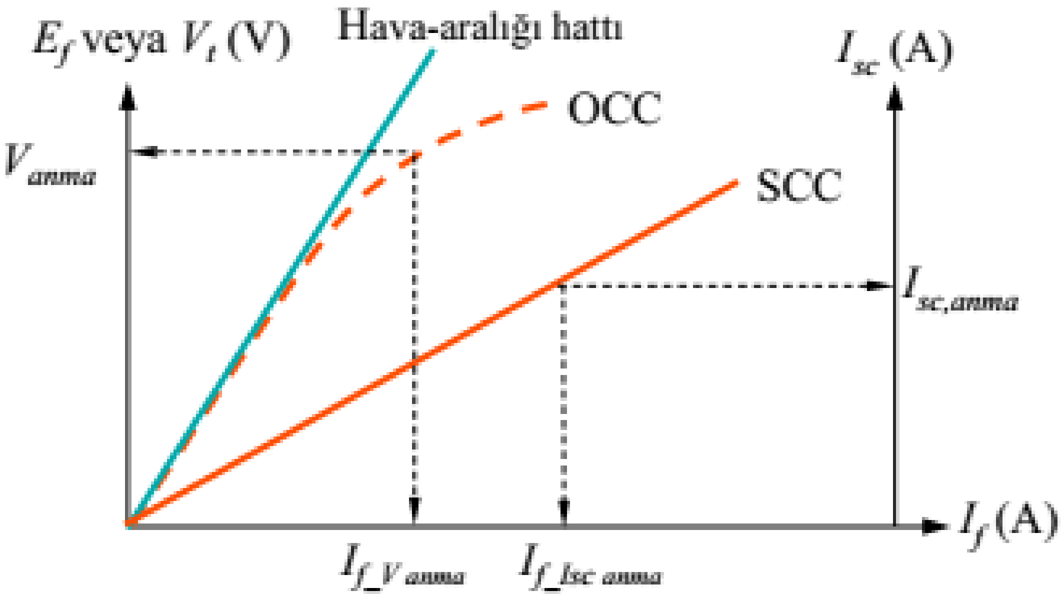


FIGURE 5–19
A sketch of the approximate synchronous reactance of a synchronous generator as a function of the field current in the machine. The constant value of reactance found at low values of field current is the *unsaturated* synchronous reactance of the machine.

Kısa-devre oranı (Short Circuit Ratio, SCR)

Senkron generatörü tanımlamak için kullanılan diğer bir parametre de kısa-devre oranıdır. Bir generatörün kısa-devre oranı (SCR); açık devrede anma gerilimi için gerekli uyartım akımının kısa devrede anma akımı için gerekli uyartım akımına oranıdır. Kısa devre oranı SCR, doymuş senkron reaktansın per unit değerinin tersidir ve izleyen denklem ile hesaplanır:

$$SCR = \frac{I_{f\_V_{anma}}}{I_{f\_Isc_{anma}}} = \frac{1}{X_{s\_doymuş}\left[p.u.olarak\right]}$$



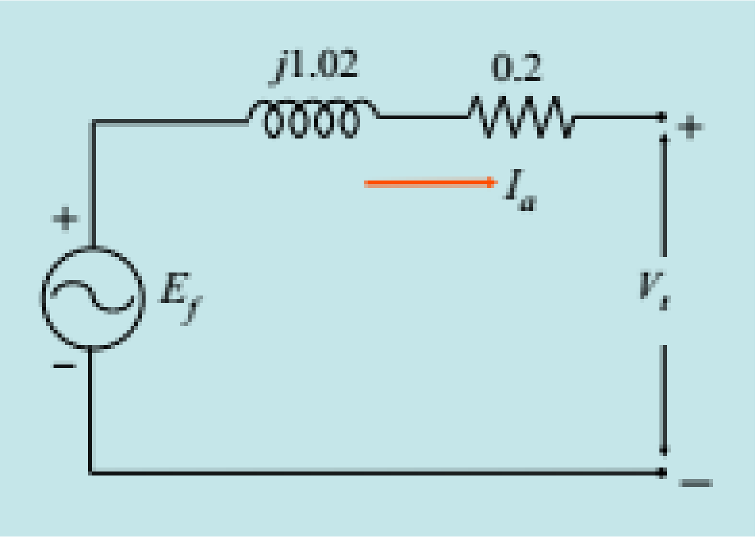
Örnek 1

200kVA, 480V, 60Hz, 4-kutup, Y-bağlı senkron generatörün 5A anma uyartım akımıyla deneyi yapılmış ve izleyen veriler alınmıştır.

a) Açık-devre (OC) deneyinden – Anma uyartım akımında, terminal (uç) gerilimi = 540V,
b) Kısa-devre (SC) deneyinden – anma uyartım akımında, hat akımı=300A ,
c) DA deneyinden – 10V DA gerilimi iki çıkış ucuna uygulandığında, 25A akım ölçülmüştür.

1) Dönme hızını d/d olarak hesaplayınız.
2) Üretilen emk ile doymuş eşdeğer devre parametrelerini (endüvi direnci ve senkron reaktansı) hesaplayınız.

Örnek 1 için çözüm



1)

f_e = elektrik frekansı = $Pn_m/120$

f_e = 60Hz

P = kutup sayısı = 4

n_m = mekanik dönme hızı, d/d

Bu durumda, n_m = (120 x 60)/4 = 1800 d/d

2) Açık-devre deneyinde, $I_a = 0$ *ve* $E_f = V_t$

E_f = 540/1.732=311.8V (Makina Y-bağlı olduğundan)

Kısa-devre deneyinde, çıkış uçları kısa devredir, $V_t = 0$

$E_f = I_a Z_s$ veya $Z_s = E_f / I_a$ =311.8/300=1.04ohm

DA deneyinde, $R_a = V_{DA}/(2I_{DA})$ = 10/(2X25) = 0.2 ohm

Synchronous empedans ve reactance, $Z_{s,doymuş} = \sqrt{R_a^2 + X_{s,doymuş}^2}$

$$X_{s,doymuş} = \sqrt{Z_{s,doymuş}^2 - R_a^2} = \sqrt{1.04^2 - 0.2^2} = 1.02\,\text{ohm}$$

Synchronous generator under load

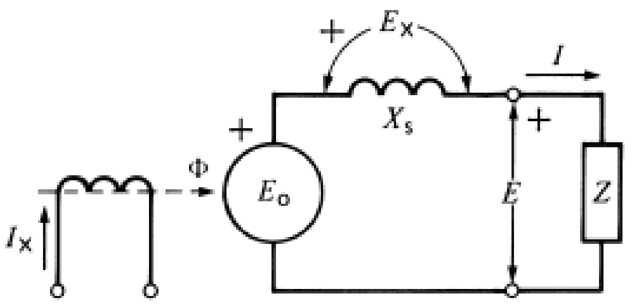
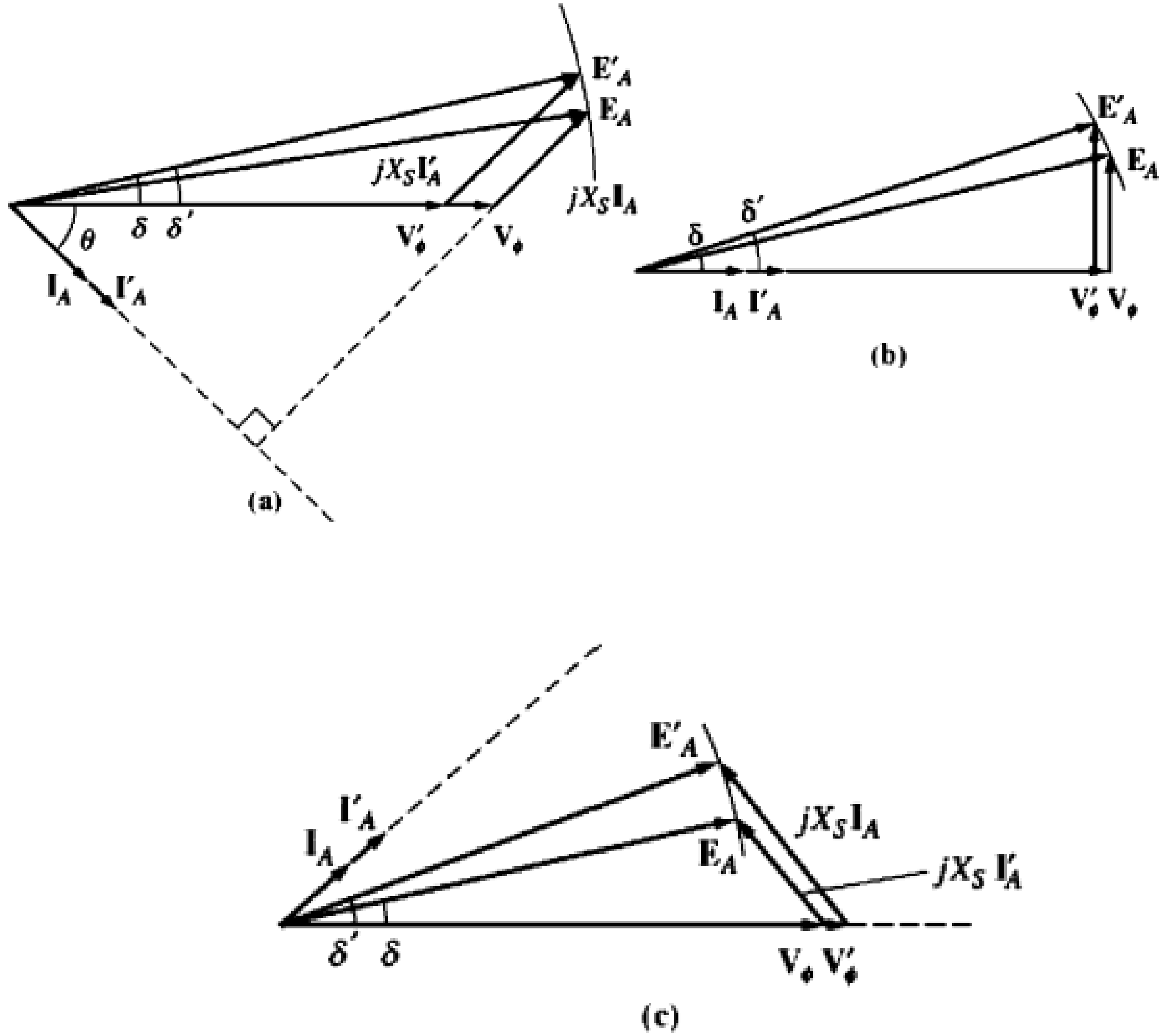


Figure 19
Equivalent circuit of a generator under load.

1. Isolated loads, supplied by a single generator
2. The infinite bus



General conclusions from this discussion of synchronous generator behavior are

1. If lagging loads ($+Q$ or inductive reactive power loads) are added to a generator, $\mathbf{V}_\phi$ and the terminal voltage V_T decrease significantly.
2. If unity-power-factor loads (no reactive power) are added to a generator, there is a slight decrease in $\mathbf{V}_\phi$ and the terminal voltage.
3. If leading loads ($-Q$ or capacitive reactive power loads) are added to a generator, $\mathbf{V}_\phi$ and the terminal voltage will rise.

A convenient way to compare the voltage behavior of two generators is by their *voltage regulation*. The voltage regulation (VR) of a generator is defined by the equation

$$\boxed{\mathrm{VR} = \frac{V_{\mathrm{nl}} - V_{\mathrm{fl}}}{V_{\mathrm{fl}}} \times 100\%}$$

Example Problems

Example 5–2. A 480-V, 60-Hz, Δ-connected, four-pole synchronous generator has the OCC shown in Figure 5–23a. This generator has a synchronous reactance of 0.1 Ω and an armature resistance of 0.015 Ω. At full load, the machine supplies 1200 A at 0.8 PF lagging. Under full-load conditions, the friction and windage losses are 40 kW, and the core losses are 30 kW. Ignore any field circuit losses.

(a) What is the speed of rotation of this generator?
(b) How much field current must be supplied to the generator to make the terminal voltage 480 V at no load?
(c) If the generator is now connected to a load and the load draws 1200 A at 0.8 PF lagging, how much field current will be required to keep the terminal voltage equal to 480 V?
(d) How much power is the generator now supplying? How much power is supplied to the generator by the prime mover? What is this machine's overall efficiency?
(e) If the generator's load were suddenly disconnected from the line, what would happen to its terminal voltage?
(f) Finally, suppose that the generator is connected to a load drawing 1200 A at 0.8 PF *leading*. How much field current would be required to keep V_T at 480 V?

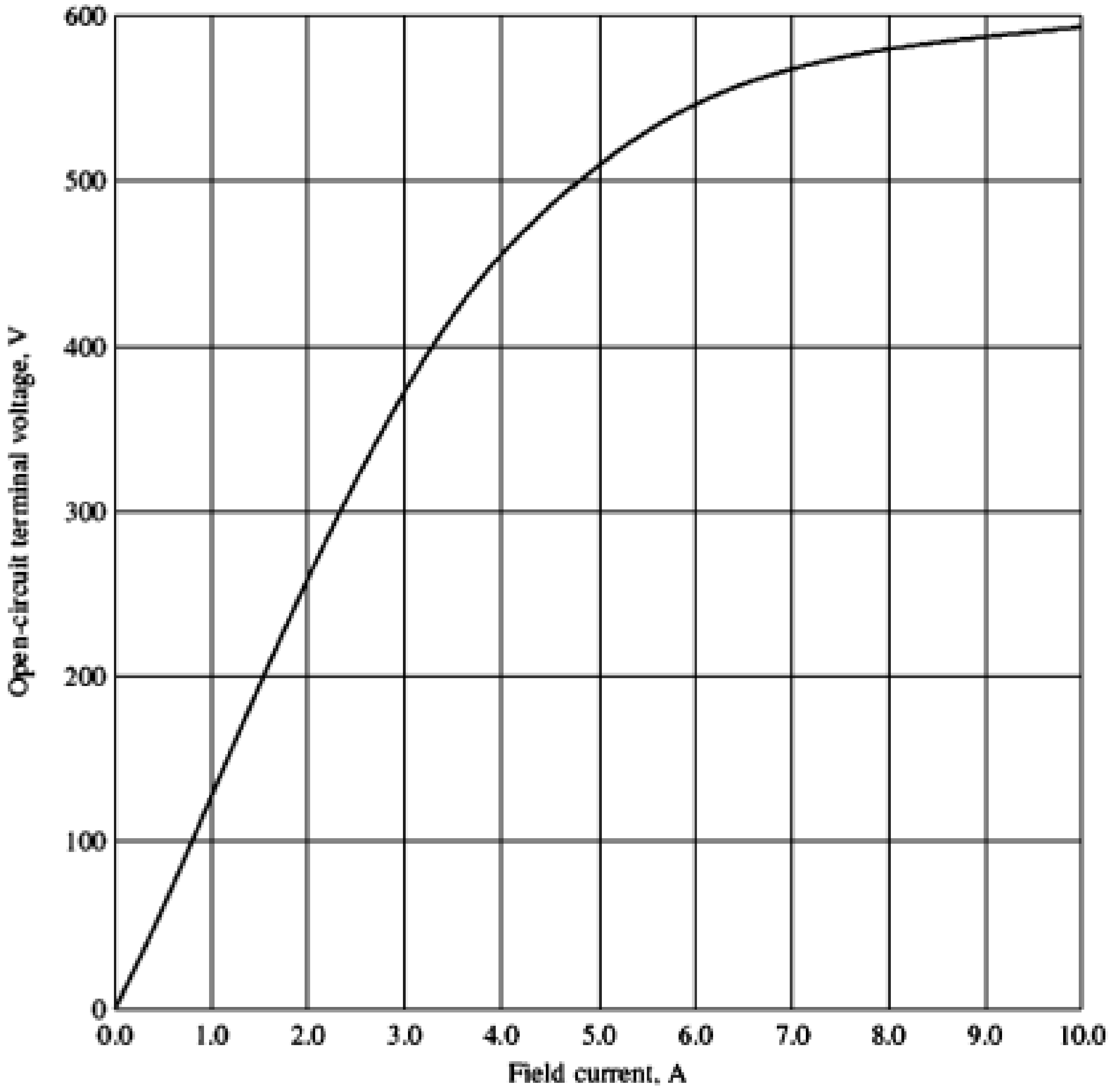


FIGURE 5–23
(a) Open-circuit characteristic of the generator in Example 5–2.

Solution

This synchronous generator is Δ-connected, so its phase voltage is equal to its line voltage $V_\phi = V_T$, while its phase current is related to its line current by the equation $I_L = \sqrt{3} I_\phi$.

(a) The relationship between the electrical frequency produced by a synchronous generator and the mechanical rate of shaft rotation is given by Equation (4–34):

$$f_e = \frac{n_m P}{120} \tag{4–34}$$

Therefore,

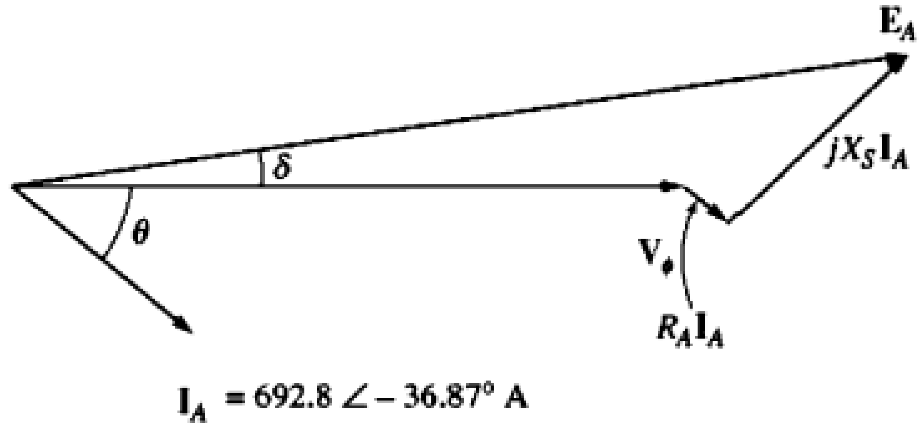
$$n_m = \frac{120 f_e}{P}$$

$$= \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

(b) In this machine, $V_T = V_\phi$. Since the generator is at no load, $\mathbf{I}_A = 0$ and $\mathbf{E}_A = \mathbf{V}_\phi$. Therefore, $V_T = V_\phi = E_A = 480$ V, and from the open-circuit characteristic, $I_F = 4.5$ A.

(c) If the generator is supplying 1200 A, then the armature current in the machine is

$$I_A = \frac{1200\text{ A}}{\sqrt{3}} = 692.8\text{ A}$$



$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$= 480\angle 0^\circ \text{ V} + (0.015\ \Omega)(692.8\angle -36.87^\circ \text{ A}) + (j0.1\ \Omega)(692.8\angle -36.87^\circ \text{ A})$$
$$= 480\angle 0^\circ \text{ V} + 10.39\angle -36.87^\circ \text{ V} + 69.28\angle 53.13^\circ \text{ V}$$
$$= 529.9 + j49.2 \text{ V} = 532\angle 5.3^\circ \text{ V}$$

To keep the terminal voltage at 480 V, $\mathbf{E}_A$ must be adjusted to 532 V. From Figure 5–23, the required field current is 5.7 A.

(d) The power that the generator is now supplying can be found from Equation (5–16):

$$P_{\text{out}} = \sqrt{3}V_T I_L \cos\theta$$
$$= \sqrt{3}(480 \text{ V})(1200 \text{ A}) \cos 36.87^\circ$$
$$= 798 \text{ kW}$$

$$P_{\text{elec loss}} = 3I_A^2R_A$$
$$= 3(692.8 \text{ A})^2(0.015\ \Omega) = 21.6 \text{ kW}$$

$$P_{\text{in}} = 798 \text{ kW} + 21.6 \text{ kW} + 30 \text{ kW} + 40 \text{ kW} = 889.6 \text{ kW}$$

Therefore, the machine's overall efficiency is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{798 \text{ kW}}{889.6 \text{ kW}} \times 100\% = 89.75\%$$

(*e*) If the generator's load were suddenly disconnected from the line, the current $\mathbf{I}_A$ would drop to zero, making $\mathbf{E}_A = \mathbf{V}_\phi$. Since the field current has not changed, $|\mathbf{E}_A|$ has not changed and $\mathbf{V}_\phi$ and V_T must rise to equal $\mathbf{E}_A$. Therefore, if the load were suddenly dropped, the terminal voltage of the generator would rise to 532 V.

(*f*) If the generator were loaded down with 1200 A at 0.8 PF leading while the terminal voltage was 480 V, then the internal generated voltage would have to be

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$

$$\begin{aligned}
&= 480\angle 0^\circ \text{ V} + (0.015\ \Omega)(692.8\angle 36.87^\circ \text{ A}) + (j0.1\ \Omega)(692.8\angle 36.87^\circ \text{ A}) \\
&= 480\angle 0^\circ \text{ V} + 10.39\angle 36.87^\circ \text{ V} + 69.28\angle 126.87^\circ \text{ V} \\
&= 446.7 + j61.7 \text{ V} = 451\angle 7.1^\circ \text{ V}
\end{aligned}$$

Therefore, the internal generated voltage E_A must be adjusted to provide 451 V if V_T is to remain 480 V. Using the open-circuit characteristic, the field current would have to be adjusted to 4.1 A.

Example 5–3. A 480-V, 50-Hz, Y-connected, six-pole synchronous generator has a per-phase synchronous reactance of 1.0 Ω. Its full-load armature current is 60 A at 0.8 PF lagging. This generator has friction and windage losses of 1.5 kW and core losses of 1.0 kW at 60 Hz at full load. Since the armature resistance is being ignored, assume that the I^2R losses are negligible. The field current has been adjusted so that the terminal voltage is 480 V at no load.

(*a*) What is the speed of rotation of this generator?

(*b*) What is the terminal voltage of this generator if the following are true?

1. It is loaded with the rated current at 0.8 PF lagging.
2. It is loaded with the rated current at 1.0 PF.
3. It is loaded with the rated current at 0.8 PF leading.

(*c*) What is the efficiency of this generator (ignoring the unknown electrical losses) when it is operating at the rated current and 0.8 PF lagging?

(*d*) How much shaft torque must be applied by the prime mover at full load? How large is the induced countertorque?

(*e*) What is the voltage regulation of this generator at 0.8 PF lagging? At 1.0 PF? At 0.8 PF leading?

(*a*) The speed of rotation of a synchronous generator in revolutions per minute is given by Equation (4–34):

$$f_e = \frac{n_m P}{120} \qquad (4\text{–}34)$$

Therefore,

$$n_m = \frac{120 f_e}{P}$$

$$= \frac{120(50\ \text{Hz})}{6\ \text{poles}} = 1000\ \text{r/min}$$

Alternatively, the speed expressed in radians per second is

$$\omega_m = (1000\ \text{r/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi}{}\right.$$

$$= 104.7\ \text{rad/s}$$

(*b*) 1. If the generator is loaded down wit
sulting phasor diagram looks like
phasor diagram, we know that $\mathbf{V}_\phi$ is at an angle of 0°, that the magnitude of $\mathbf{E}_A$ is 277 V, and that the quantity $jX_S\mathbf{I}_A$ is

$$jX_S\mathbf{I}_A = j(1.0\ \Omega)(60\angle -36.87°\ \text{A}) = 60\angle 53.13°\ \text{V}$$

The two quantities not known on the voltage diagram are the magnitude of $\mathbf{V}_\phi$ and the angle δ of $\mathbf{E}_A$. To find these values, the easiest approach is to construct a right triangle on the phasor diagram, as shown in the figure. From Figure 5–24a, the right triangle gives

$$E_A^2 = (V_\phi + X_S I_A \sin\theta)^2 + (X_S I_A \cos\theta)^2$$

$$(277\ \text{V})^2 = [V_\phi + (1.0\ \Omega)(60\ \text{A})\sin 36.87°]^2 + [(1.0\ \Omega)(60\ \text{A})\cos 36.87°]^2$$

$$76{,}729 = (V_\phi + 36)^2 + 2304$$

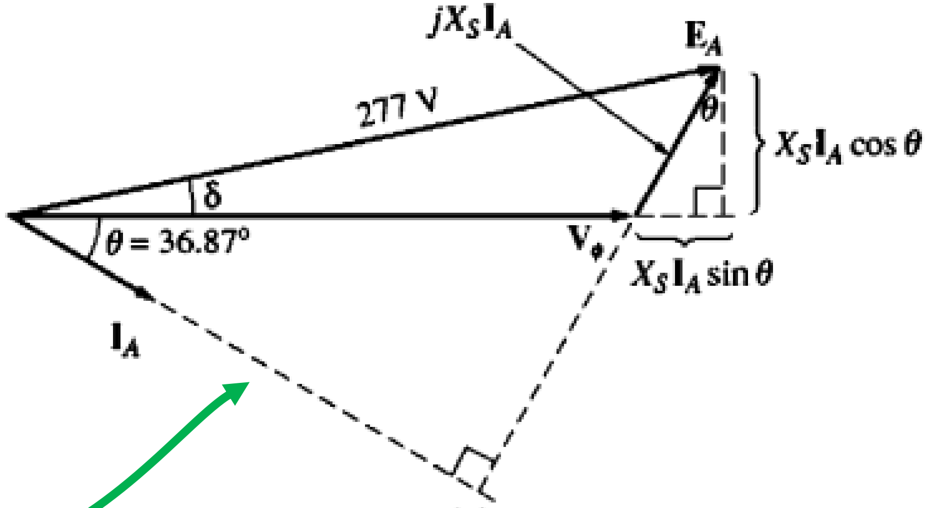
$$74{,}425 = (V_\phi + 36)^2$$

$$272.8 = V_\phi + 36$$

$$V_\phi = 236.8\ \text{V}$$

Since the generator is Y-connected, $V_T = \sqrt{3}V_\phi = 410$ V.

60 ∠ 53.13°



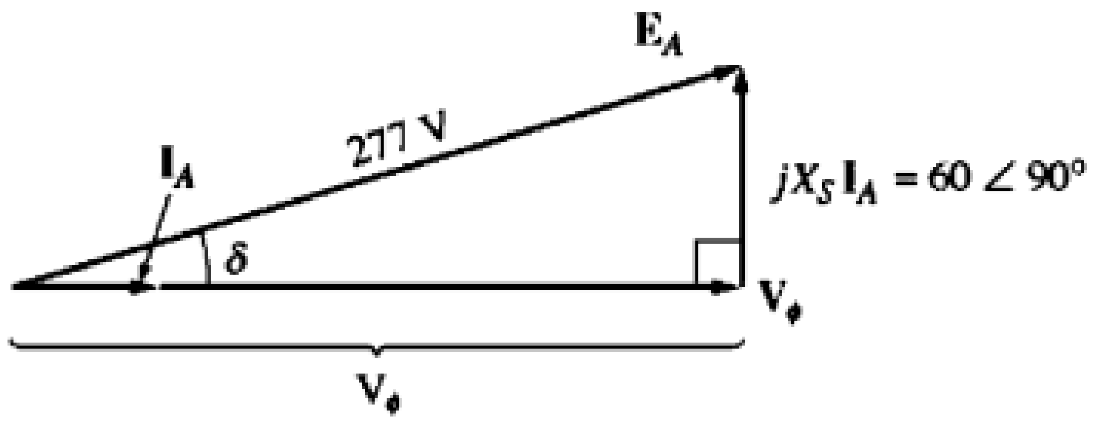
(a)

$$(277\text{ V})^2 = [V_\phi + (1.0\ \Omega)(60\text{ A})\sin 36.87°]^2 + [(1.0\ \Omega)(60\text{ A})\cos 36.87°]^2$$
$$76{,}729 = (V_\phi + 36)^2 + 2304$$
$$74{,}425 = (V_\phi + 36)^2$$
$$272.8 = V_\phi + 36$$
$$V_\phi = 236.8\text{ V}$$

Since the generator is Y-connected, $V_T = \sqrt{3}V_\phi = 410$ V.

2. If the generator is loaded with the rated current at unity power factor, then the phasor diagram will look like Figure 5–24b. To find $\mathbf{V}_\phi$ here the right triangle is

$$E_A^2 = V_\phi^2 + (X_S I_A)^2$$
$$(277\text{ V})^2 = V_\phi^2 + [(1.0\ \Omega)(60\text{ A})]^2$$
$$76{,}729 = V_\phi^2 + 3600$$
$$V_\phi^2 = 73{,}129$$
$$V_\phi = 270.4\text{ V}$$



Therefore, $V_T = \sqrt{3}V_\phi = 468.4$ V.

3. When the generator is loaded with the rated current at 0.8 PF leading, the resulting phasor diagram is the one shown in Figure 5–24c. To find $\mathbf{V}_\phi$ in this situation, we construct the triangle *OAB* shown in the figure. The resulting equation is

$$E_A^2 = (V_\phi - X_S I_A)^2 + (X_S I_A \cos\theta)^2$$

Therefore, the phase voltage at the rated load and 0.8 PF leading is

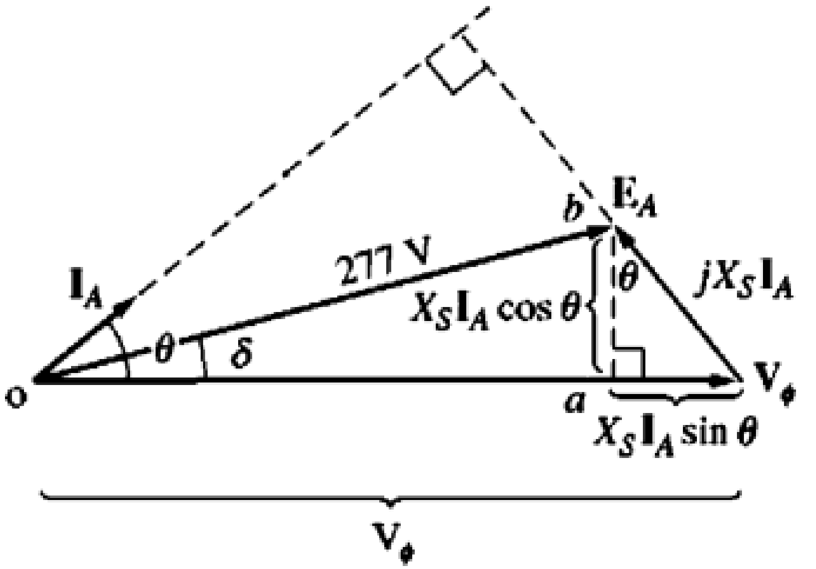
$$(277\text{ V})^2 = [V_\phi - (1.0\,\Omega)(60\text{ A})\sin 36.87°]^2 + [(1.0\,\Omega)(60\text{ A})\cos 36.87°]^2$$

$$76{,}729 = (V_\phi - 36)^2 + 2304$$

$$74{,}425 = (V_\phi - 36)^2$$

$$272.8 = V_\phi - 36$$

$$V_\phi = 308.8\text{ V}$$



Since the generator is Y-connected, $V_T = \sqrt{3}V_\phi = 535$ V.

(c) The output power of this generator at 60 A and 0.8 PF lagging is

$$P_{\text{out}} = 3V_\phi I_A \cos\theta$$
$$= 3(236.8\text{ V})(60\text{ A})(0.8) = 34.1\text{ kW}$$

The mechanical input power is given by

$$P_{\text{in}} = P_{\text{out}} + P_{\text{elec loss}} + P_{\text{core loss}} + P_{\text{mech loss}}$$
$$= 34.1\text{ kW} + 0 + 1.0\text{ kW} + 1.5\text{ kW} = 36.6\text{ kW}$$

The efficiency of the generator is thus

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{34.1\text{ kW}}{36.6\text{ kW}} \times 100\% = 93.2\%$$

(d) The input torque to this generator is given by the equation

$$P_{\text{in}} = \tau_{\text{app}}\omega_{\text{m}}$$

so
$$\tau_{\text{app}} = \frac{P_{\text{in}}}{\omega_m} = \frac{36.6\text{ kW}}{125.7\text{ rad/s}} = 291.2\text{ N}\cdot\text{m}$$

The induced countertorque is given by

$$P_{\text{conv}} = \tau_{\text{app}}\omega_{\text{m}}$$

so
$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_V} = \frac{34.1\text{ kW}}{125.7\text{ rad/s}} = 271.3\text{ N}\cdot\text{m}$$

(e) The voltage regulation of a generator is defined as

$$\text{VR} = \frac{V_{\text{nl}} - V_{\text{fl}}}{V_{\text{fl}}} \times 100\% \qquad (4\text{–}67)$$

By this definition, the voltage regulation for the lagging, unity, and leading power-factor cases are

1. Lagging case: $\text{VR} = \dfrac{480\text{ V} - 410\text{ V}}{410\text{ V}} \times 100\% = 17.1\%$
2. Unity case: $\text{VR} = \dfrac{480\text{ V} - 468\text{ V}}{468\text{ V}} \times 100\% = 2.6\%$
3. Leading case: $\text{VR} = \dfrac{480\text{ V} - 535\text{ V}}{535\text{ V}} \times 100\% = -10.3\%$

Synchronization of a generator

We often have to connect two or more generators in parallel to supply a common load. For example, as the power requirements of a large utility system build up during the day, generators are successively connected to the system to provide the extra power. Later, when the power demand falls, selected generators are temporarily disconnected from the system until power again builds up the following day. Synchronous generators are therefore regularly being connected and disconnected from a large power grid in response to customer demand. Such a grid is said to be an *infinite bus* because it contains so many generators essentially connected in parallel that neither the voltage nor the frequency of the grid can be altered.

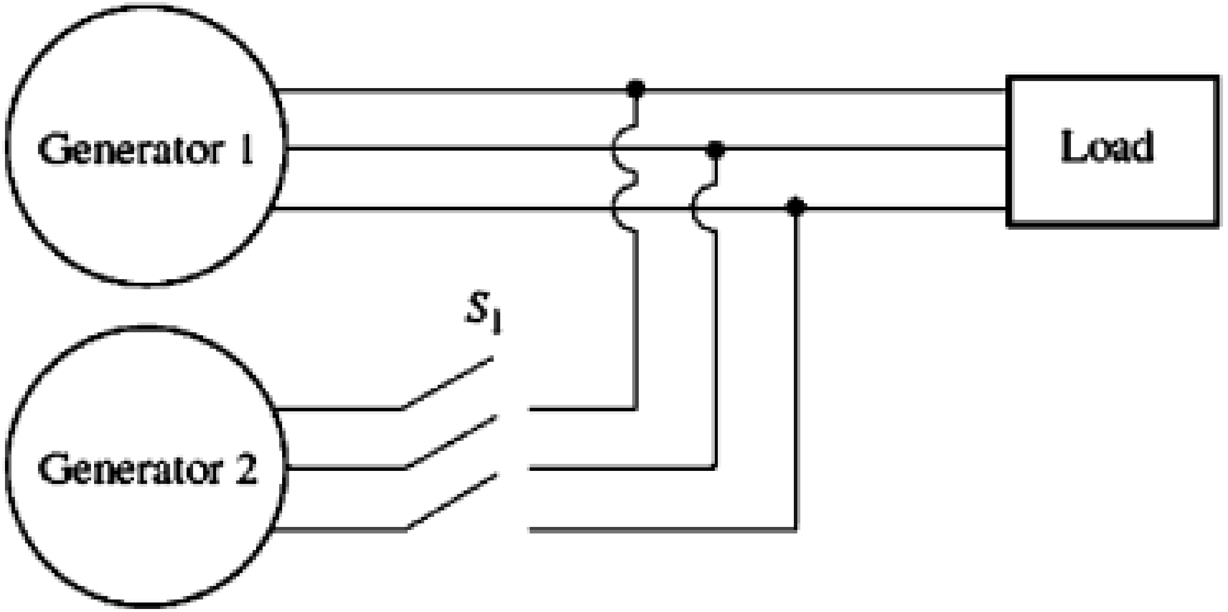
Before connecting a generator to an infinite bus (or in parallel with another generator), it must be *synchronized*. A generator is said to be synchronized when it meets all the following conditions:

1. The generator frequency is equal to the system frequency.
2. The generator voltage is equal to the system voltage.
3. The generator voltage is in phase with the system voltage.
4. The phase sequence of the generator is the same as that of the system.

Why are synchronous generators operated in parallel? There are several major advantages to such operation:

1. Several generators can supply a bigger load than one machine by itself.
2. Having many generators increases the reliability of the power system, since the failure of any one of them does not cause a total power loss to the load.
3. Having many generators operating in parallel allows one or more of them to be removed for shutdown and preventive maintenance.
4. If only one generator is used and it is not operating at near full load, then it will be relatively inefficient. With several smaller machines in parallel, it is possible to operate only a fraction of them. The ones that do operate are operating near full load and thus more efficiently.

The Conditions Required for Paralleling



1. The rms *line voltages* of the two generators must be equal.
2. The two generators must have the same *phase sequence*.
3. The phase angles of the two *a* phases must be equal.
4. The frequency of the new generator, called the *oncoming generator*, must be slightly higher than the frequency of the running system.

Condition 2 ensures that the sequence in which the phase voltages peak in the two generators is the same. If the phase sequence is different (as shown in Figure 5–27a), then even though one pair of voltages (the *a* phases) are in phase, the other two pairs of voltages are 120° out of phase. If the generators were connected in this manner, there would be no problem with phase *a*, but huge currents would flow in phases *b* and *c*, damaging both machines. To correct a phase sequence problem, simply swap the connections on any two of the three phases on one of the machines.

If the frequencies of the generators are not very nearly equal when they are connected together, large power transients will occur until the generators stabilize at a common frequency. The frequencies of the two machines must be very nearly equal, but they cannot be exactly equal. They must differ by a small amount so that the phase angles of the oncoming machine will change slowly with respect to the phase angles of the running system. In that way, the angles between the voltages can be observed and switch S_1 can be closed when the systems are exactly in phase.

To synchronize an alternator, we proceed as follows:

1. Adjust the speed regulator of the turbine so that the generator frequency is close to the system frequency.
2. Adjust the excitation so that the generator voltage E_o is equal to the system voltage E.

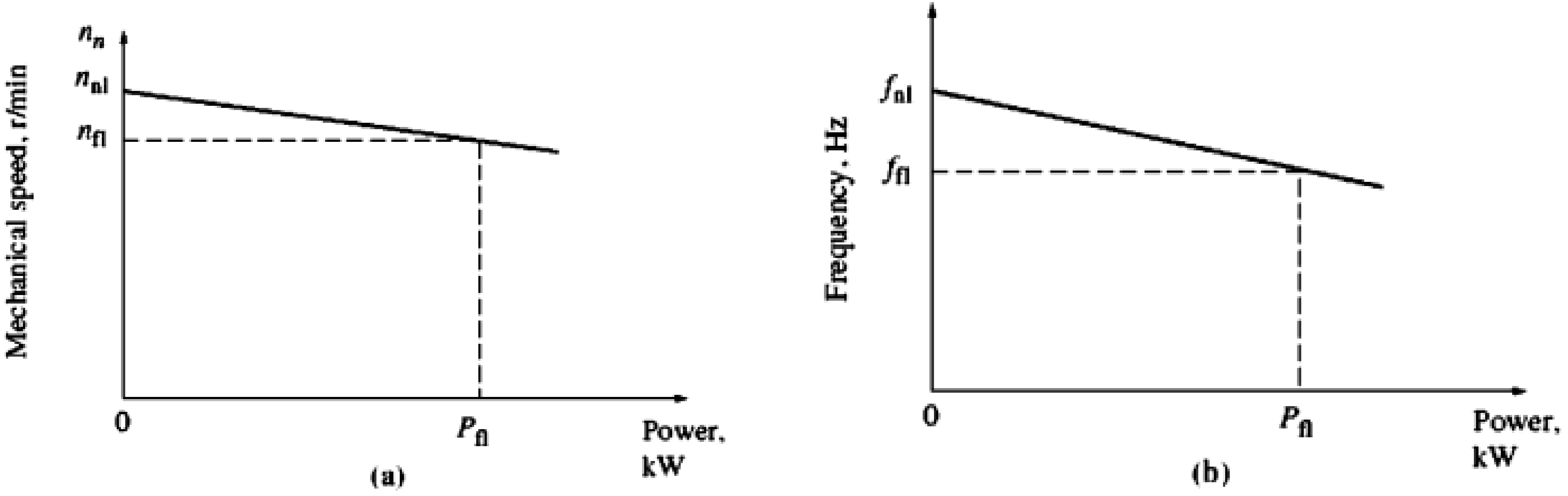
Figure 24
Synchroscope.

3. Observe the phase angle between E_o and E by means of a *synchroscope* (Fig. 24). This instrument has a pointer that continually indicates the phase angle between the two voltages, covering the entire range from zero to 360 degrees. Although the degrees are not shown, the dial has a zero marker to indicate when the voltages are in phase. In practice, when we synchronize an alternator, the pointer rotates slowly as it tracks the phase angle between the alternator and system voltages. If the generator frequency is slightly higher than the system frequency, the pointer rotates clockwise, indicating that the generator has a tendency to lead the system frequency. Conversely, if the generator frequency is slightly low, the pointer rotates counterclockwise. The turbine speed regulator is fine-tuned accordingly, so that the pointer barely creeps across the dial. A final check is made to see that the alternator voltage is still equal to the system voltage. Then, at the moment the pointer crosses the zero marker . . .

Frequency–Power and Voltage–Reactive Power Characteristics of a Synchronous Generator

Whatever governor mechanism is present on a prime mover, it will always be adjusted to provide a slight drooping characteristic with increasing load. The speed droop (SD) of a prime mover is defined by the equation

$$\boxed{SD = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\%} \qquad (5\text{–}27)$$



Since the shaft speed is related to the resulting electrical frequency by Equation (4–34),

$$f_e = \frac{n_m P}{120} \tag{4–34}$$

The relationship between frequency and power can be described quantitatively by the equation

$$\boxed{P = s_P(f_{\text{nl}} - f_{\text{sys}})} \tag{5–28}$$

where P = power output of the generator
f_{nl} = no-load frequency of the generator
f_{sys} = operating frequency of system
s_P = slope of curve, in kW/Hz or MW/Hz

A similar relationship can be derived for the reactive power Q and terminal voltage V_T. As previously seen, when a lagging load is added to a synchronous

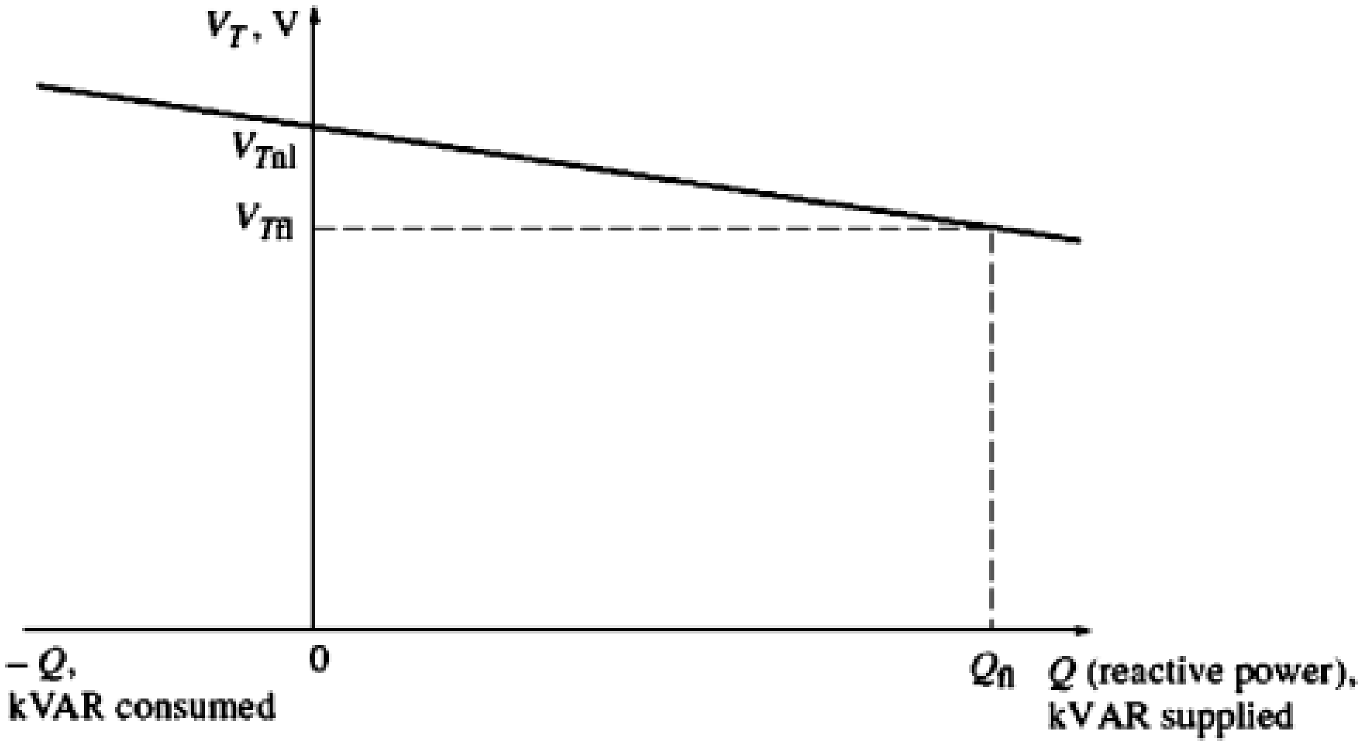


FIGURE 5–30
The curve of terminal voltage (V_T) versus reactive power (Q) for a synchronous generator.

Example 5–5. Figure 5–31 shows a generator supplying a load. A second load is to be connected in parallel with the first one. The generator has a no-load frequency of 61.0 Hz and a slope s_P of 1 MW/Hz. Load 1 consumes a real power of 1000 kW at 0.8 PF lagging, while load 2 consumes a real power of 800 kW at 0.707 PF lagging.

(a) Before the switch is closed, what is the operating frequency of the system?
(b) After load 2 is connected, what is the operating frequency of the system?
(c) After load 2 is connected, what action could an operator take to restore the system frequency to 60 Hz?

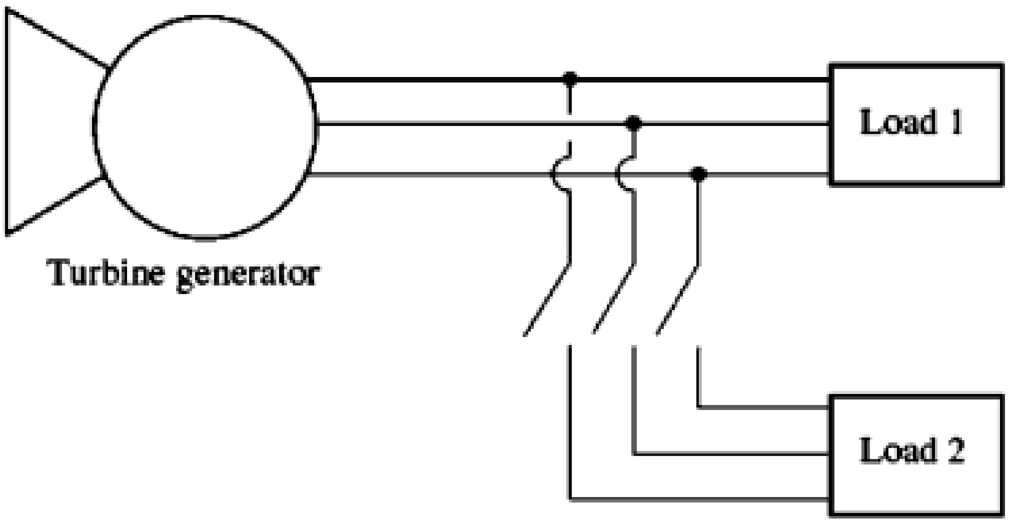


FIGURE 5–31
The power system in Example 5–5.

Solution

This problem states that the slope of the generator's characteristic is 1 MW/Hz and that its no-load frequency is 61 Hz. Therefore, the power produced by the generator is given by

$$P = s_P(f_{\text{nl}} - f_{\text{sys}}) \tag{5–28}$$

so

$$f_{\text{sys}} = f_{\text{nl}} - \frac{P}{s_P}$$

(a) The initial system frequency is given by

$$\begin{aligned} f_{\text{sys}} &= f_{\text{nl}} - \frac{P}{s_P} \\ &= 61\ \text{Hz} - \frac{1000\ \text{kW}}{1\ \text{MW/Hz}} = 61\ \text{Hz} - 1\ \text{Hz} = 60\ \text{Hz} \end{aligned}$$

(b) After load 2 is connected,

$$\begin{aligned} f_{\text{sys}} &= f_{\text{nl}} - \frac{P}{s_P} \\ &= 61\ \text{Hz} - \frac{1800\ \text{kW}}{1\ \text{MW/Hz}} = 61\ \text{Hz} - 1.8\ \text{Hz} = 59.2\ \text{Hz} \end{aligned}$$

(c) After the load is connected, the system frequency falls to 59.2 Hz. To restore the system to its proper operating frequency, the operator should increase the governor no-load set points by 0.8 Hz, to 61.8 Hz. This action will restore the system frequency to 60 Hz.

To summarize, when a generator is operating by itself supplying the system loads, then

1. The real and reactive power supplied by the generator will be the amount demanded by the attached load.
2. The governor set points of the generator will control the operating frequency of the power system.
3. The field current (or the field regulator set points) control the terminal voltage of the power system.

This is the situation found in isolated generators in remote field environments.

Operation of Generators in Parallel with Large Power Systems

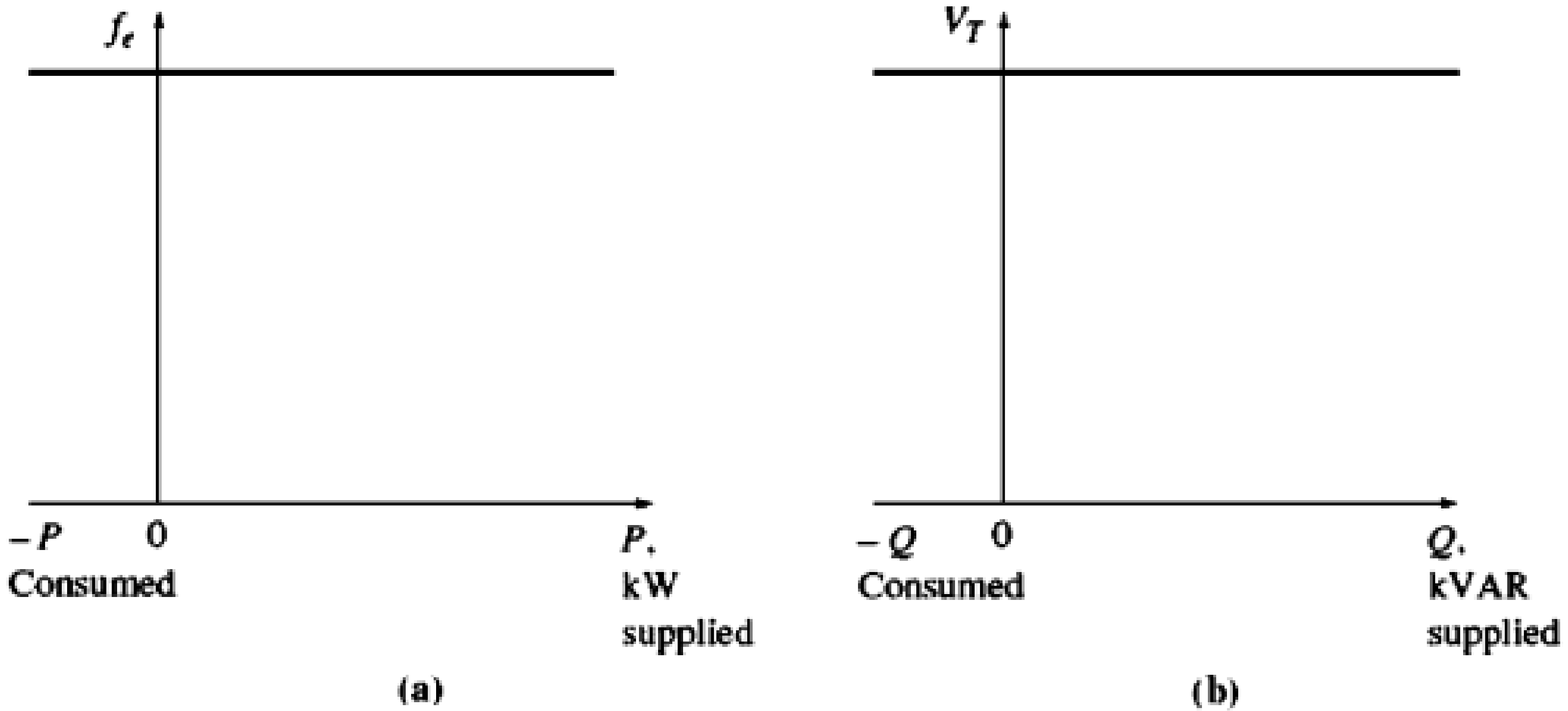


FIGURE 5–32
Curves for an infinite bus: (a) frequency versus power and (b) terminal voltage versus reactive power.

When a generator is connected in parallel with another generator or a large system, *the frequency and terminal voltage of all the machines must be the same*,

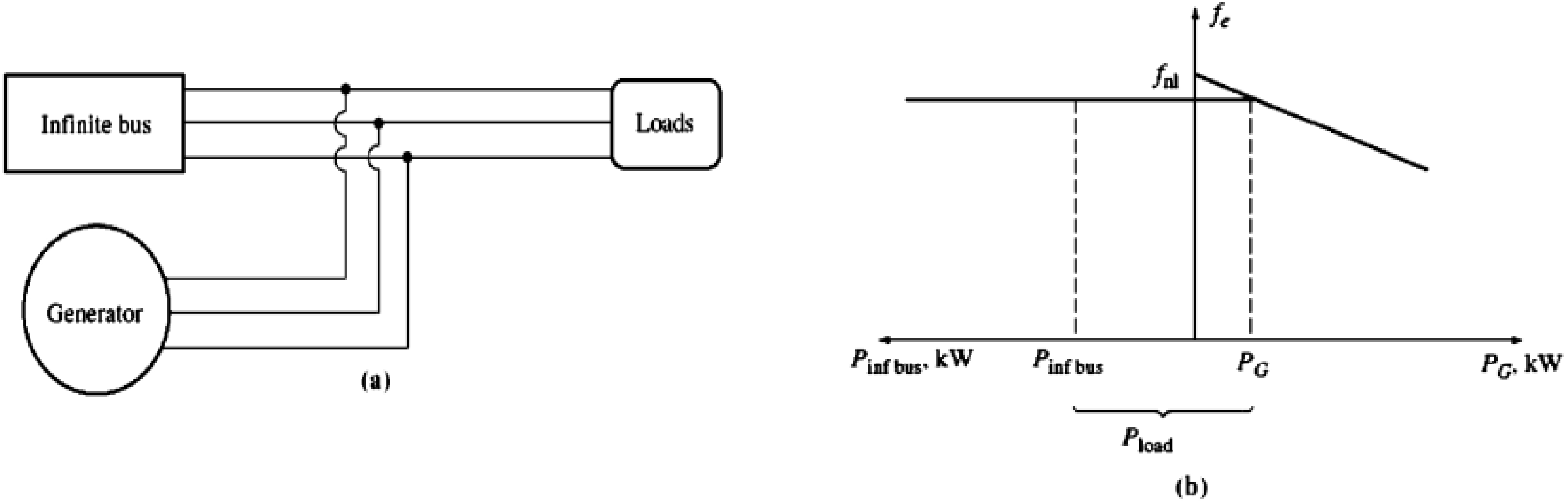


FIGURE 5–33
(a) A synchronous generator operating in parallel with an infinite bus. (b) The frequency-versus-power diagram (or *house diagram*) for a synchronous generator in parallel with an infinite bus.

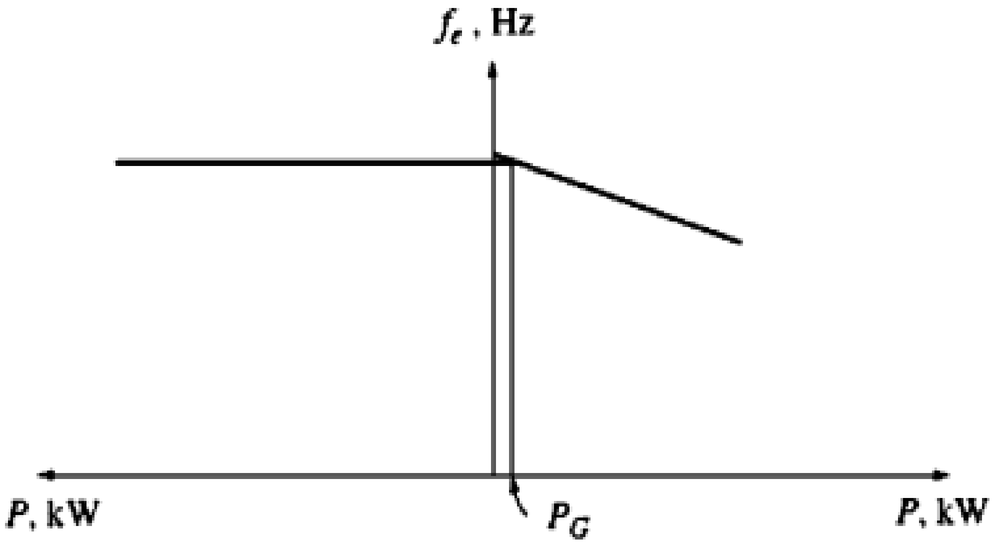


FIGURE 5–34
The frequency-versus-power diagram at the moment just after paralleling.

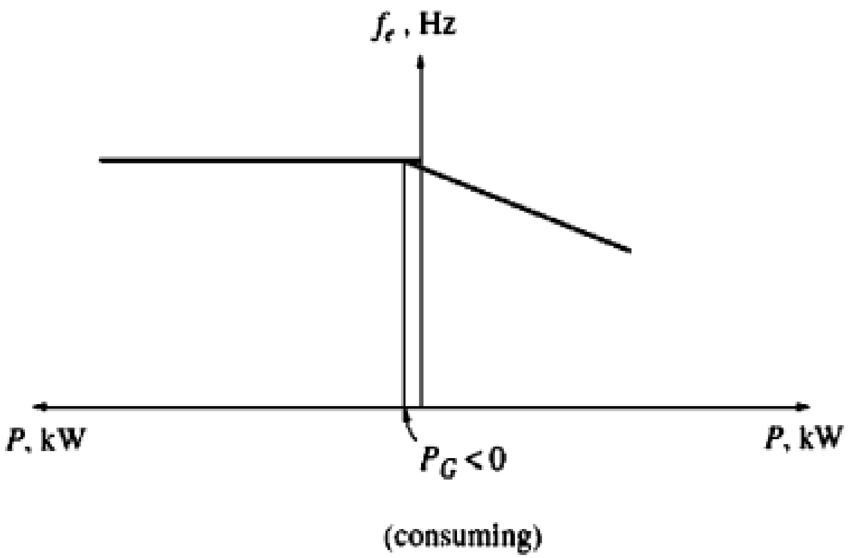


FIGURE 5–35
The frequency-versus-power diagram if the no-load frequency of the generator were slightly *less* than system frequency before paralleling.

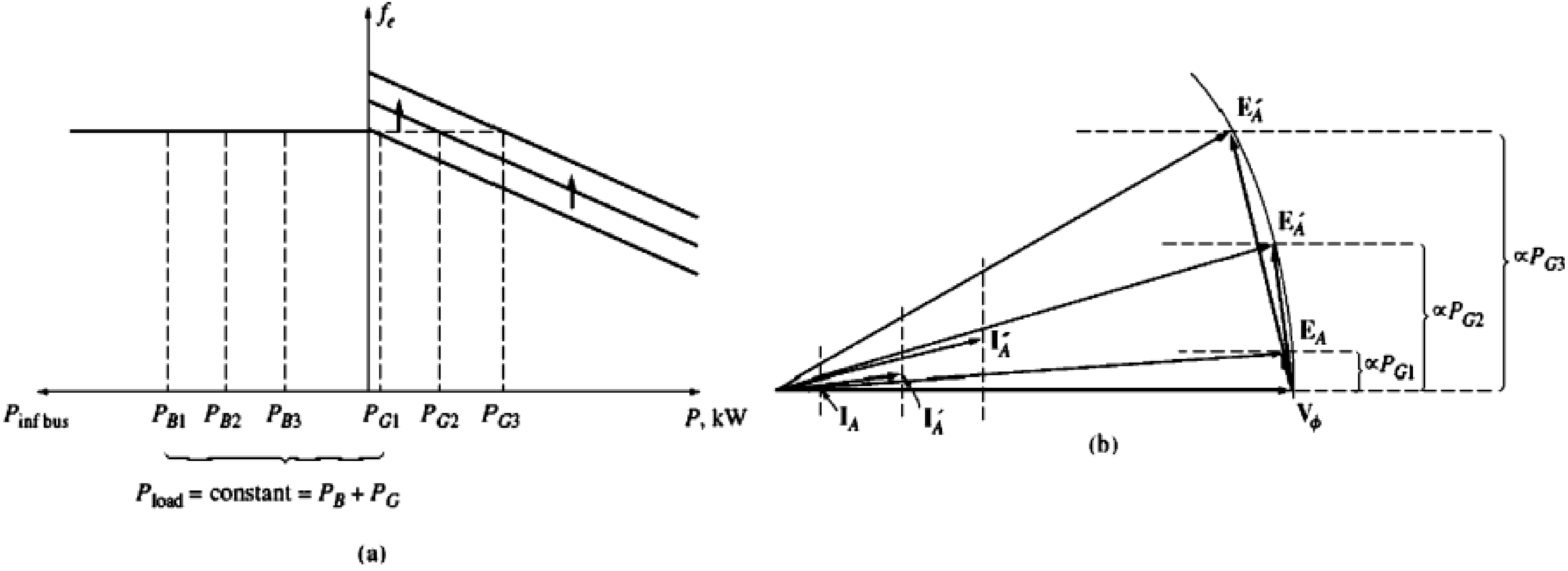


FIGURE 5–36

The effect of increasing the governor's set points on (a) the house diagram; (b) the phasor diagram.

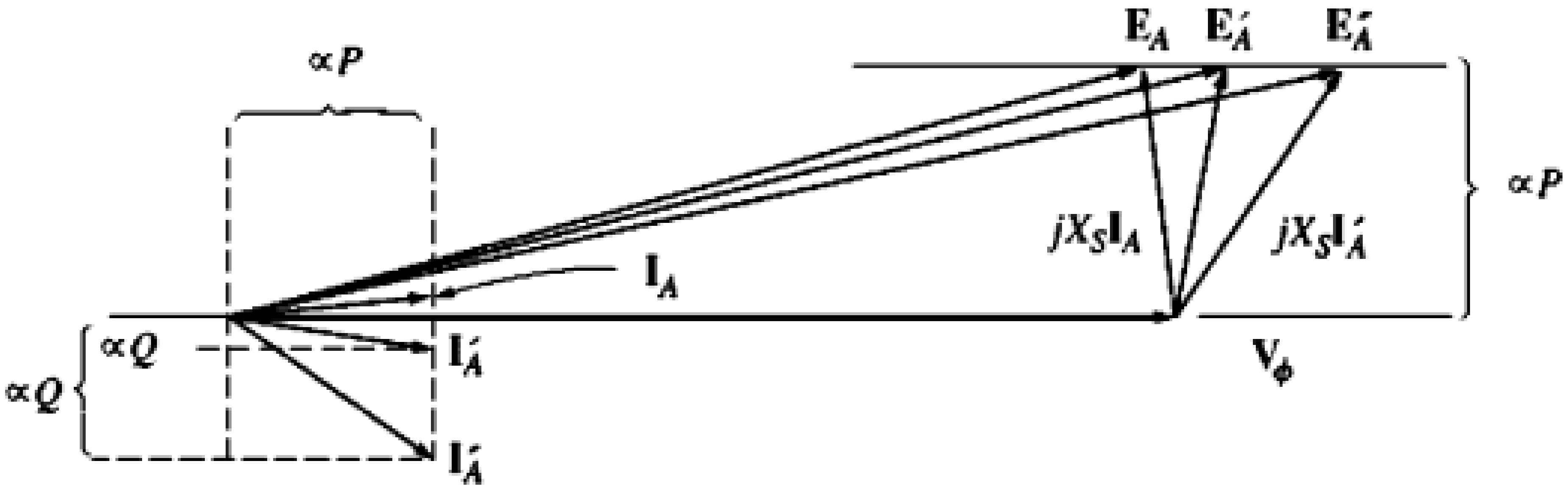


FIGURE 5–37
The effect of increasing the generator's field current on the phasor diagram of the machine.

Synchronous generator on an infinite bus

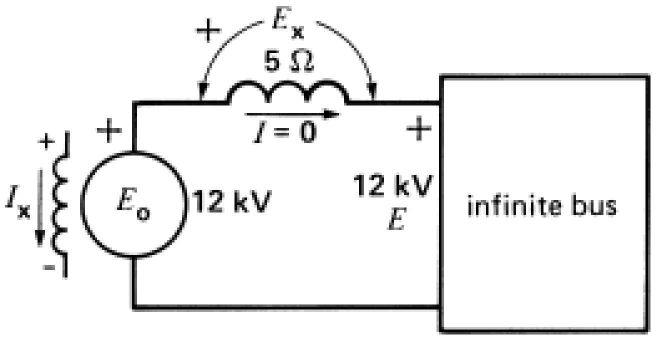


Figure 26a
Generator *floating* on an infinite bus.

An infinite bus is a system so powerful that it imposes its own voltage and frequency upon any apparatus connected to its terminals. Once connected to a large system (infinite bus), a synchronous generator becomes part of a network comprising hundreds of other generators that deliver power to thousands of loads. It is impossible, therefore, to specify the nature of the load (large or small, resistive or capacitive) connected to the terminals of this particular generator. What, then, determines the power the machine delivers? To answer this question, we must remember that both the value and the frequency of the terminal voltage across the generator are fixed. Consequently, we can vary only two machine parameters:

1. The exciting current I_x
2. The mechanical torque exerted by the turbine

Let us see how a change in these parameters affects the performance of the machine.

Infinite bus—effect of varying the exciting current

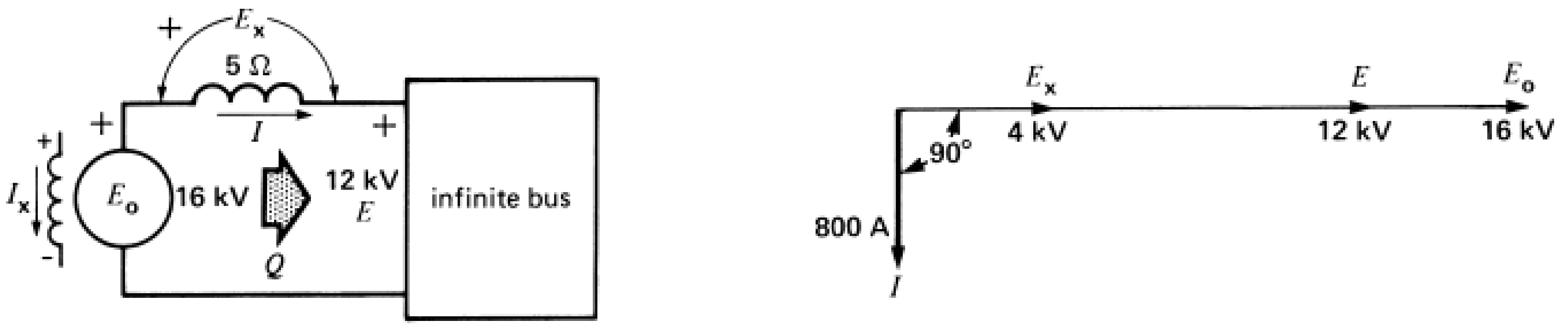


Figure 26b
Over-excited generator on an infinite bus.

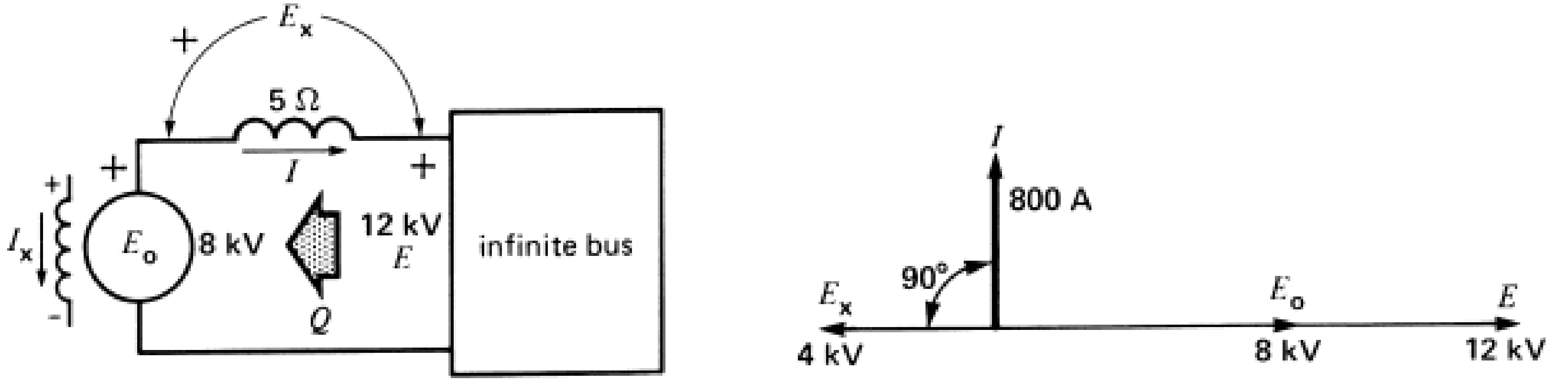


Figure 26c
Under-excited generator on an infinite bus.

Infinite bus—effect of varying the mechanical torque

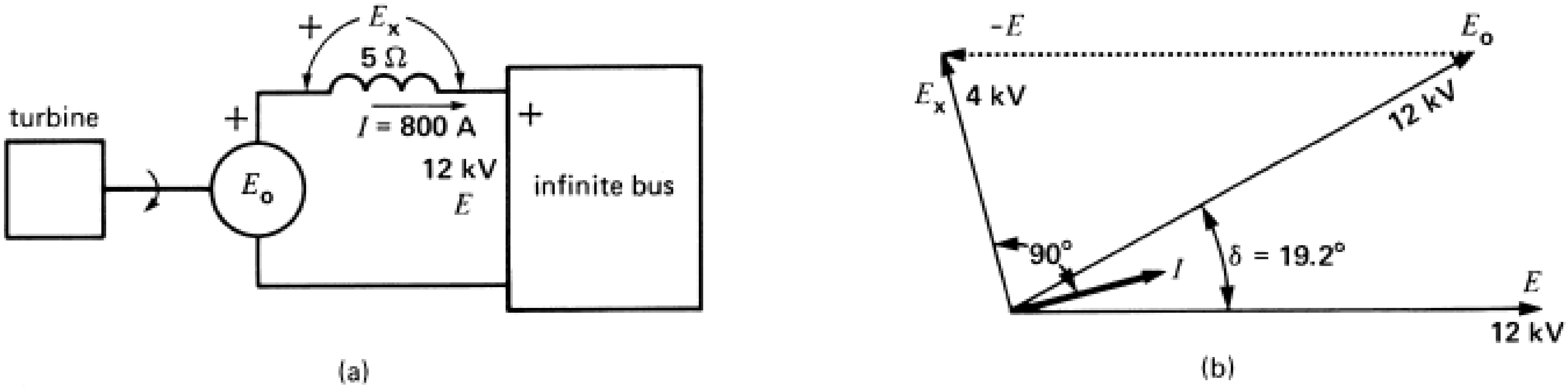


Figure 27

a. Turbine driving the generator.

b. Phasor diagram showing the torque angle δ.

To summarize, when a generator is operating in parallel with an infinite bus:

1. The frequency and terminal voltage of the generator are controlled by the system to which it is connected.
2. The governor set points of the generator control the real power supplied by the generator to the system.
3. The field current in the generator controls the reactive power supplied by the generator to the system.

This situation is much the way real generators operate when connected to a very large power system.

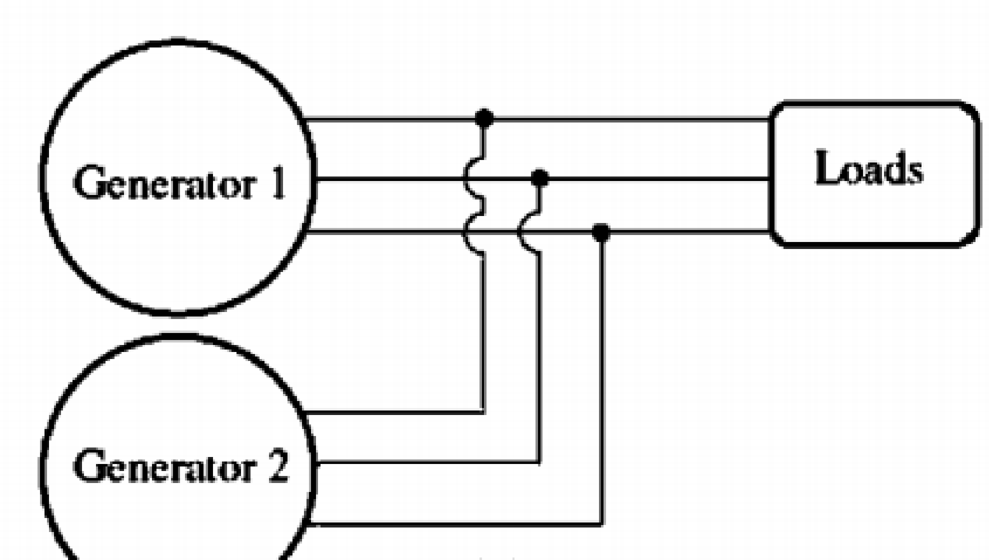
Operation of Generators in Parallel with Other Generators of the Same Size

If a generator is connected in parallel with another one of the same size, the resulting system is as shown in Figure 5–38a. In this system, the basic constraint is that *the sum of the real and reactive powers supplied by the two generators must equal the P and Q demanded by the load*. The system frequency is not constrained to be constant, and neither is the power of a given generator constrained to be constant. The power-frequency diagram for such a system immediately after G_2 has been paralleled to the line is shown in Figure 5–38b. Here, the total power P_{tot} (which is equal to P_{load}) is given by

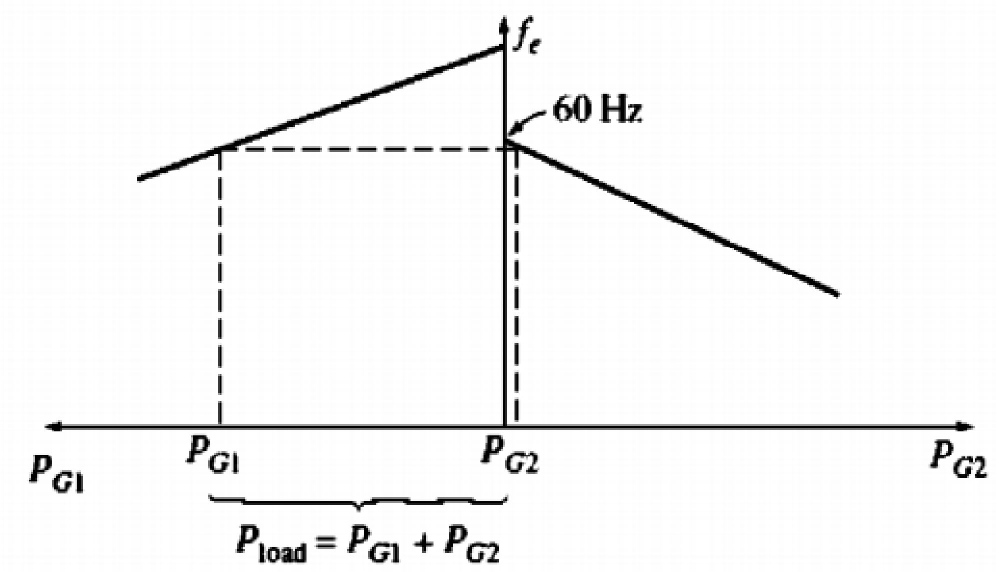
$$P_{tot} = P_{load} = P_{G1} + P_{G2} \tag{5–29a}$$

and the total reactive power is given by

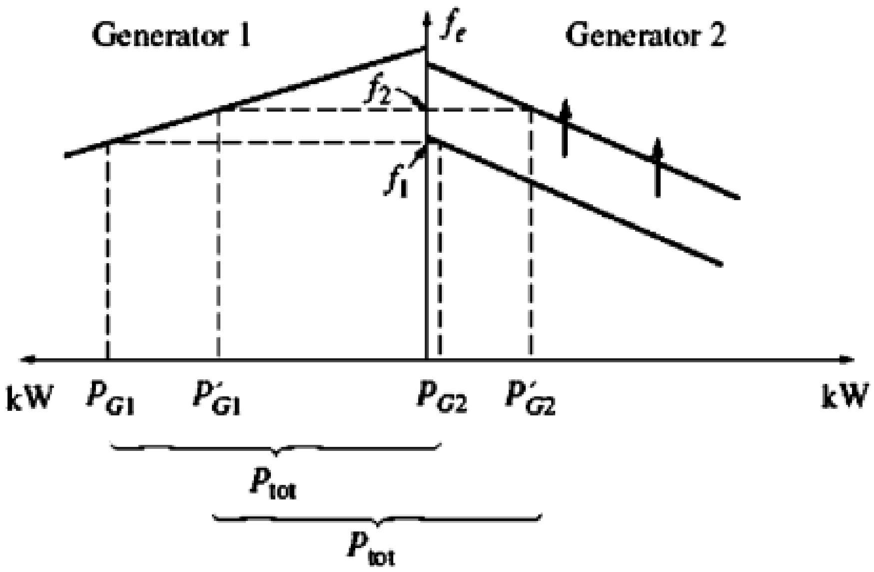
$$Q_{tot} = Q_{load} = Q_{G1} + Q_{G2} \tag{5–29b}$$


Generator 1
Loads
Generator 2

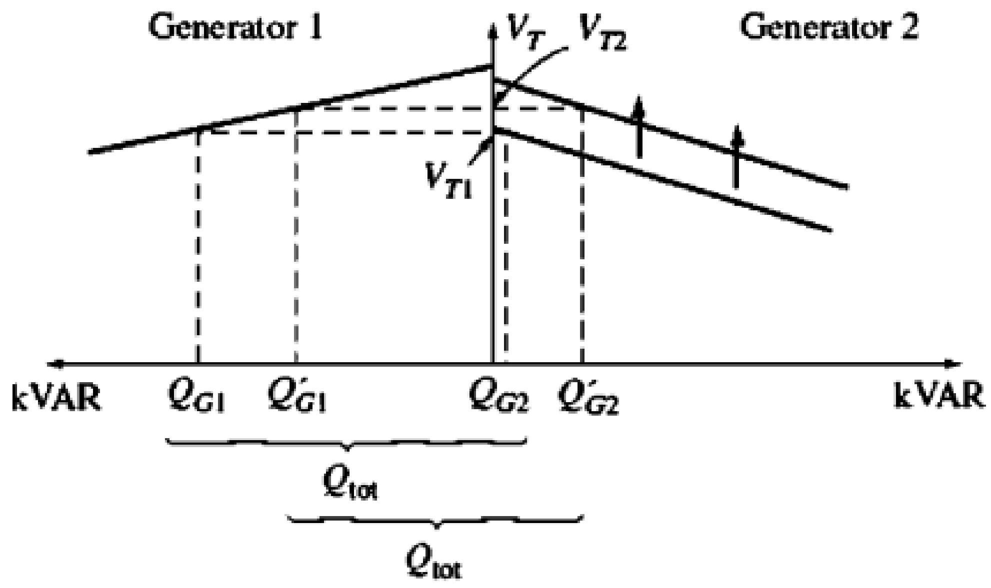
(a)


f_e
60 Hz
P_{G1}
P_{G1}
P_{G2}
P_{G2}
$P_{load} = P_{G1} + P_{G2}$

(b)


Generator 1
f_e
Generator 2
f_2
f_1
kW
P_{G1}
P'_{G1}
P_{G2}
P'_{G2}
kW
P_{tot}
P_{tot}

(c)


Generator 1
V_T
V_{T2}
Generator 2
V_{T1}
kVAR
Q_{G1}
Q'_{G1}
Q_{G2}
Q'_{G2}
kVAR
Q_{tot}
Q_{tot}

(d)

Therefore, when two generators are operating together, an increase in governor set points on one of them

1. *Increases the system frequency.*
2. *Increases the power supplied by that generator, while reducing the power supplied by the other one.*

What happens if the field current of G_2 is increased? The resulting behavior is analogous to the real-power situation and is shown in Figure 5–38d. When two generators are operating together and the field current of G_2 is increased,

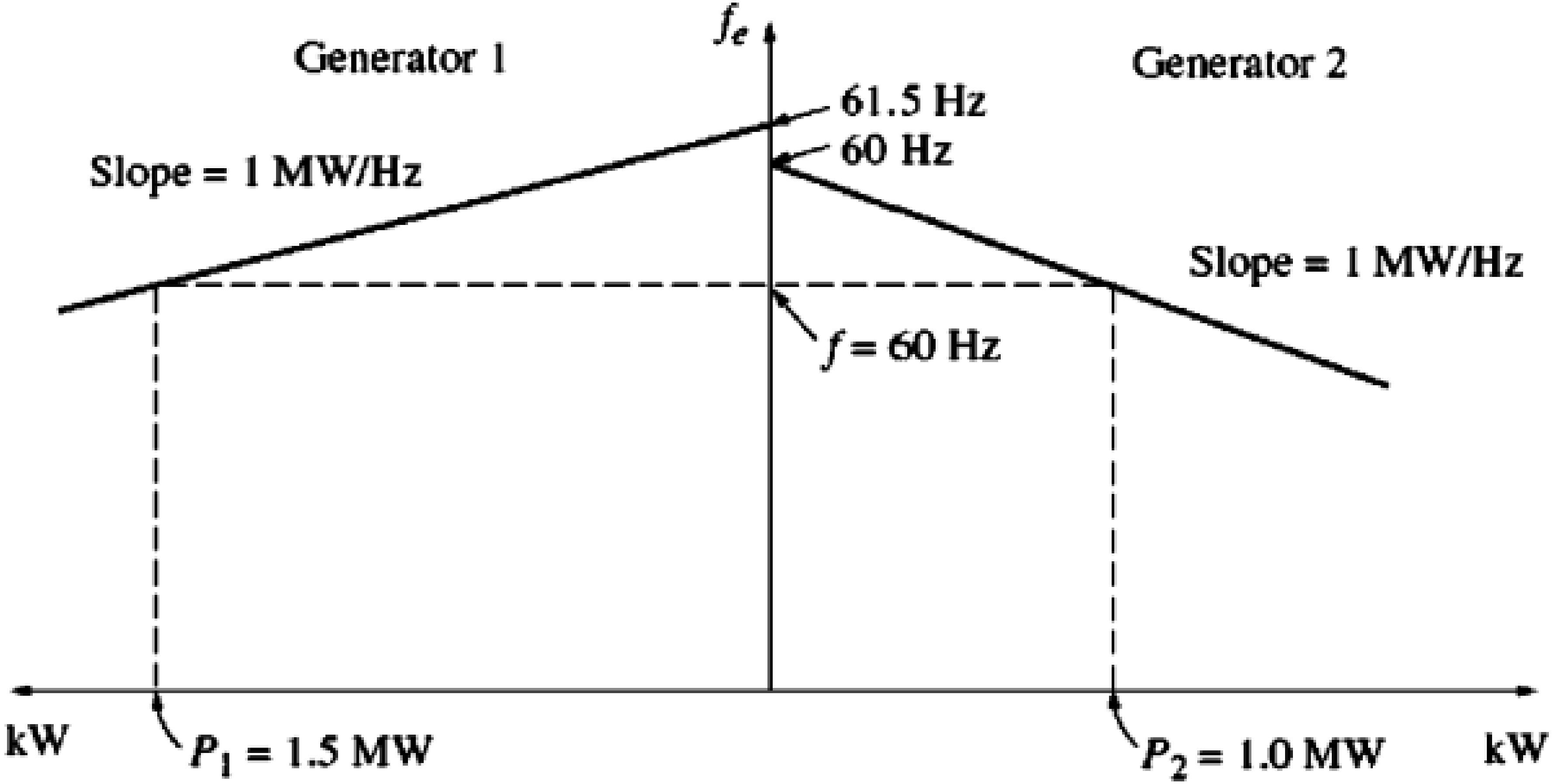
1. *The system terminal voltage is increased.*
2. *The reactive power Q supplied by that generator is increased, while the reactive power supplied by the other generator is decreased.*

Example 5–6. Figure 5–38a shows two generators supplying a load. Generator 1 has a no-load frequency of 61.5 Hz and a slope s_{P1} of 1 MW/Hz. Generator 2 has a no-load frequency of 61.0 Hz and a slope s_{P2} of 1 MW/Hz. The two generators are supplying a real load totaling 2.5 MW at 0.8 PF lagging. The resulting system power-frequency or house diagram is shown in Figure 5–39.

(a) At what frequency is this system operating, and how much power is supplied by each of the two generators?

(b) Suppose an additional 1-MW load were attached to this power system. What would the new system frequency be, and how much power would G_1 and G_2 supply now?

(c) With the system in the configuration described in part *b*, what will the system frequency and generator powers be if the governor set points on G_2 are increased by 0.5 Hz?



f_e
Generator 1
Generator 2
61.5 Hz
60 Hz
Slope = 1 MW/Hz
Slope = 1 MW/Hz
$f = 60$ Hz
kW
$P_1 = 1.5$ MW
$P_2 = 1.0$ MW
kW

(a) In the first case, both generators have a slope of 1 MW/Hz, and G_1 has a no-load frequency of 61.5 Hz, while G_2 has a no-load frequency of 61.0 Hz. The total load is 2.5 MW. Therefore, the system frequency can be found as follows:

$$\begin{aligned}
P_{\text{load}} &= P_1 + P_2 \\
&= s_{P1}(f_{\text{nl},1} - f_{\text{sys}}) + s_{P2}(f_{\text{nl},2} - f_{\text{sys}}) \\
2.5\text{ MW} &= (1\text{ MW/Hz})(61.5\text{ Hz} - f_{\text{sys}}) + (1\text{ MW/Hz})(61\text{ Hz} - f_{\text{sys}}) \\
&= 61.5\text{ MW} - (1\text{ MW/Hz})f_{\text{sys}} + 61\text{ MW} - (1\text{ MW/Hz})f_{\text{sys}} \\
&= 122.5\text{ MW} - (2\text{ MW/Hz})f_{\text{sys}}
\end{aligned}$$

therefore $$f_{\text{sys}} = \frac{122.5\text{ MW} - 2.5\text{ MW}}{(2\text{MW/Hz})} = 60.0\text{ Hz}$$

The resulting powers supplied by the two generators are

$$\begin{aligned}
P_1 &= s_{P1}(f_{\text{nl},1} - f_{\text{sys}}) \\
&= (1\text{ MW/Hz})(61.5\text{ Hz} - 60.0\text{ Hz}) = 1.5\text{ MW} \\
P_2 &= s_{P2}(f_{\text{nl},2} - f_{\text{sys}}) \\
&= (1\text{ MW/Hz})(61.0\text{ Hz} - 60.0\text{ Hz}) = 1\text{ MW}
\end{aligned}$$

(b) When the load is increased by 1 MW, the total load becomes 3.5 MW. The new system frequency is now given by

$$P_{\text{load}} = s_{P1}(f_{\text{nl},1} - f_{\text{sys}}) + s_{P2}(f_{\text{nl},2} - f_{\text{sys}})$$

$$3.5\text{ MW} = (1\text{ MW/Hz})(61.5\text{ Hz} - f_{\text{sys}}) + (1\text{ MW/Hz})(61\text{ Hz} - f_{\text{sys}})$$

$$= 61.5\text{ MW} - (1\text{ MW/Hz})f_{\text{sys}} + 61\text{ MW} - (1\text{ MW/Hz})f_{\text{sys}}$$

$$= 122.5\text{ MW} - (2\text{ MW/Hz})f_{\text{sys}}$$

therefore $$f_{\text{sys}} = \frac{122.5\text{ MW} - 3.5\text{ MW}}{(2\text{MW/Hz})} = 59.5\text{ Hz}$$

The resulting powers are

$$P_1 = s_{P1}(f_{\text{nl},1} - f_{\text{sys}})$$

$$= (1\text{ MW/Hz})(61.5\text{ Hz} - 59.5\text{ Hz}) = 2.0\text{ MW}$$

$$P_2 = s_{P2}(f_{\text{nl},2} - f_{\text{sys}})$$

$$= (1\text{ MW/Hz})(61.0\text{ Hz} - 59.5\text{ Hz}) = 1.5\text{ MW}$$

(c) If the no-load governor set points of G_2 are increased by 0.5 Hz, the new system frequency becomes

$$
\begin{aligned}
P_{\text{load}} &= s_{P1}(f_{\text{nl},1} - f_{\text{sys}}) + s_{P2}(f_{\text{nl},2} - f_{\text{sys}}) \\
3.5\text{ MW} &= (1\text{ MW/Hz})(61.5\text{ Hz} - f_{\text{sys}}) + (1\text{ MW/Hz})(61.5\text{ Hz} - f_{\text{sys}}) \\
&= 123\text{ MW} - (2\text{ MW/Hz})f_{\text{sys}} \\
f_{\text{sys}} &= \frac{123\text{ MW} - 3.5\text{ MW}}{(2\text{MW/Hz})} = 59.75\text{ Hz}
\end{aligned}
$$

The resulting powers are

$$
\begin{aligned}
P_1 = P_2 &= s_{P1}(f_{\text{nl},1} - f_{\text{sys}}) \\
&= (1\text{ MW/Hz})(61.5\text{ Hz} - 59.75\text{ Hz}) = 1.75\text{ MW}
\end{aligned}
$$

To summarize, in the case of two generators operating together:

1. The system is constrained in that the total power supplied by the two generators together must equal the amount consumed by the load. Neither f_{sys} nor V_T is constrained to be constant.
2. To adjust the real power sharing between generators without changing f_{sys}, simultaneously increase the governor set points on one generator while decreasing the governor set points on the other. The machine whose governor set point was increased will assume more of the load.
3. To adjust f_{sys} without changing the real power sharing, simultaneously increase or decrease both generators' governor set points.
4. To adjust the reactive power sharing between generators without changing V_T, simultaneously increase the field current on one generator while decreasing the field current on the other. The machine whose field current was increased will assume more of the reactive load.
5. To adjust V_T without changing the reactive power sharing, simultaneously increase or decrease both generators' field currents.

It is very important that any synchronous generator intended to operate in parallel with other machines have a *drooping* frequency–power characteristic. If two generators have flat or nearly flat characteristics, then the power sharing between

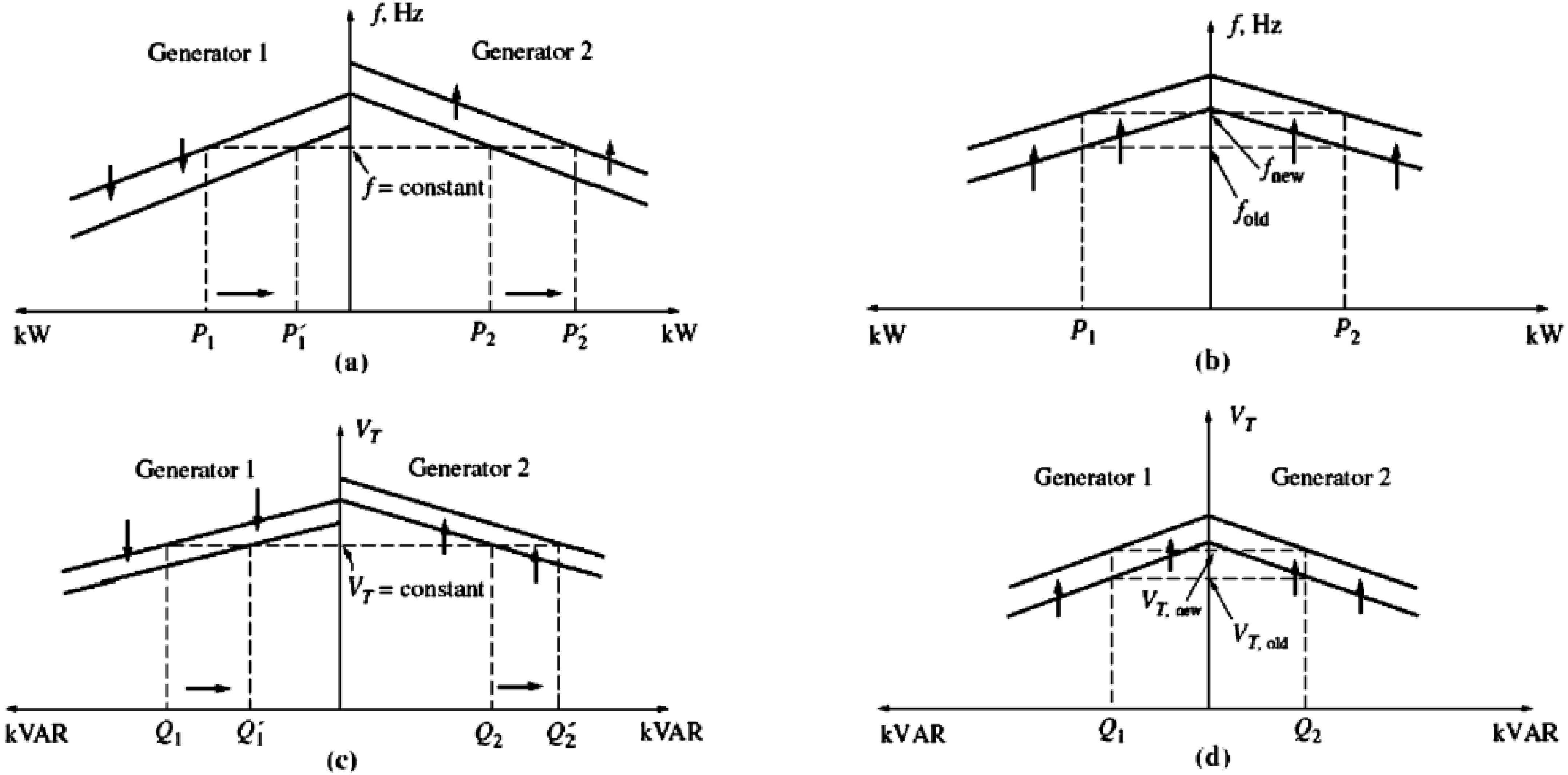


FIGURE 5–40
(a) Shifting power sharing without affecting system frequency. (b) Shifting system frequency without affecting power sharing. (c) Shifting reactive power sharing without affecting terminal voltage. (d) Shifting terminal voltage without affecting reactive power sharing.

Example 7

A 36 MVA, 21 kV, 1800 r/min, 3-phase generator connected to a power grid has a synchronous reactance of 9 Ω per phase. If the exciting voltage is 12 kV (line-to-neutral), and the system voltage is 17.3 kV (line-to-line), calculate the following:

a. The active power which the machine delivers when the torque angle δ is 30° (electrical)
b. The peak power that the generator can deliver before it falls out of step (loses synchronism)

Solution

a. We have

$$E_o = 12\ \text{kV}$$
$$E = 17.3\ \text{kV}/\sqrt{3} = 10\ \text{kV}$$
$$\delta = 30°$$

The active power delivered to the power grid is

$$\begin{aligned} P &= (E_oE/X_s)\sin\delta \\ &= (12 \times 10/9) \times 0.5 \\ &= 6.67\ \text{MW} \end{aligned}$$

The total power delivered by all three phases is

$$(3 \times 6.67) = 20\ \text{MW}$$

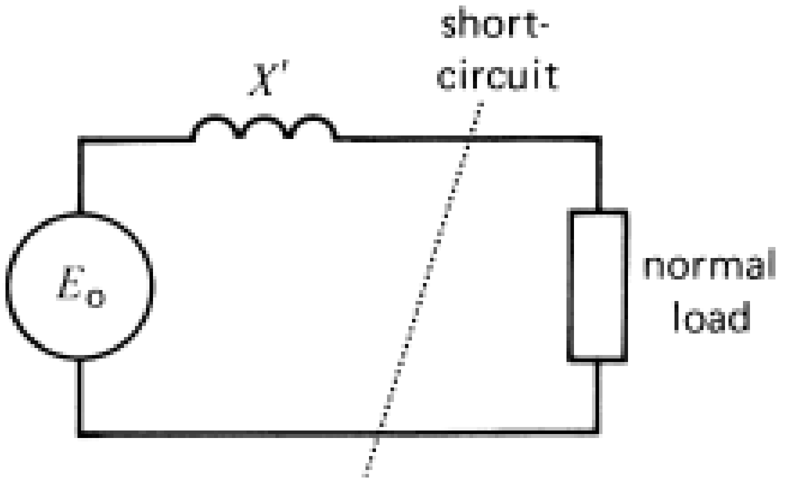
b. The maximum power, per phase, is attained when δ = 90°.

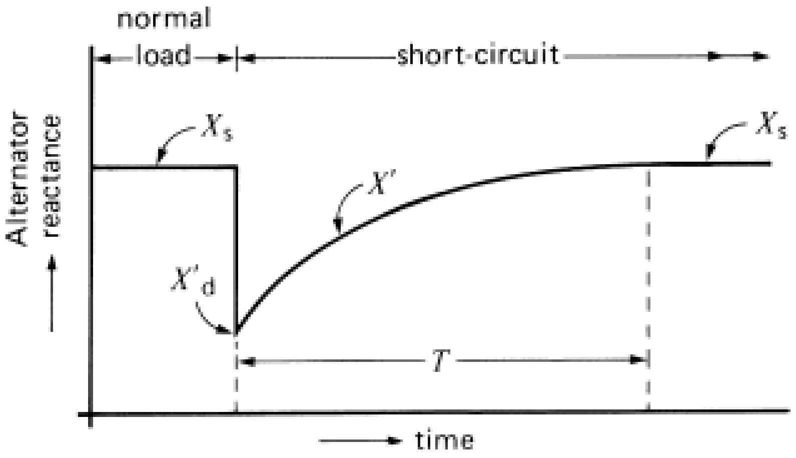
$$\begin{aligned} P &= (E_oE/X_s)\sin 90 \\ &= (12 \times 10/9) \times 1 \\ &= 13.3\ \text{MW} \end{aligned}$$

The peak power output of the alternator is, therefore,

$$(3 \times 13.3) = 40\ \text{MW}$$

Transient reactance





Example 8

A 250 MVA, 25 kV, 3-phase, steam-turbine generator has a synchronous reactance of 1.6 pu and a transient reactance X'_d of 0.23 pu. It delivers its rated output at a power factor of 100%. A short-circuit suddenly occurs on the line, close to the generating station.

Calculate

a. The induced voltage E_o prior to the short-circuit
b. The initial value of the short-circuit current
c. The final value of the short-circuit current if the circuit breakers should fail to open

Solution

a. The base impedance of the generator is

$$Z_B = E_B{}^2/S_B$$
$$= 25\,000^2/(250 \times 10^6)$$
$$= 2.5\ \Omega$$

The synchronous reactance is

$$X_s = X_s(\text{pu})\, Z_B$$
$$= 1.6 \times 2.5$$
$$= 4\ \Omega$$

The rated line-to-neutral voltage per phase is

$$E = 25/\sqrt{3} = 14.4\ \text{kV}$$

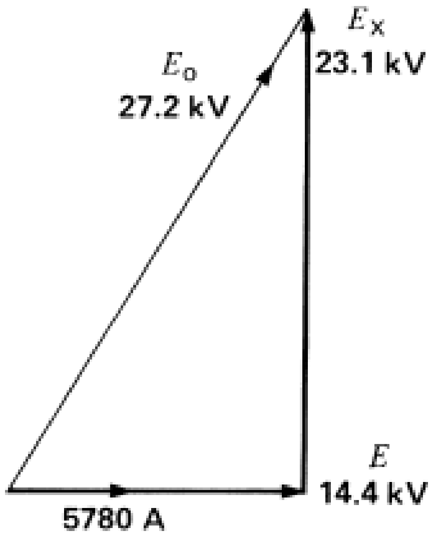
The rated load current per phase is

$$I = S/\sqrt{3}\, E$$
$$= 250 \times 10^6/(1.73 \times 25\,000)$$
$$= 5774\ \text{A}$$

The internal voltage drop E_x is

$$E_x = IX_s = 5774 \times 4$$
$$= 23.1\ \text{kV}$$

The current is in phase with E because the power factor of the load is unity. Thus, referring to the phasor diagram (Fig. 31), E_o is



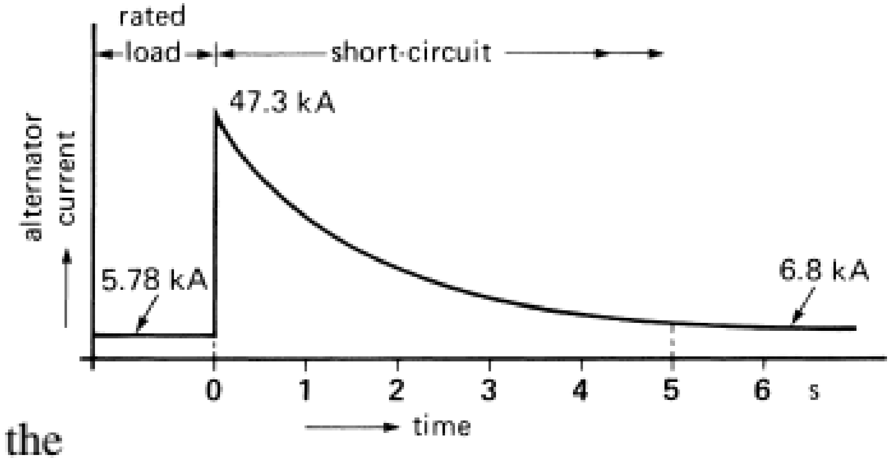
$$E_o = \sqrt{E^2 + E_X^2}$$
$$= \sqrt{14.4^2 + 23.1^2}$$
$$= 27.2\ \text{kV}$$

b. The transient reactance is

$$X'_d = X'_d(\text{pu})\, Z_B$$
$$= 0.23 \times 2.5$$
$$= 0.575\ \Omega$$

The initial short-circuit current is

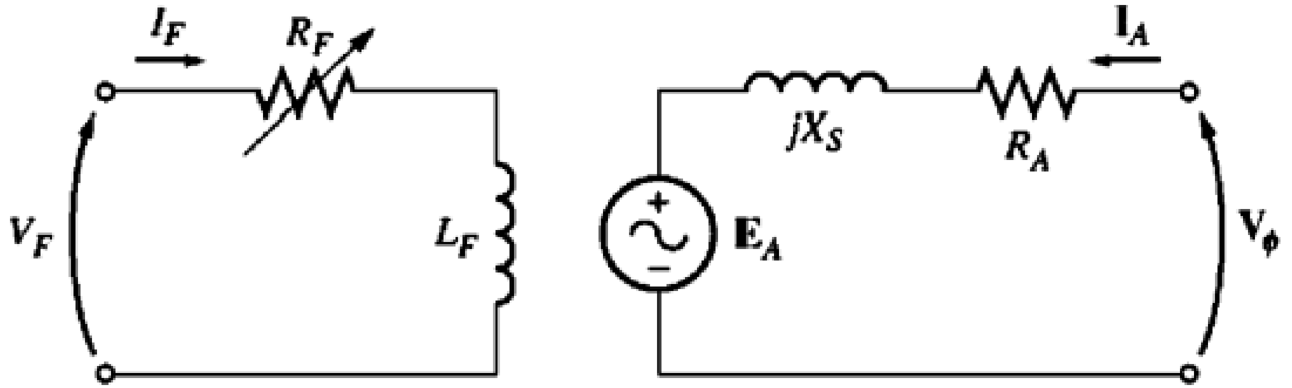
$$I_{sc} = E_o/X'_d$$
$$= 27.2/0.575$$
$$= 47.3\ \text{kA}$$



c. If the short-circuit is sustained and the excitation is unchanged, the current will eventually level off at a steady-state value:

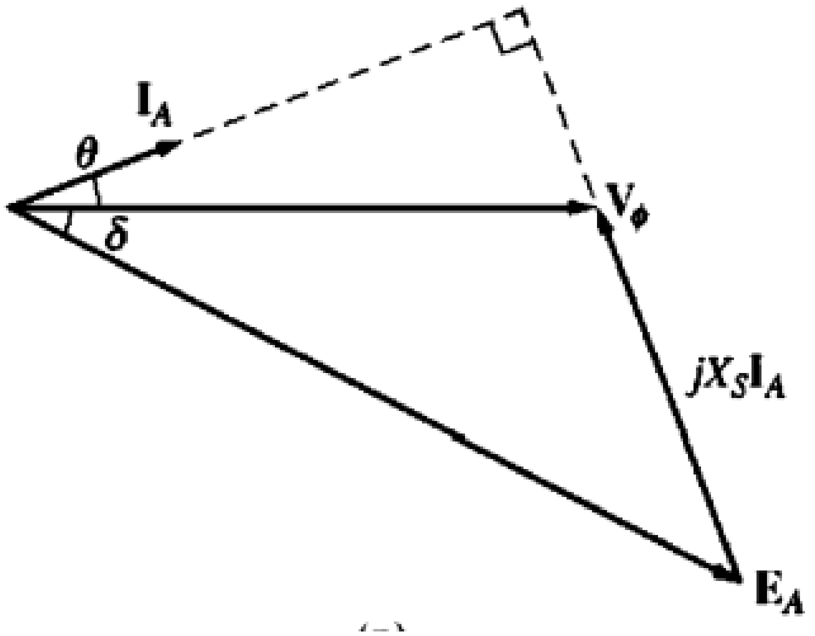
$$I = E_o/X_s = 27.2/4$$
$$= 6.8\ \text{kA}$$

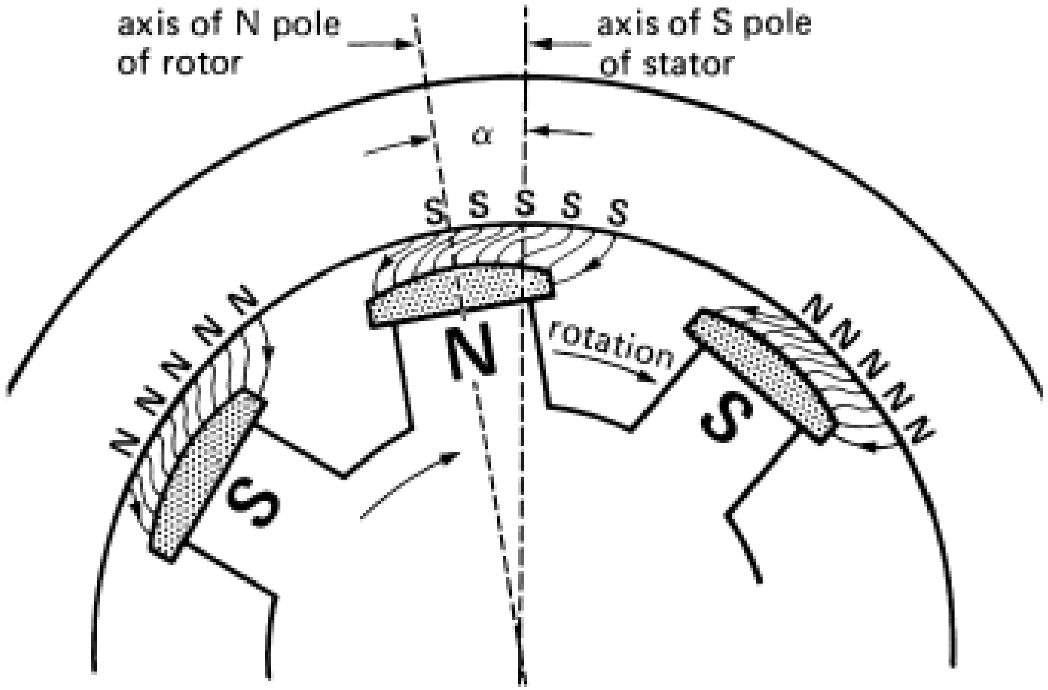
The Equivalent Circuit of a Synchronous Motor



$$\boxed{\mathbf{E}_A = \mathbf{V}_\phi - jX_S\mathbf{I}_A - R_A\mathbf{I}_A}$$

$$\boxed{\mathbf{V}_\phi = \mathbf{E}_A + jX_S\mathbf{I}_A + R_A\mathbf{I}_A}$$





Motor under load—simple calculations

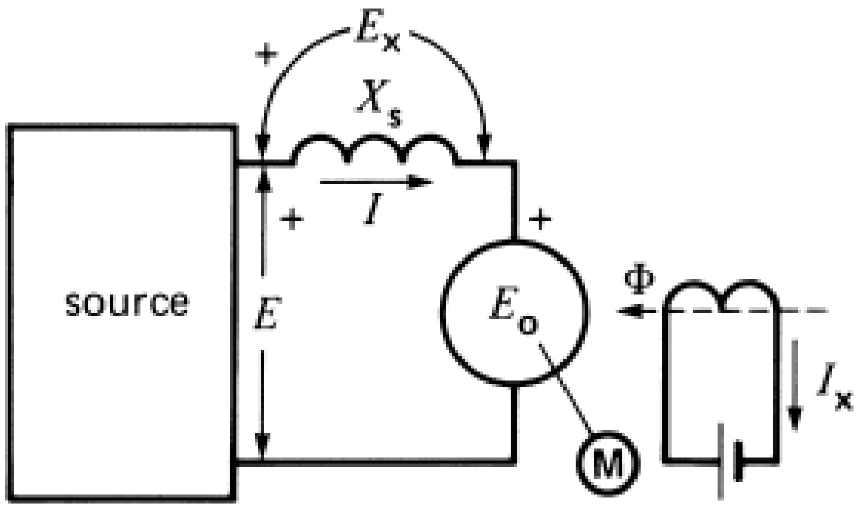


Figure 7a
Equivalent circuit of a synchronous motor, showing one phase.

Figure 7b
Motor at no-load, with E_o adjusted to equal E.

$$E_x = E - E_o$$

Consequently, a current I must flow in the circuit, given by

$$jIX_s = E_x$$

from which

$$I = -jE_x/X_s$$
$$= -j(E - E_o)/X_s$$

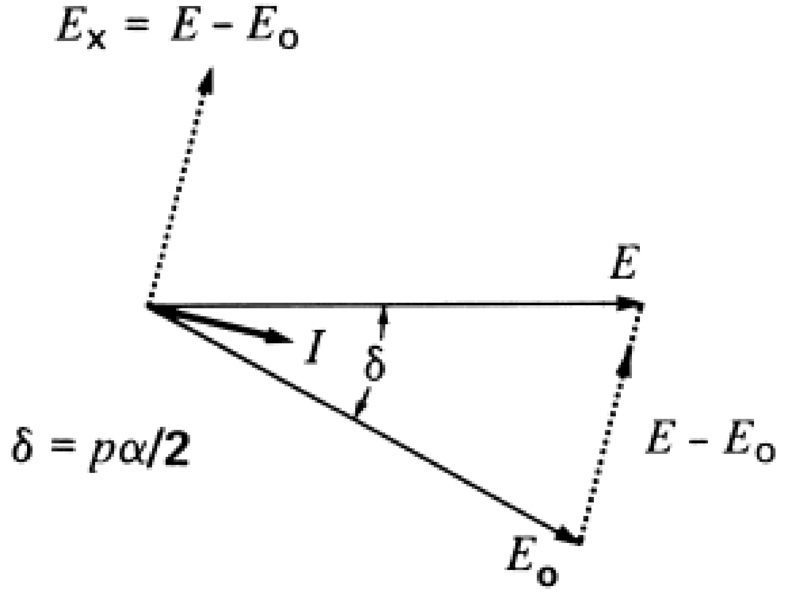


Figure 7c
Motor under load E_o has the same value as in Fig. 7b, but it lags behind E.

Example 2a

A 500 hp, 720 r/min synchronous motor connected to a 3980 V, 3-phase line generates an excitation voltage E_o of 1790 V (line-to-neutral) when the dc exciting current is 25 A. The synchronous reactance is 22 Ω and the torque angle between E_o and E is 30°.

Calculate

a. The value of E_x
b. The ac line current
c. The power factor of the motor
d. The approximate horsepower developed by the motor
e. The approximate torque developed at the shaft

Solution

This problem can best be solved by using vector notation.

a. The voltage E (line-to-neutral) applied to the motor has a value

$$E = E_L/\sqrt{3} = 3980/\sqrt{3}$$
$$= 2300 \text{ V}$$

Let us select E as the reference phasor, whose angle with respect to the horizontal axis is assumed to be zero. Thus,

$$E = 2300 \angle 0°$$

It follows that E_o is given by the phasor

$$E_o = 1790 \angle -30°$$

The equivalent circuit per phase is given in Fig. 8a.

Moving clockwise around the circuit and applying Kirchhoff's voltage law we can write

$$-E + E_x + E_o = 0$$
$$E_x = E - E_o$$
$$= 2300\angle 0° - 1790\angle -30°$$
$$= 2300\,(\cos 0° + j \sin 0°) - 1790\,(\cos - 30° + j \sin - 30°)$$
$$= 2300 - 1550 + j\,895$$
$$= 750 + j\,895$$
$$= 1168\angle 50°$$

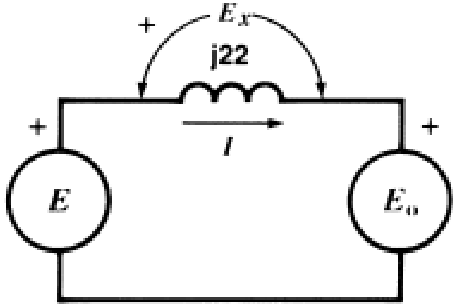


Figure 8a
Equivalent circuit of a synchronous motor connected to a source *E*.

b. The line current I is given by

$$j\,22\,I = E_x$$
$$I = \frac{1168\angle 50°}{22\angle 90°}$$
$$= 53\angle -40°$$

Thus, phasor I has a value of 53 A and it lags 40° behind phasor E.

c. The power factor of the motor is given by the cosine of the angle between the line-to-neutral voltage E across the motor terminals and the current I. Hence,

$$\text{power factor} = \cos\theta = \cos 40°$$
$$= 0.766, \text{ or } 76.6\%$$

The power factor is lagging because the current lags behind the voltage.
The complete phasor diagram is shown in Fig. 8b.

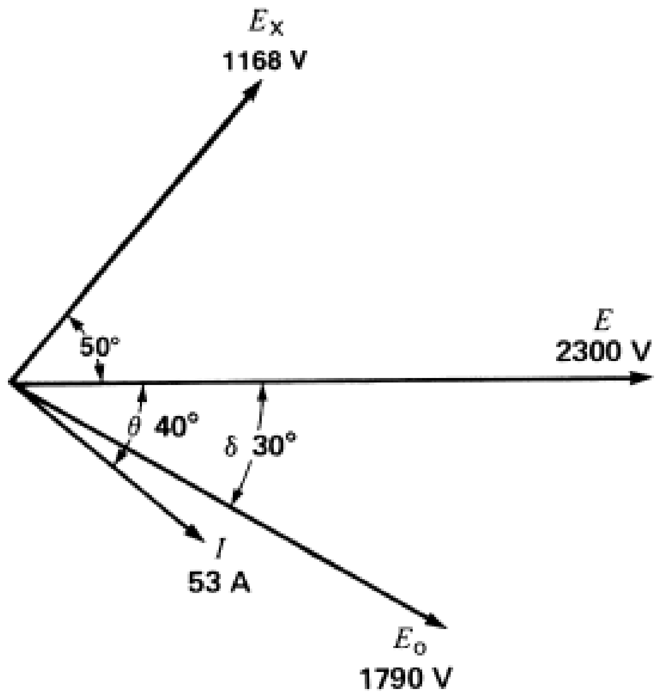


Figure 8b

d. Total active power input to the stator

$$P_i = 3 \times E_{LN} I_L \cos\theta$$
$$= 3 \times 2300 \times 53 \times \cos 40°$$
$$= 280\,142 \text{ W} = 280.1 \text{ kW}$$

Neglecting the I^2R losses and iron losses in the stator, the electrical power transmitted across the airgap to the rotor is 280.1 kW.
Approximate horsepower developed

$$P = 280.1 \times 10^3/746 = 375 \text{ hp}$$

e. Approximate torque

$$T = \frac{9.55 \times P}{n} = \frac{9.55 \times 280.1 \times 10^3}{720}$$
$$= 3715 \text{ N·m}$$

Example 2b

The motor in Example 2a has a stator resistance of 0.64 Ω per phase and possesses the following losses:

I_2R losses in the rotor:	3.2 kW
Stator core loss:	3.3 kW
Windage and friction loss:	1.5 kW

Calculate

a. The actual horsepower developed
b. The actual torque developed at the shaft
c. The efficiency of the motor

Solution

a. Power input to the stator is 280.1 kW

Stator I^2R losses = $3 \times 53^2 \times 0.64\ \Omega = 5.4$ kW
Total stator losses = 5.4 + 3.3 = 8.7 kW
Power transmitted to the rotor = 280.1 − 8.7
= 271.4 kW

The power at the shaft is the power to the rotor minus the windage and friction losses. The rotor I^2R losses are supplied by an external dc source and so they do not affect the mechanical power.

Power available at the shaft is

$$P_o = 271.4 - 1.5 = 269.9 \text{ kW}$$

$$= \frac{269.9 \times 10^3}{746} = 361.8 \text{ hp}$$

This power is very close to the approximate value calculated in Example 2a.

b. The corresponding torque is

$$T = \frac{9.55 \times P}{n} = \frac{9.55 \times 269.9 \times 10^3}{720}$$

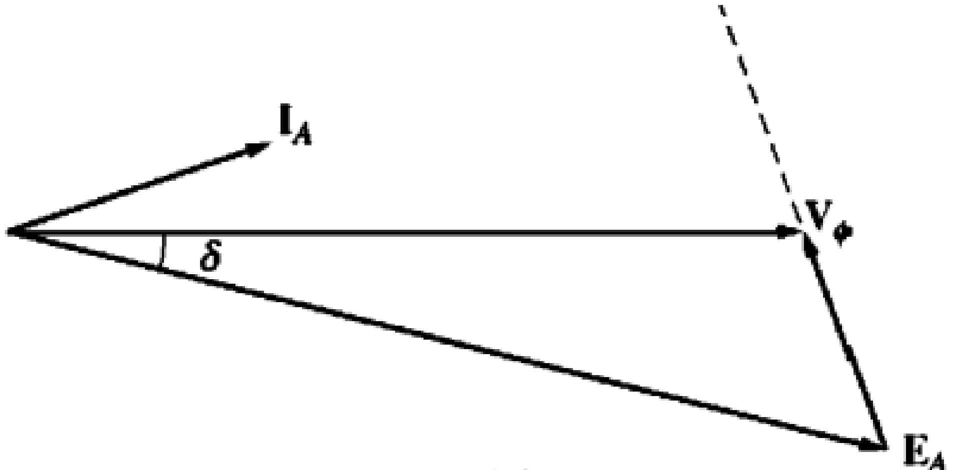
$$= 3580 \text{ N·m}$$

c. Total losses = 5.4 + 3.3 + 3.2 + 1.5 = 13.4 kW

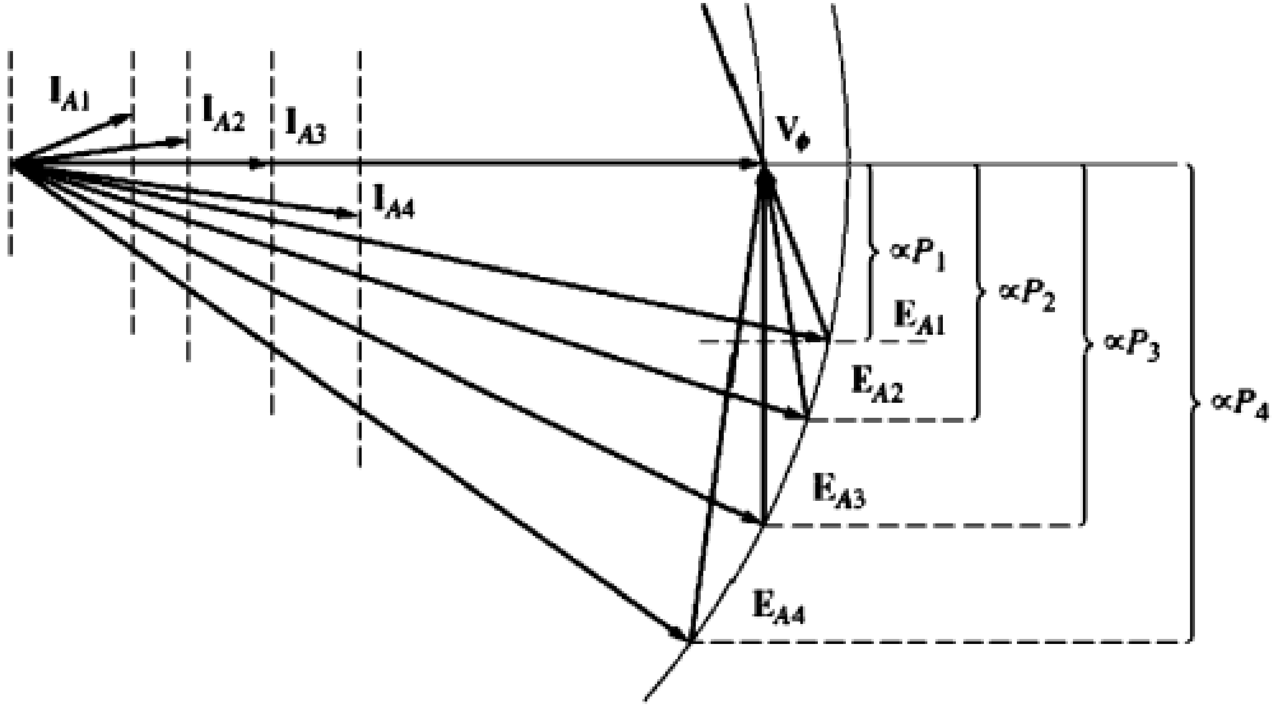
Total power input = 280.1 + 3.2 = 283.3 kW
Total power output = 269.9 kW
Efficiency = 269.9/283.3 = 0.9527 = 95.3%

Note that the stator resistance of 0.64 Ω is very small compared to the reactance of 22 Ω. Consequently, the true phasor diagram is very close to the phasor diagram of Fig. 8b.

The Effect of Load Changes on a Synchronous Motor



(a)



(b)

Example 6–1. A 208-V, 45-kVA, 0.8-PF-leading, Δ-connected, 60-Hz synchronous machine has a synchronous reactance of 2.5 Ω and a negligible armature resistance. Its friction and windage losses are 1.5 kW, and its core losses are 1.0 kW. Initially, the shaft is supplying a 15-hp load, and the motor's power factor is 0.80 leading.

(a) Sketch the phasor diagram of this motor, and find the values of $\mathbf{I}_A$, I_L, and $\mathbf{E}_A$.

(b) Assume that the shaft load is now increased to 30 hp. Sketch the behavior of the phasor diagram in response to this change.

(c) Find $\mathbf{I}_A$, I_L, and $\mathbf{E}_A$ after the load change. What is the new motor power factor?

Solution

(*a*) Initially, the motor's output power is 15 hp. This corresponds to an output of

$$P_{out} = (15 \text{ hp})(0.746 \text{ KW/hp}) = 11.19 \text{ kW}$$

Therefore, the electric power supplied to the machine is

$$P_{in} = P_{out} + P_{mech\ loss} + P_{core\ loss} + P_{elec\ loss}$$
$$= 11.19 \text{ kW} + 1.5 \text{ kW} + 1.0 \text{ kW} + 0 \text{ kW} = 13.69 \text{ kW}$$

Since the motor's power factor is 0.80 leading, the resulting line current flow is

$$I_L = \frac{P_{in}}{\sqrt{3} V_T \cos\theta}$$
$$= \frac{13.69 \text{ kW}}{\sqrt{3}(208 \text{ V})(0.80)} = 47.5 \text{ A}$$

and the armature current is $I_L/\sqrt{3}$, with 0.8 leading power factor, which gives the result

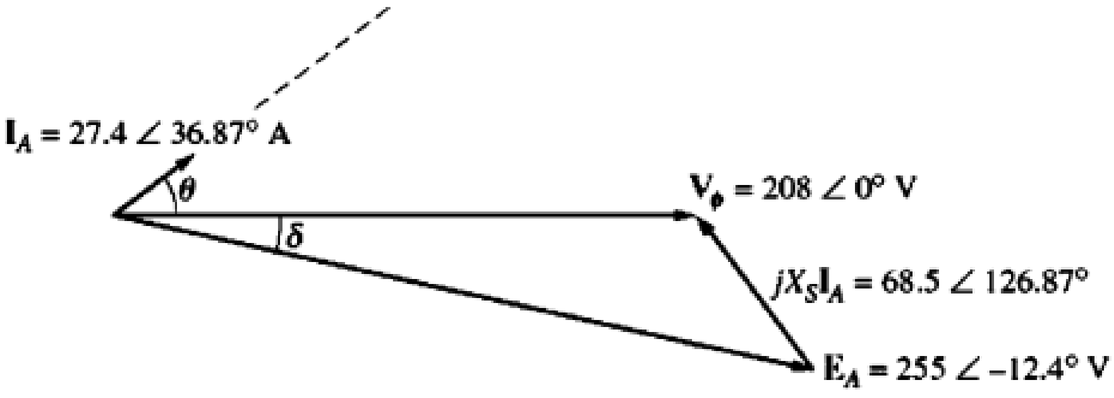
$$\mathbf{I}_A = 27.4 \angle 36.87° \text{ A}$$

To find $\mathbf{E}_A$, apply Kirchhoff's voltage law [Equation (6–2)]:

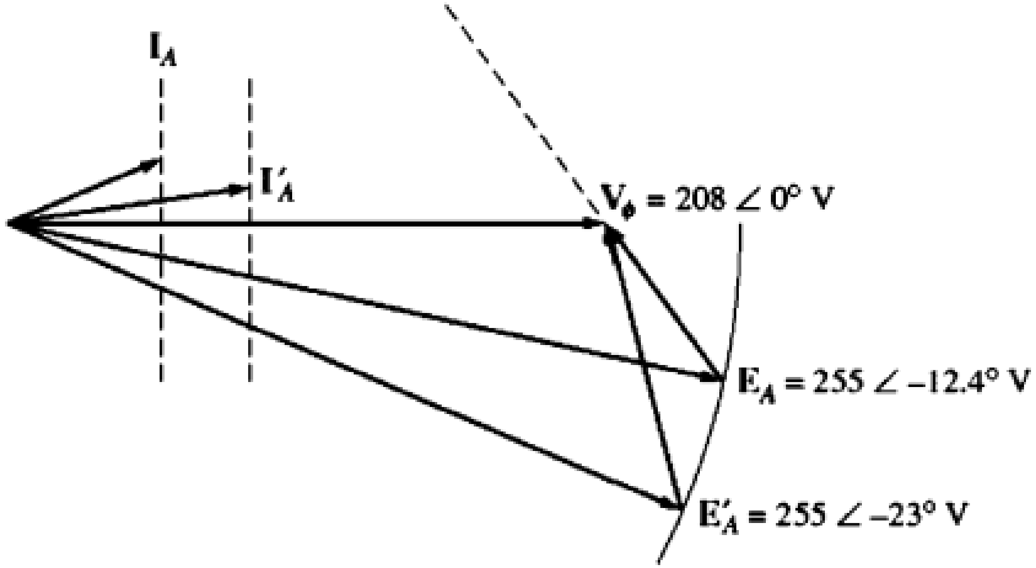
$$\mathbf{E}_A = \mathbf{V}_\phi - jX_S\mathbf{I}_A$$
$$= 208 \angle 0° \text{ V} - (j2.5\ \Omega)(27.4 \angle 36.87° \text{ A})$$
$$= 208 \angle 0° \text{ V} - 68.5 \angle 126.87° \text{ V}$$
$$= 249.1 - j54.8 \text{ V} = 255 \angle -12.4° \text{ V}$$

The resulting phasor diagram is shown in Figure 6–7a.

(*b*) As the power on the shaft is increased to 30 hp, the shaft slows momentarily, and the internal generated voltage $\mathbf{E}_A$ swings out to a larger angle δ while maintaining a constant magnitude. The resulting phasor diagram is shown in Figure 6–7b.



(a)



(b)

(c) After the load changes, the electric input power of the machine becomes

$$\begin{aligned} P_{\text{in}} &= P_{\text{out}} + P_{\text{mech loss}} + P_{\text{core loss}} + P_{\text{elec loss}} \\ &= (30\ \text{hp})(0.746\ \text{kW/hp}) + 1.5\ \text{kW} + 1.0\ \text{kW} + 0\ \text{kW} \\ &= 24.88\ \text{kW} \end{aligned}$$

From the equation for power in terms of torque angle [Equation (5-20)], it is possible to find the magnitude of the angle δ (remember that the magnitude of $\mathbf{E}_A$ is constant):

$$P = \frac{3V_\phi E_A \sin\delta}{X_S} \qquad (5\text{–}20)$$

so

$$\begin{aligned} \delta &= \sin^{-1}\frac{X_S P}{3V_\phi E_A} \\ &= \sin^{-1}\frac{(2.5\ \Omega)(24.88\ \text{kW})}{3(208\ \text{V})(255\ \text{V})} \\ &= \sin^{-1} 0.391 = 23° \end{aligned}$$

The internal generated voltage thus becomes $\mathbf{E}_A = 355\angle -23°$ V. Therefore, $\mathbf{I}_A$ will be given by

$$\begin{aligned} \mathbf{I}_A &= \frac{\mathbf{V}_\phi - \mathbf{E}_A}{jX_S} \\ &= \frac{208\angle 0°\ \text{V} - 255\angle -23°\ \text{V}}{j2.5\ \Omega} \\ &= \frac{103.1\angle 105°\ \text{V}}{j2.5\ \Omega} = 41.2\angle 15°\ \text{A} \end{aligned}$$

and I_L will become

$$I_L = \sqrt{3} I_A = 71.4\ \text{A}$$

The final power factor will be cos (−15°) or 0.966 leading.

Power and torque

When a synchronous motor operates under load, it draws active power from the line. The power is given by the same equation used for the synchronous generator:

$$P = (E_o E/X_s) \sin \delta$$

E_o = line-to-neutral voltage induced by I_x [V]

E = line-to-neutral voltage of the source [V]

X_s = synchronous reactance per phase [Ω]

δ = torque angle between E_o and E [electrical degrees]

the stator. The peak power P_{max} (per phase) is given by

$$P_{max} = \frac{E_o E}{X_s} \quad (3)$$

$$T = \frac{9.55\,P}{n_s} \quad (4)$$

where

T = torque, per phase [N·m]

P = mechanical power, per phase [W]

n_s = synchronous speed [r/min]

9.55 = a constant [exact value = $60/2\pi$]

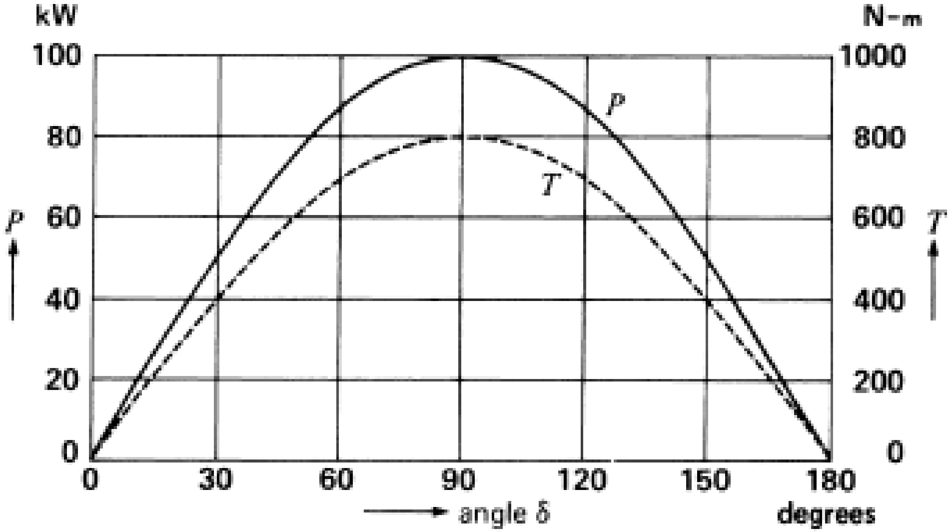


Figure 9
Power and torque per phase as a function of the torque angle δ. Synchronous motor rated 150 kW (200 hp), 1200 r/min, 3-phase, 60 Hz. See Example 3.

Example 3

A 150 kW, 1200 r/min, 460 V, 3-phase synchronous motor has a synchronous reactance of 0.8 Ω, per phase. If the excitation voltage E_o is fixed at 300 V, per phase, determine the following:

a. The power versus δ curve
b. The torque versus δ curve
c. The pull-out torque of the motor

Solution

a. The line-to-neutral voltage is

$$E = E_L/\sqrt{3} = 460/\sqrt{3}$$
$$= 266 \text{ V}$$

The mechanical power per phase is

$$P = (E_o E/X_s) \sin \delta \qquad (2)$$
$$= (266 \times 300/0.8) \sin \delta$$
$$= 99\,750 \sin \delta \text{ [W]}$$
$$= 100 \sin \delta \text{ [kW]}$$

By selecting different values for δ, we can calculate the corresponding values of P and T, per phase.

δ [°]	P [kW]	T [N•m]
0	0	0
30	50	400
60	86.6	693
90	100	800
120	86.6	693
150	50	400
180	0	0

These values are plotted in Fig. 9.

b. The torque curve can be found by applying Eq. 4:

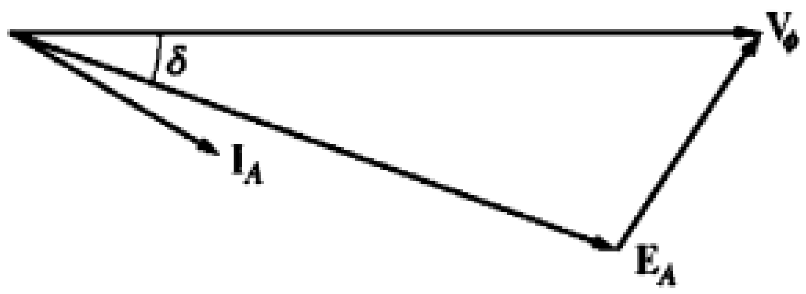
$$T = 9.55\, P/n_s$$
$$= 9.55\, P/1200$$
$$= P/125$$

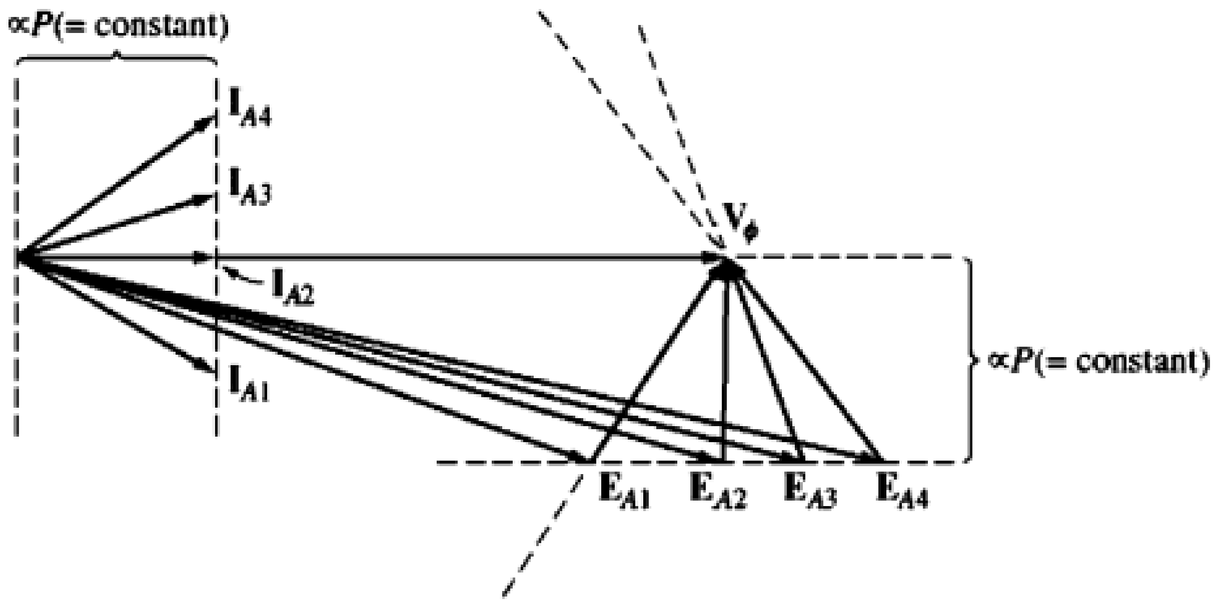
c. The pull-out torque T_{max} coincides with the maximum power output:

$$T_{max} = 800 \text{ N·m}$$

The actual pull-out torque is 3 times as great (2400 N·m) because this is a 3-phase machine. Similarly, the power and torque values given in Fig. 9 must also be multiplied by 3. Consequently, this 150 kW motor can develop a maximum output of 300 kW, or about 400 hp.

The Effect of Field Current Changes on a Synchronous Motor





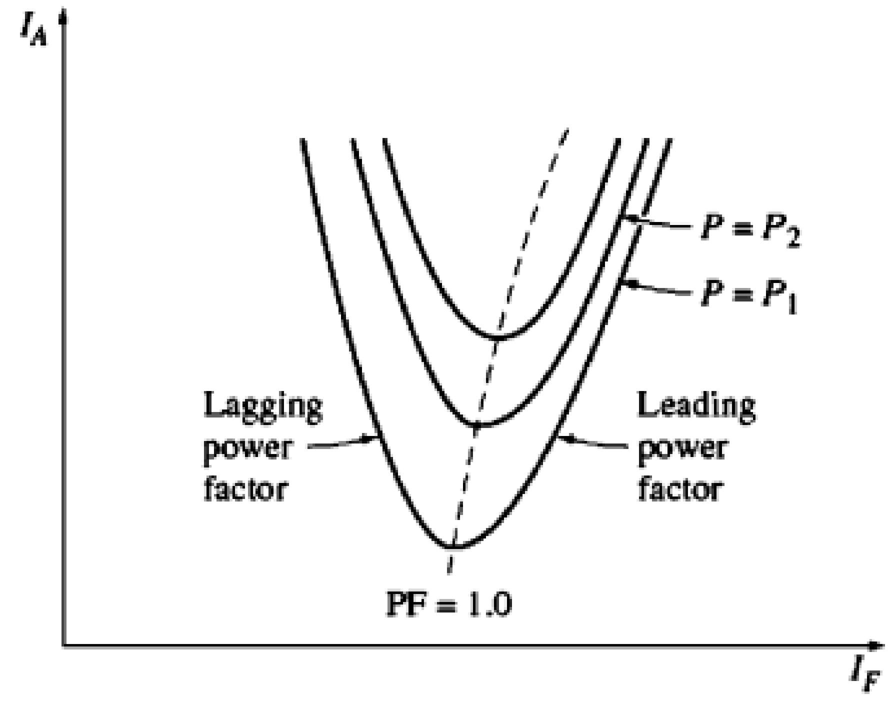


FIGURE 6–9
Synchronous motor V curves.

Example 6–2. The 208-V, 45-kVA, 0.8-PF-leading, Δ-connected, 60-Hz synchronous motor of the previous example is supplying a 15-hp load with an initial power factor of 0.85 PF lagging. The field current I_F at these conditions is 4.0 A.

(a) Sketch the initial phasor diagram of this motor, and find the values $\mathbf{I}_A$ and $\mathbf{E}_A$.
(b) If the motor's flux is increased by 25 percent, sketch the new phasor diagram of the motor. What are $\mathbf{E}_A$, $\mathbf{I}_A$, and the power factor of the motor now?
(c) Assume that the flux in the motor varies linearly with the field current I_F. Make a plot of I_A versus I_F for the synchronous motor with a 15-hp load.

Solution

(a) From the previous example, the electric input power with all the losses included is $P_{\text{in}} = 13.69$ kW. Since the motor's power factor is 0.85 lagging, the resulting armature current flow is

$$I_A = \frac{P_{\text{in}}}{3V_\phi \cos\theta}$$

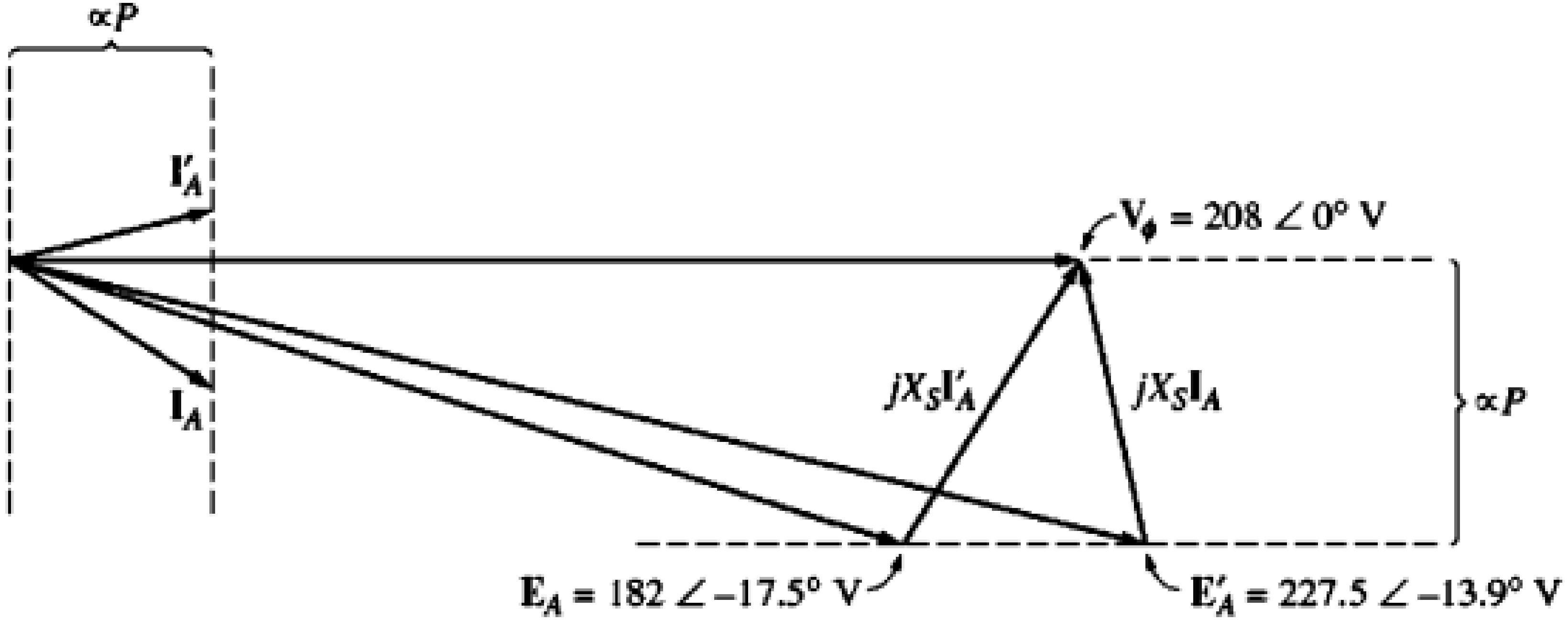
$$= \frac{13.69\text{ kW}}{3(208\text{ V})(0.85)} = 25.8\text{ A}$$

The angle θ is $\cos^{-1} 0.85 = 31.8°$, so the phasor current I_A is equal to

$$\mathbf{I}_A = 25.8 \angle -31.8°\text{ A}$$

To find $\mathbf{E}_A$, apply Kirchhoff's voltage law [Equation (6–2)]:

$$\begin{aligned}
\mathbf{E}_A &= \mathbf{V}_\phi - jX_S\mathbf{I}_A \\
&= 208 \angle 0^\circ\ \text{V} - (j2.5\ \Omega)(25.8 \angle -31.8^\circ\ \text{A}) \\
&= 208 \angle 0^\circ\ \text{V} - 64.5 \angle 58.2^\circ\ \text{V} \\
&= 182 \angle -17.5^\circ\ \text{V}
\end{aligned}$$



(*b*) If the flux ϕ is increased by 25 percent, then $E_A = K\phi\omega$ will increase by 25 percent too:

$$E_{A2} = 1.25\, E_{A1} = 1.25(182\ \text{V}) = 227.5\ \text{V}$$

$$E_{A1} \sin \delta_1 = E_{A2} \sin \delta_2$$

$$\begin{aligned}
\delta_2 &= \sin^{-1}\left(\frac{E_{A1}}{E_{A2}} \sin \delta_1\right) \\
&= \sin^{-1}\left[\frac{182\ \text{V}}{227.5\ \text{V}} \sin(-17.5^\circ)\right] = -13.9^\circ
\end{aligned}$$

$$\mathbf{I}_{A2} = \frac{\mathbf{V}_\phi - \mathbf{E}_{A2}}{jX_S}$$

$$\begin{aligned}
\mathbf{I}_A &= \frac{208 \angle 0^\circ\ \text{V} - 227.5 \angle -13.9^\circ\ \text{V}}{j2.5\ \Omega} \\
&= \frac{56.2 \angle 103.2^\circ\ \text{V}}{j2.5\ \Omega} = 22.5 \angle 13.2^\circ\ \text{A}
\end{aligned}$$

Finally, the motor's power factor is now

$$\text{PF} = \cos(13.2^\circ) = 0.974 \qquad \text{leading}$$

Mechanical and electrical angles

$$\delta = p\alpha/2$$

Example 4

A 3-phase, 6000 kW, 4 kV, 180 r/min, 60 Hz motor has a synchronous reactance of 1.2 Ω. At full-load the rotor poles are displaced by a *mechanical* angle of 1° from their no-load position. If the line-to-neutral excitation E_o = 2.4 kV, calculate the mechanical power developed.

Solution

The number of poles is

$$p = 120\,f/n_s = 120 \times 60/180 = 40$$

The electrical torque angle is

$$\delta = p\alpha/2 = (40 \times 1)/2 = 20°$$

Assuming a wye connection, the voltage E applied to the motor is

$$E = E_L/\sqrt{3} = 4\ \text{kV}/\sqrt{3}$$
$$= 2.3\ \text{kV}$$
$$= 2309\ \text{V}$$

and the excitation voltage is

$$E_o = 2400\ \text{V}$$

The mechanical power developed per phase is

$$P = (E_oE/X_s)\sin\delta$$
$$= (2400 \times 2309/1.2)\sin 20°$$
$$= 1\,573\,300$$
$$= 1573\ \text{kW}$$

$$\text{Total power} = 3 \times 1573$$
$$= 4719\ \text{kW}\ (\sim 6300\ \text{hp})$$

Reluctance torque

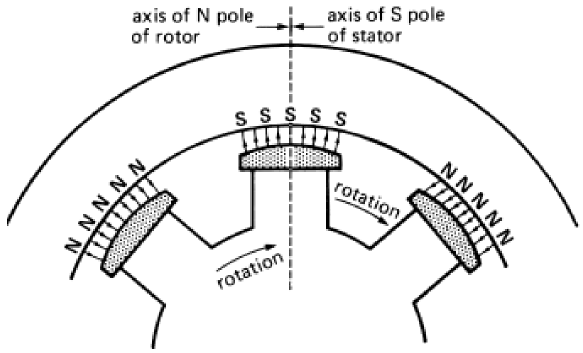


Figure 10a
The flux produced by the stator flows across the air gap through the salient poles.

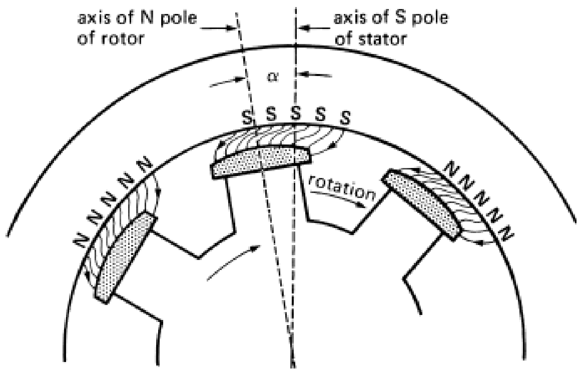


Figure 10b
The salient poles are attracted to the stator poles, thus producing a reluctance torque.

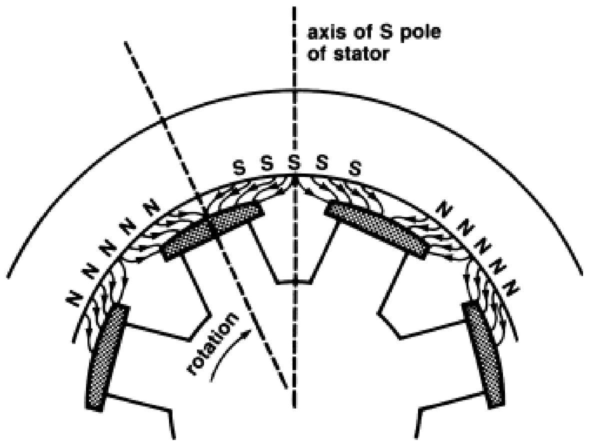


Figure 10c
The reluctance torque is zero when the salient poles are midway between the stator poles.

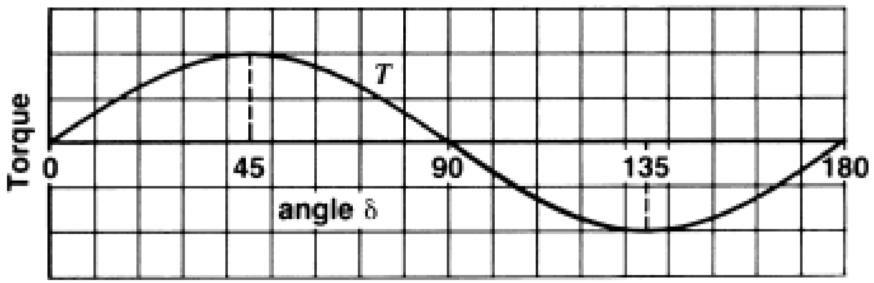


Figure 11
Reluctance torque versus the torque angle.

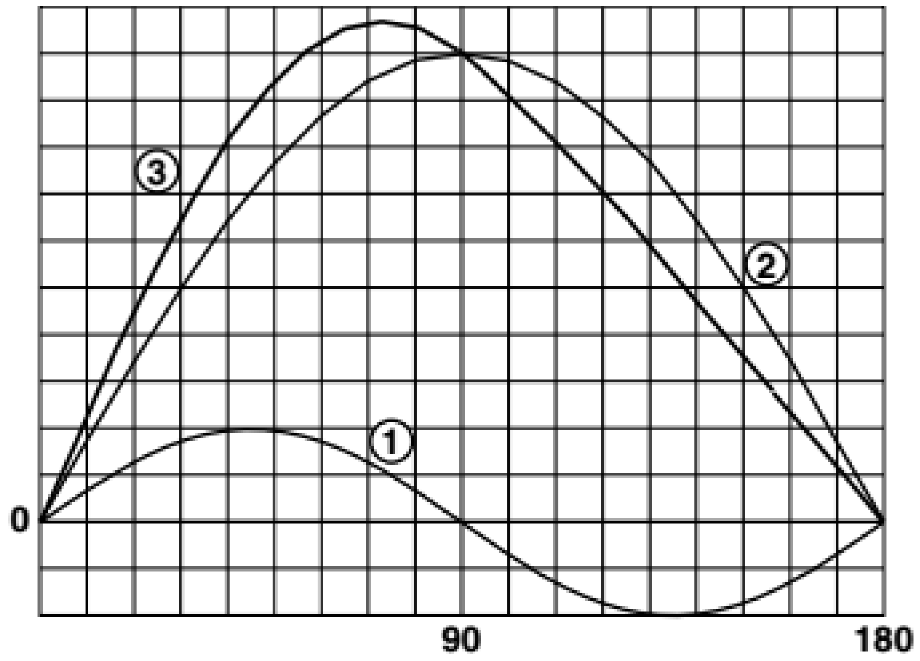


Figure 12
In a synchronous motor the reluctance torque (1) plus the smooth-rotor torque (2) produce the resultant torque (3). Torque (2) is due to the dc excitation of the rotor.

The Synchronous Motor and Power-Factor Correction

Figure 14
Unity power factor synchronous motor and phasor diagram at full-load.

The active power absorbed per phase is, therefore,

$$P = E_{ab} I_p \tag{6}$$

Figure 15
80 percent power factor synchronous motor and phasor diagram at full-load.

$$I_p = 0.8\ I_s$$
$$I_q = 0.6\ I_s$$

The active power P is given by

$$P = E_{ab} I_p = 0.8\ E_{ab} I_s$$

The reactive power delivered by the motor is

$$Q = E_{ab} I_q = 0.6\ E_{ab} I_s$$

$$Q = 0.75\ P$$
$$= 75\% \text{ of rated mechanical output}$$

$$I_s = 1.25\ I_p.$$

Example 6–3. The infinite bus in Figure 6–13 operates at 480 V. Load 1 is an induction motor consuming 100 kW at 0.78 PF lagging, and load 2 is an induction motor consuming 200 kW at 0.8 PF lagging. Load 3 is a synchronous motor whose real power consumption is 150 kW.

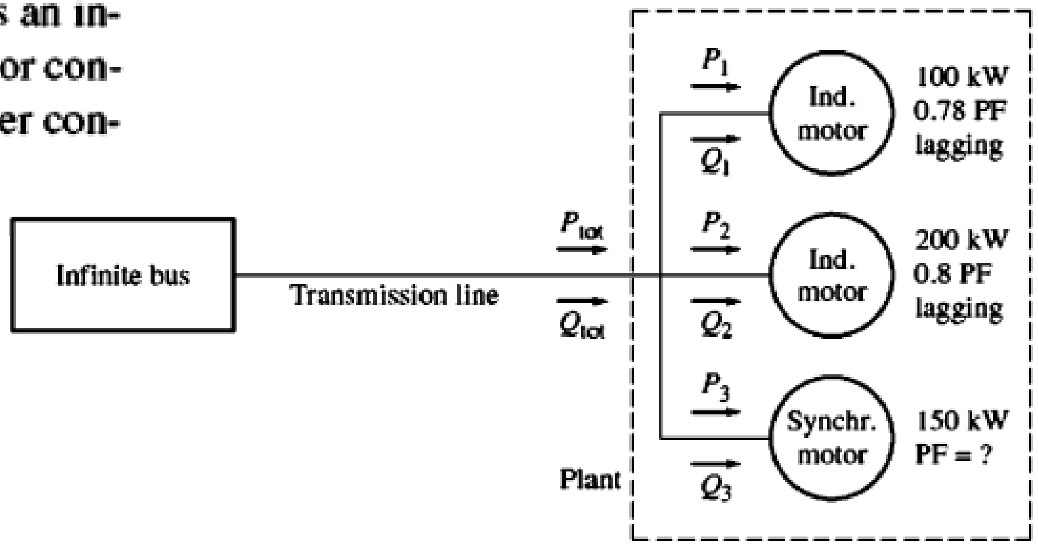
(*a*) If the synchronous motor is adjusted to operate at 0.85 PF lagging, what is the transmission line current in this system?

(*b*) If the synchronous motor is adjusted to operate at 0.85 PF leading, what is the transmission line current in this system?

(*c*) Assume that the transmission line losses are given by

$$P_{\mathrm{LL}} = 3I_L^2R_L \qquad \text{line loss}$$

where LL stands for line losses. How do the transmission losses compare in the two cases?



Solution

(*a*) In the first case, the real power of load 1 is 100 kW, and the reactive power of load 1 is

$$\begin{aligned} Q_1 &= P_1 \tan\theta \\ &= (100\text{ kW}) \tan(\cos^{-1} 0.78) = (100\text{ kW}) \tan 38.7° \\ &= 80.2\text{ kVAR} \end{aligned}$$

The real power of load 2 is 200 kW, and the reactive power of load 2 is

$$\begin{aligned} Q_2 &= P_2 \tan\theta \\ &= (200\text{ kW}) \tan(\cos^{-1} 0.80) = (200\text{ kW}) \tan 36.87° \\ &= 150\text{ kVAR} \end{aligned}$$

The real power load 3 is 150 kW, and the reactive power of load 3 is

$$\begin{aligned} Q_3 &= P_3 \tan\theta \\ &= (150\text{ kW}) \tan(\cos^{-1} 0.85) = (150\text{ kW}) \tan 31.8° \\ &= 93\text{ kVAR} \end{aligned}$$

Thus, the total real load is

$$\begin{aligned} P_{\text{tot}} &= P_1 + P_2 + P_3 \\ &= 100\text{ kW} + 200\text{ kW} + 150\text{ kW} = 450\text{ kW} \end{aligned}$$

and the total reactive load is

$$Q_{tot} = Q_1 + Q_2 + Q_3$$
$$= 80.2 \text{ kVAR} + 150 \text{ kVAR} + 93 \text{ kVAR} = 323.2 \text{ kVAR}$$

The equivalent system power factor is thus

$$\text{PF} = \cos\theta = \cos\left(\tan^{-1}\frac{Q}{P}\right) = \cos\left(\tan^{-1}\frac{323.2 \text{ kVAR}}{450 \text{ kW}}\right)$$
$$= \cos 35.7^\circ = 0.812 \text{ lagging}$$

Finally, the line current is given by

$$I_L = \frac{P_{tot}}{\sqrt{3}V_L \cos\theta} = \frac{450 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.812)} = 667 \text{ A}$$

(b) The real and reactive powers of loads 1 and 2 are unchanged, as is the real power of load 3. The reactive power of load 3 is

$$Q_3 = P_3 \tan\theta$$
$$= (150 \text{ kW}) \tan(-\cos^{-1} 0.85) = (150 \text{ kW}) \tan(-31.8^\circ)$$
$$= -93 \text{ kVAR}$$

Thus, the total real load is

$$P_{tot} = P_1 + P_2 + P_3$$
$$= 100 \text{ kW} + 200 \text{ kW} + 150 \text{ kW} = 450 \text{ kW}$$

and the total reactive load is

$$Q_{tot} = Q_1 + Q_2 + Q_3$$
$$= 80.2 \text{ kVAR} + 150 \text{ kVAR} - 93 \text{ kVAR} = 137.2 \text{ kVAR}$$

The equivalent system power factor is thus

$$\text{PF} = \cos\theta = \cos\left(\tan^{-1}\frac{Q}{P}\right) = \cos\left(\tan^{-1}\frac{137.2 \text{ kVAR}}{450 \text{ kW}}\right)$$
$$= \cos 16.96^\circ = 0.957 \text{ lagging}$$

Finally, the line current is given by

$$I_L = \frac{P_{tot}}{\sqrt{3}V_L \cos\theta} = \frac{450 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.957)} = 566 \text{ A}$$

(c) The transmission losses in the first case are

$$P_{LL} = 3I_L^2 R_L = 3(667 \text{ A})^2 R_L = 1{,}344{,}700\ R_L$$

The transmission losses in the second case are

$$P_{LL} = 3I_L^2 R_L = 3(566 \text{ A})^2 R_L = 961{,}070\ R_L$$

V-curves

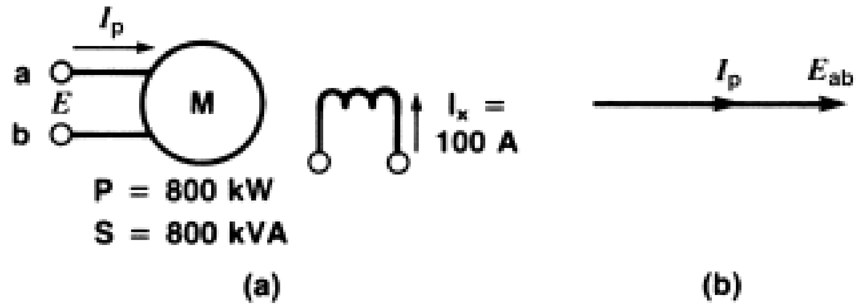


Figure 16

a. Synchronous motor operating at unity power factor with a mechanical load of 800 kW. Field excitation is 100 A.

b. Phasor diagram shows current in phase with the voltage.

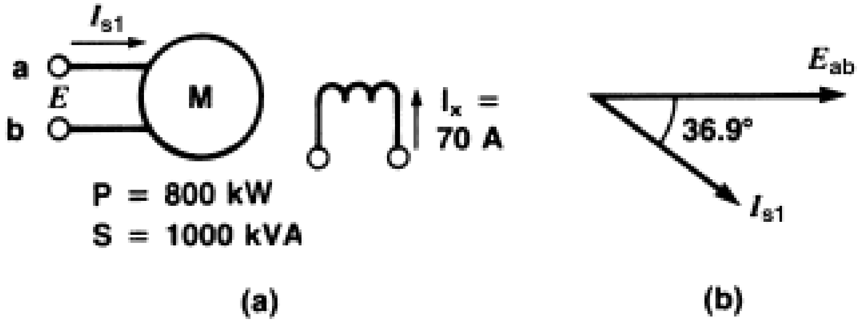


Figure 17

a. Field excitation reduced to 70 A but with same mechanical load. Motor absorbs reactive power from the line.

b. Phasor diagram shows current lagging behind the voltage.

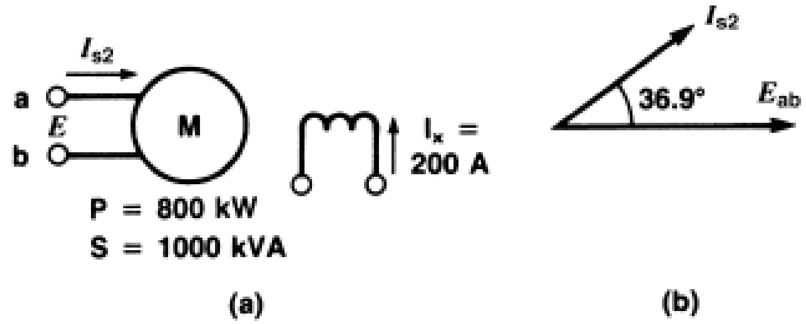


Figure 18

a. Field excitation raised to 200 A but with same mechanical load. Motor delivers reactive power to the line.

b. Phasor diagram shows current leading the voltage.

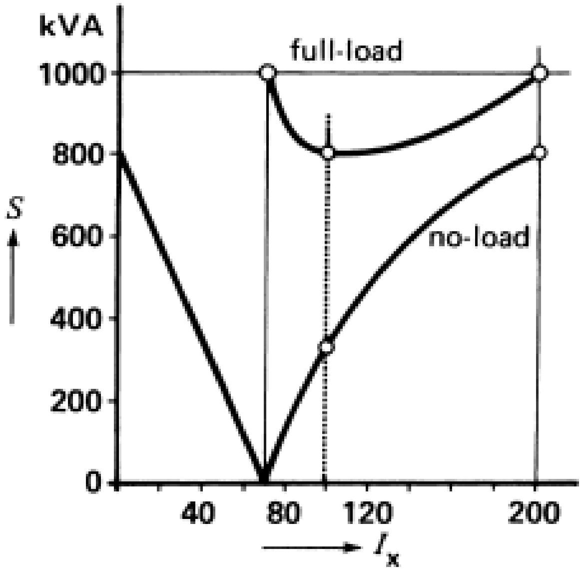


Figure 19

No-load and full-load V-curves of a 1000 hp synchronous motor.

Example 5

A 4000 hp (3000 kW), 6600 V, 60 Hz, 200 r/min synchronous motor operates at full-load at a leading power factor of 0.8. If the synchronous reactance is 11 Ω, calculate the following:

a. The apparent power of the motor, per phase
b. The ac line current
c. The value and phase of E_o
d. Draw the phasor diagram
e. Determine the torque angle δ

Note: the losses of the motor have been neglected

a. The active power per phase is

$$P = 3000/3 = 1000 \text{ kW}$$

The apparent power per phase is

$$S = P/\cos\theta = 1000/0.8 = 1250 \text{ kVA}$$

b. The line-to-neutral voltage is

$$E = E_L/\sqrt{3} = 6600/\sqrt{3} = 3811 \text{ V}$$

The line current is

$$I = S/E = 1250 \times 1000/3811 = 328 \text{ A}$$

I leads E by an angle of arcos 0.8 = 36.9°.

$$E = 3815\angle 0°$$

It follows that I is given by

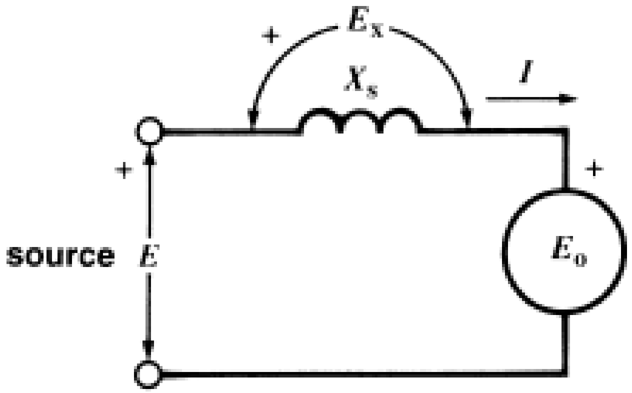
$$I = 328\angle 36.9°$$

Writing the equation for the circuit we find

$$-E + jIX_s + E_o = 0$$

thus

$$\begin{aligned} E_o &= E - jIX_s \\ &= 3811\angle 0° - j(328\angle 36.9°)11 \\ &= 3811\angle 0° - 3608\angle(36.9° + 90°) \\ &= 3811(\cos 0° + j\sin 0°) - \\ &\quad 3608(\cos 126.9° + j\sin 126.9°) \\ &= 3811 + 2166 - j\,2885 \\ &= 5977 - j\,2885 \\ &= 6637\angle -26° \end{aligned}$$



d. Consequently, E_o lags 26° behind E, and the complete phasor diagram is shown in Fig. 21.

e. The torque angle δ is 26°.

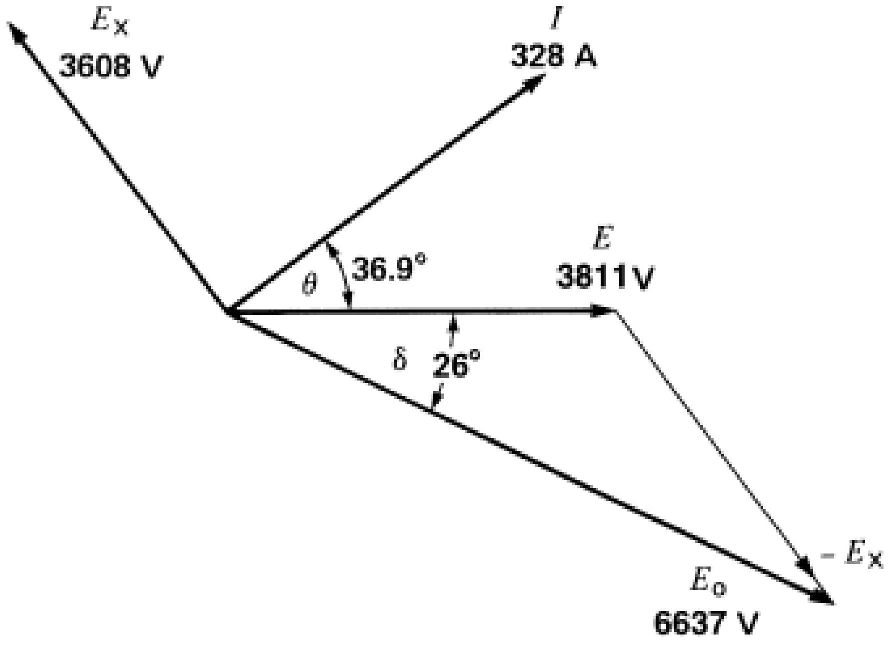


Figure 21
See Example 5.

Synchronous capacitor

The machine acts as an enormous 3-phase capacitor (or inductor) whose reactive power can be varied by changing the dc excitation.

Example 7

A synchronous capacitor is rated at 160 Mvar, 16 kV, 1200 r/min, 60 Hz. It has a synchronous reactance of 0.8 pu and is connected to a 16 kV line. Calculate the value of E_o so that the machine

a. Absorbs 160 Mvar

b. Delivers 120 Mvar

Solution

a. The nominal impedance of the machine is

$$\begin{aligned} Z_n &= E_n^2/S_n \\ &= 16\,000^2/(160 \times 10^6) \\ &= 1.6\ \Omega \end{aligned}$$

The synchronous reactance per phase is

$$\begin{aligned} X_s &= X_S(\text{pu})\, Z_n = 0.8 \times 1.6 \\ &= 1.28\ \Omega \end{aligned}$$

The line current for a reactive load of 160 Mvar is

$$\begin{aligned} I_n &= S_n/(\sqrt{3}\,E_n) \\ &= (160 \times 10^6)/(1.73 \times 16\,000) \\ &= 5780 \text{ A} \end{aligned}$$

The drop across the synchronous reactance is

$$\begin{aligned} E_x &= IX_s = 5780 \times 1.28 \\ &= 7400 \text{ V} \end{aligned}$$

The line-to-neutral voltage is

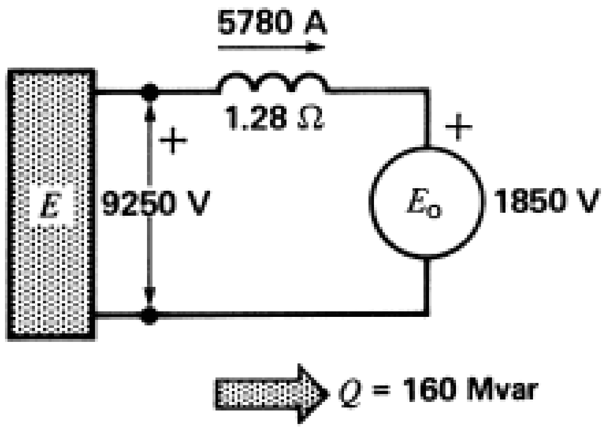
$$\begin{aligned} E &= E_L/\sqrt{3} = 16\,000/1.73 \\ &= 9250 \text{ V} \end{aligned}$$

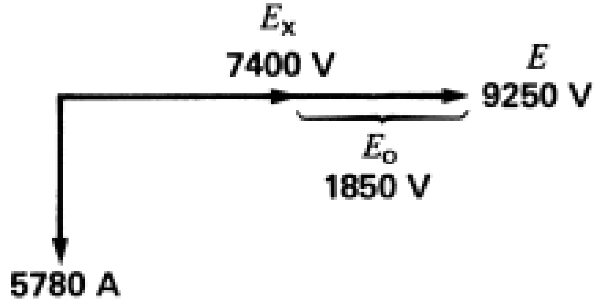
Selecting E as the reference phasor, we have

$$E = 9250\angle 0°$$

The current I lags 90° behind E because the machine is absorbing reactive power; consequently,

$$I = 5780\angle -90°$$





$$-E + jIX_s + E_o = 0$$

hence

$$\begin{aligned} E_o &= E - jIX_s \\ &= 9250\angle 0° - 5780 \times 1.28\angle(90° - 90°) \\ &= 1850\angle 0° \end{aligned}$$

Note that the excitation voltage (1850 V) is much less than the line voltage (9250 V).

b. The load current when the machine is delivering 120 Mvar is

$$I_n = Q/(\sqrt{3}\,E_n)$$
$$= (120 \times 10^6)/(1.73 \times 16\,000)$$
$$= 4335 \text{ A}$$

This time I leads E by 90° and so

$$I = 4335\angle 90°$$

From Fig. 25b we can write

$$E_o = E - jIX_s$$
$$= 9250\angle 0° - 4335 \times 1.28\angle 180°$$
$$= (9250 + 5550)\angle 0°$$
$$= 14\,800\angle 0°$$